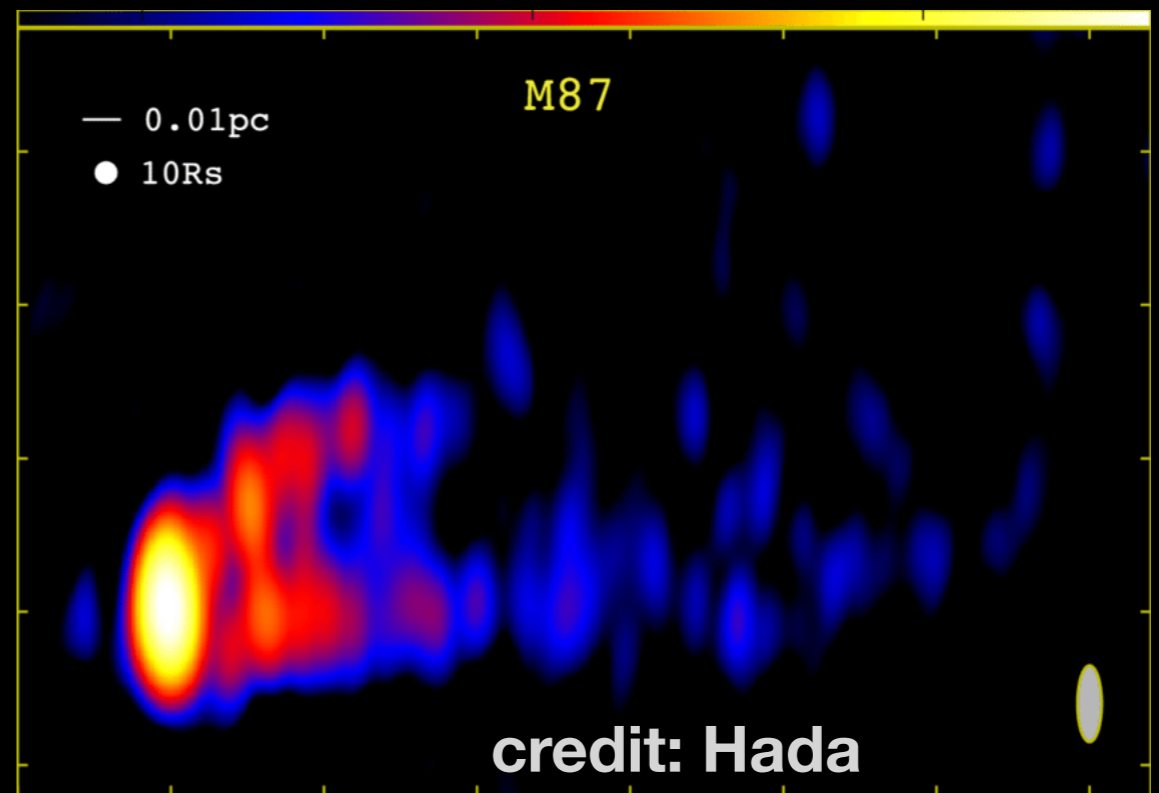


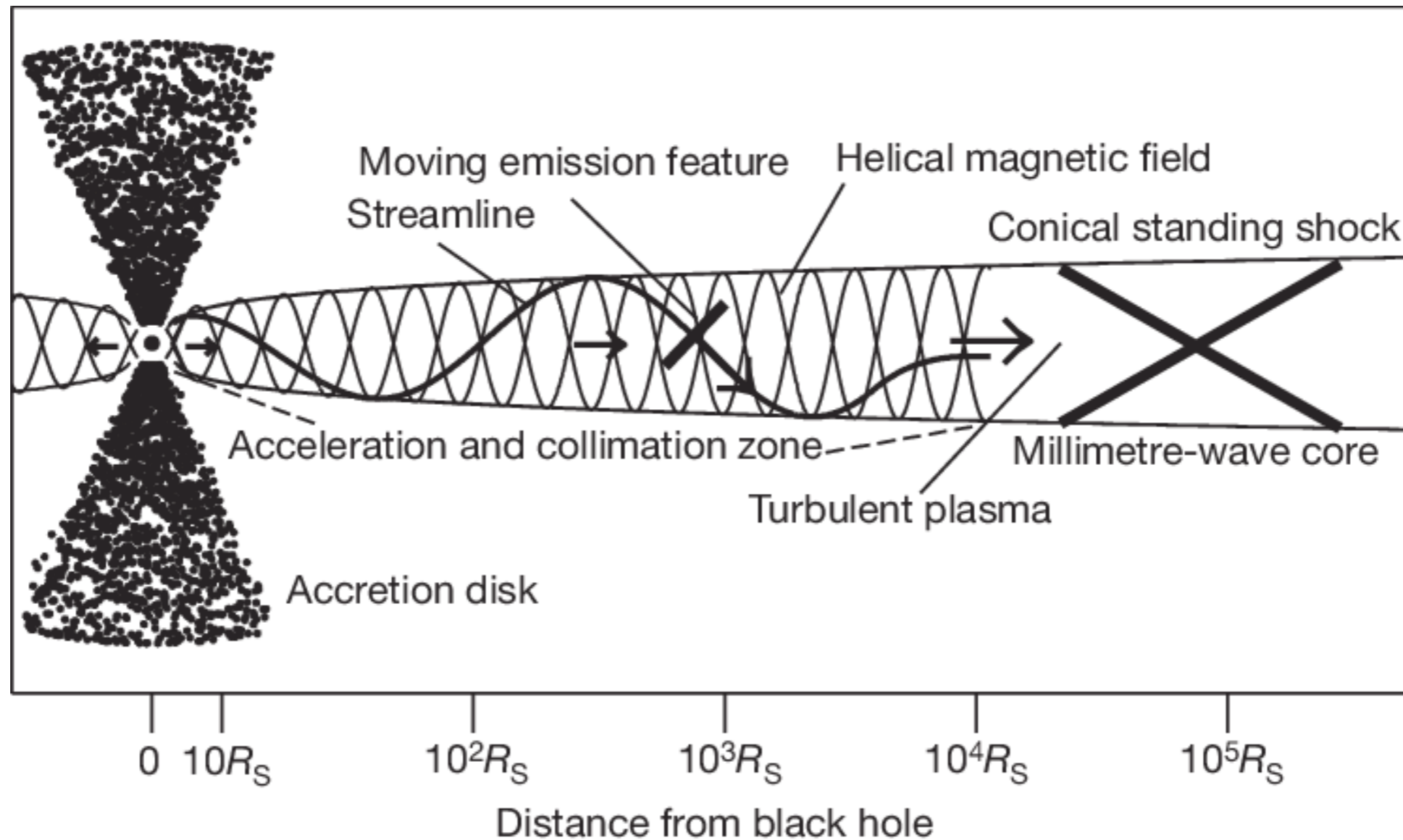
Linking VLBI observation with GRMHD theory at all scales: Phenomenological models of black hole jets/accretion



Hung-Yi Pu

BH astrophysics with VLBI workshop Jan 18th, 2021

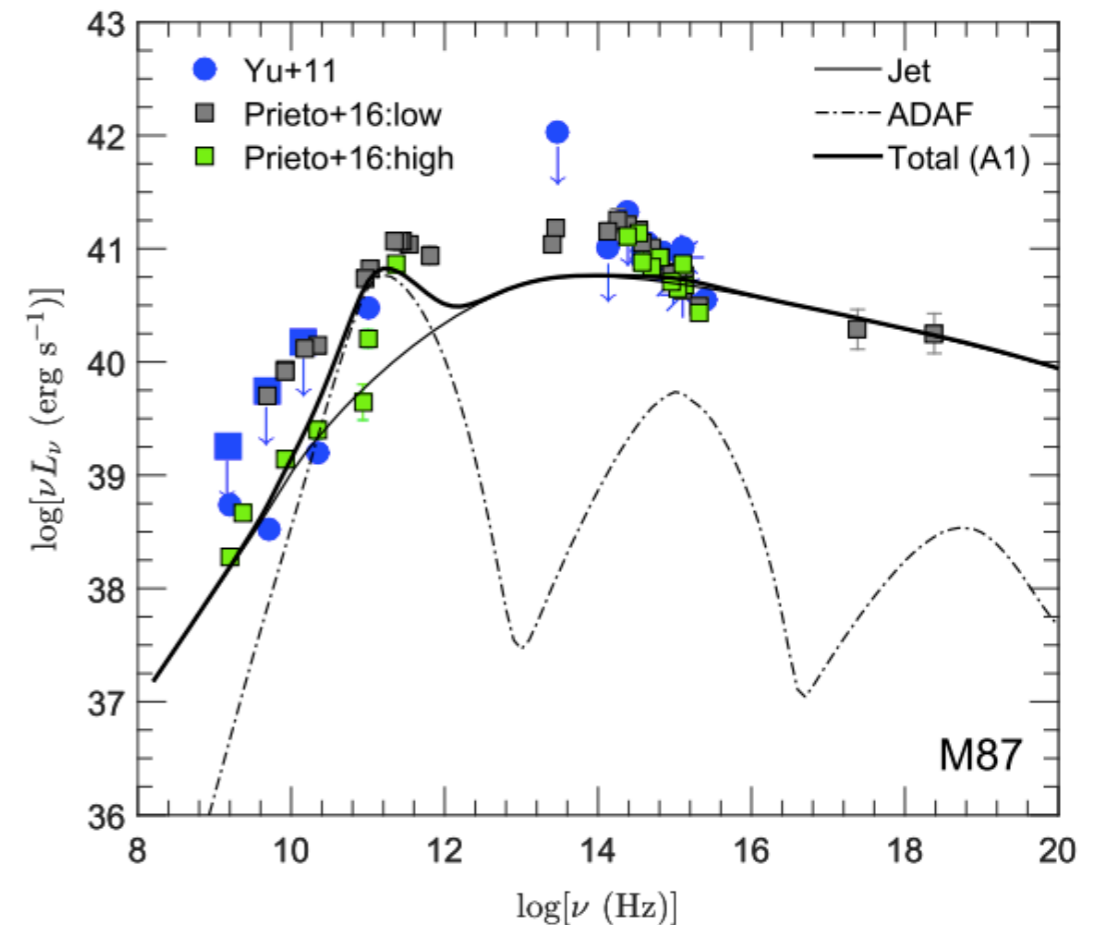
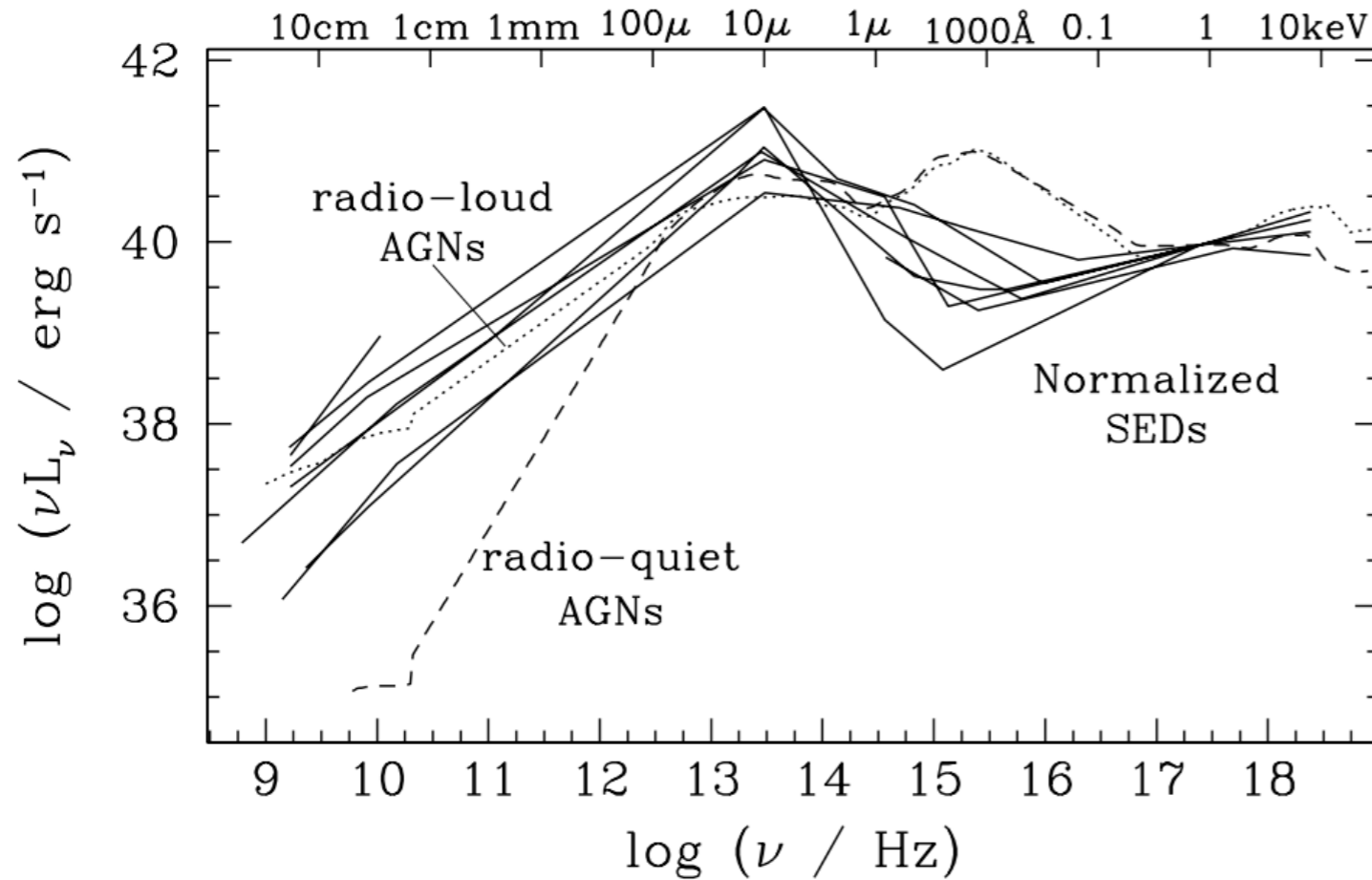
VLBI: story across many order of magnitude



LLAGN

Ho 1999

Li, Yuan, & Xie 2016



RIAF (Radiatively Inefficient Accretion Flow):

low accretion rate \rightarrow *insufficient interaction bt. ion & e-*

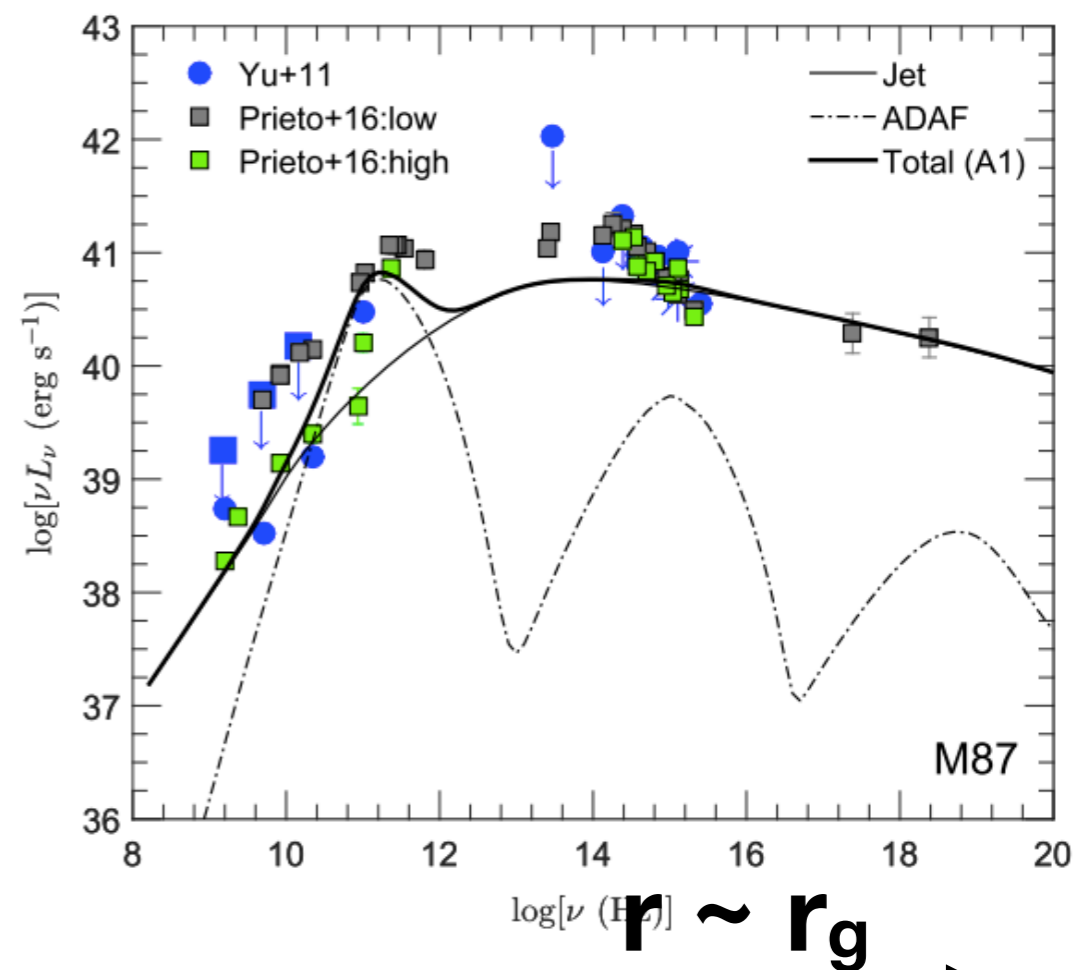
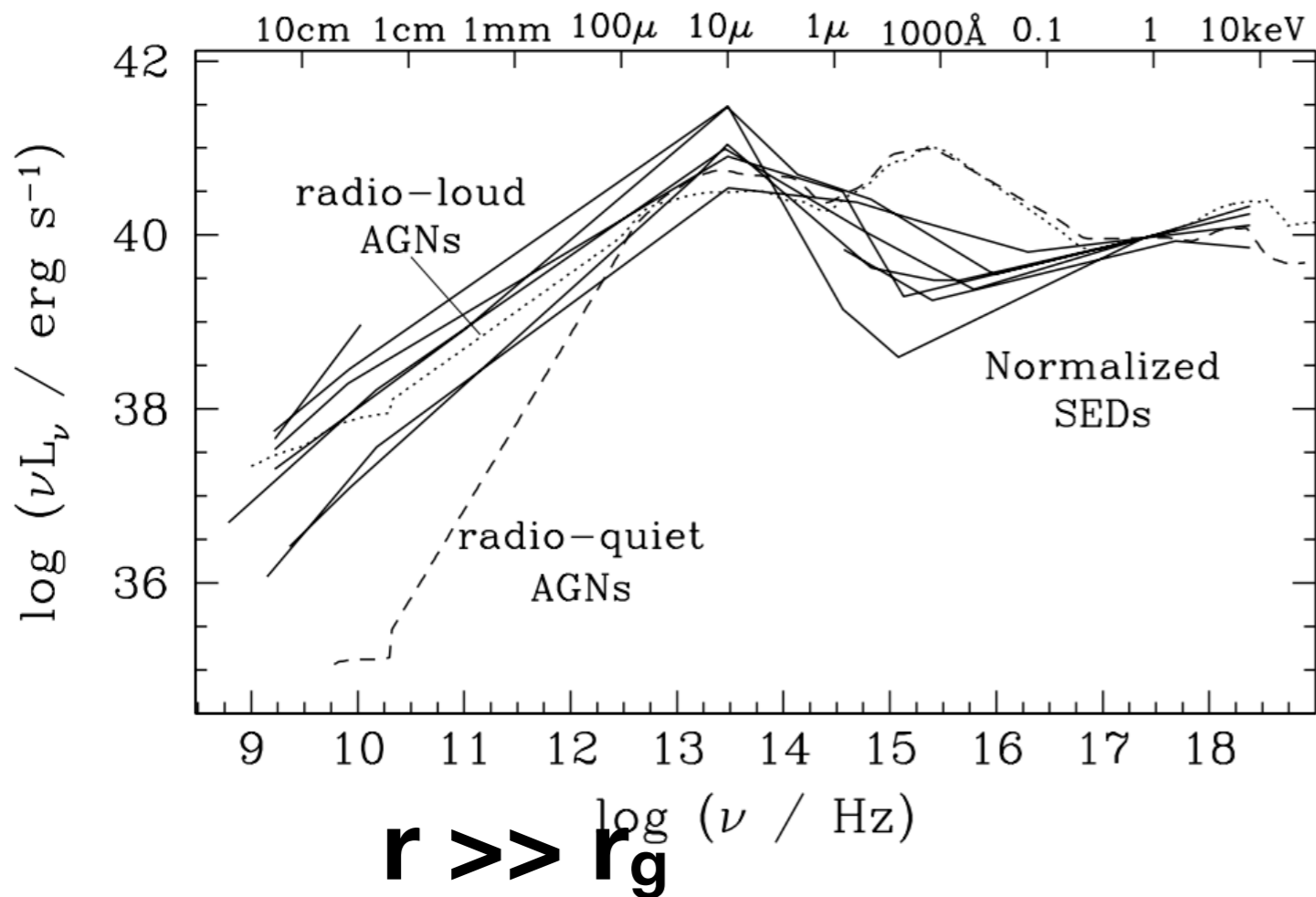
\rightarrow (1) *geometrically thick accretion flow*

\rightarrow (2) *two temperature accretion flow*

LLAGN

Ho 1999

Li, Yuan, & Xie 2016



frequency \uparrow

photon sphere \downarrow

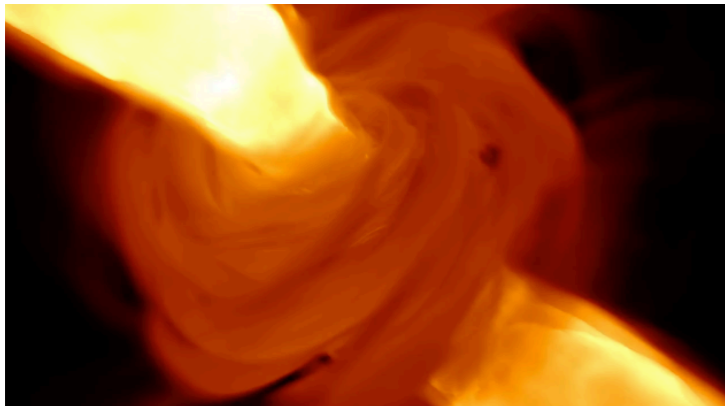
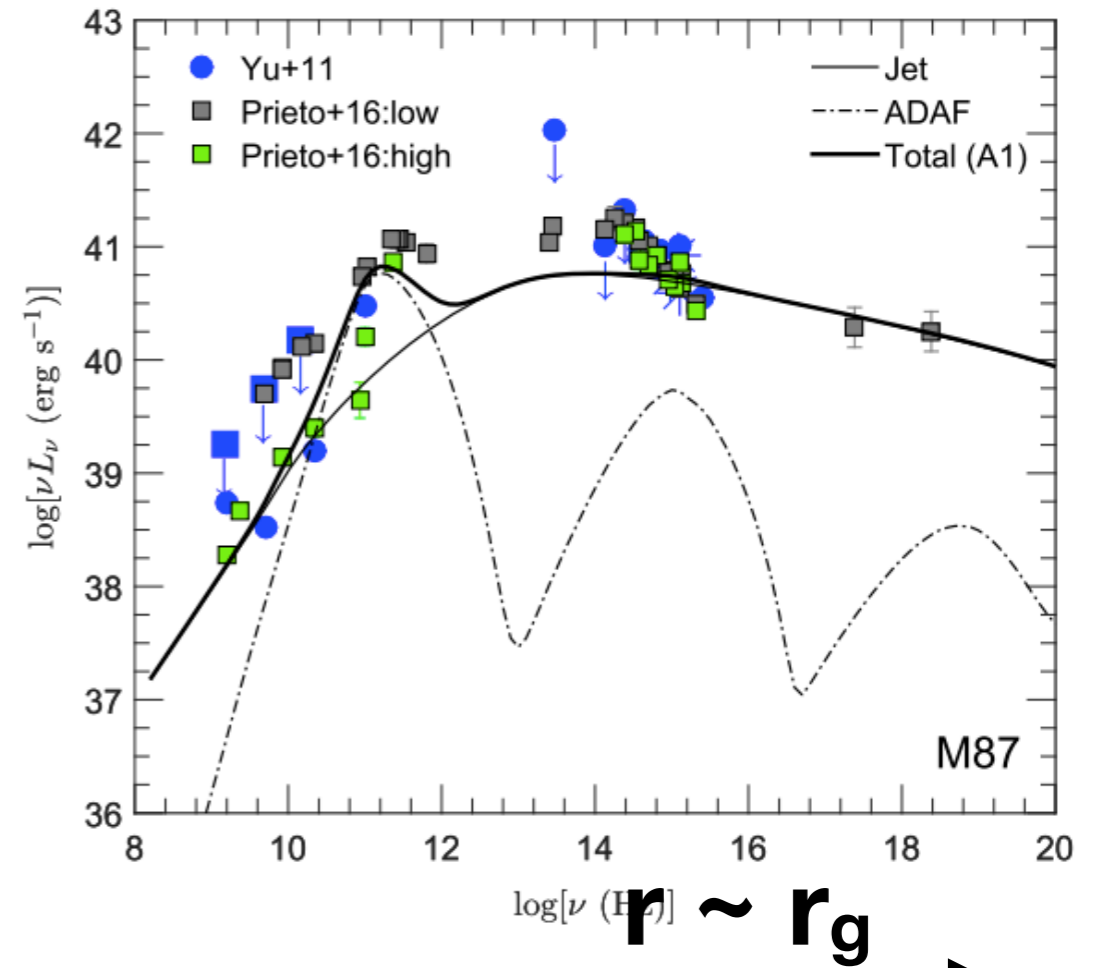
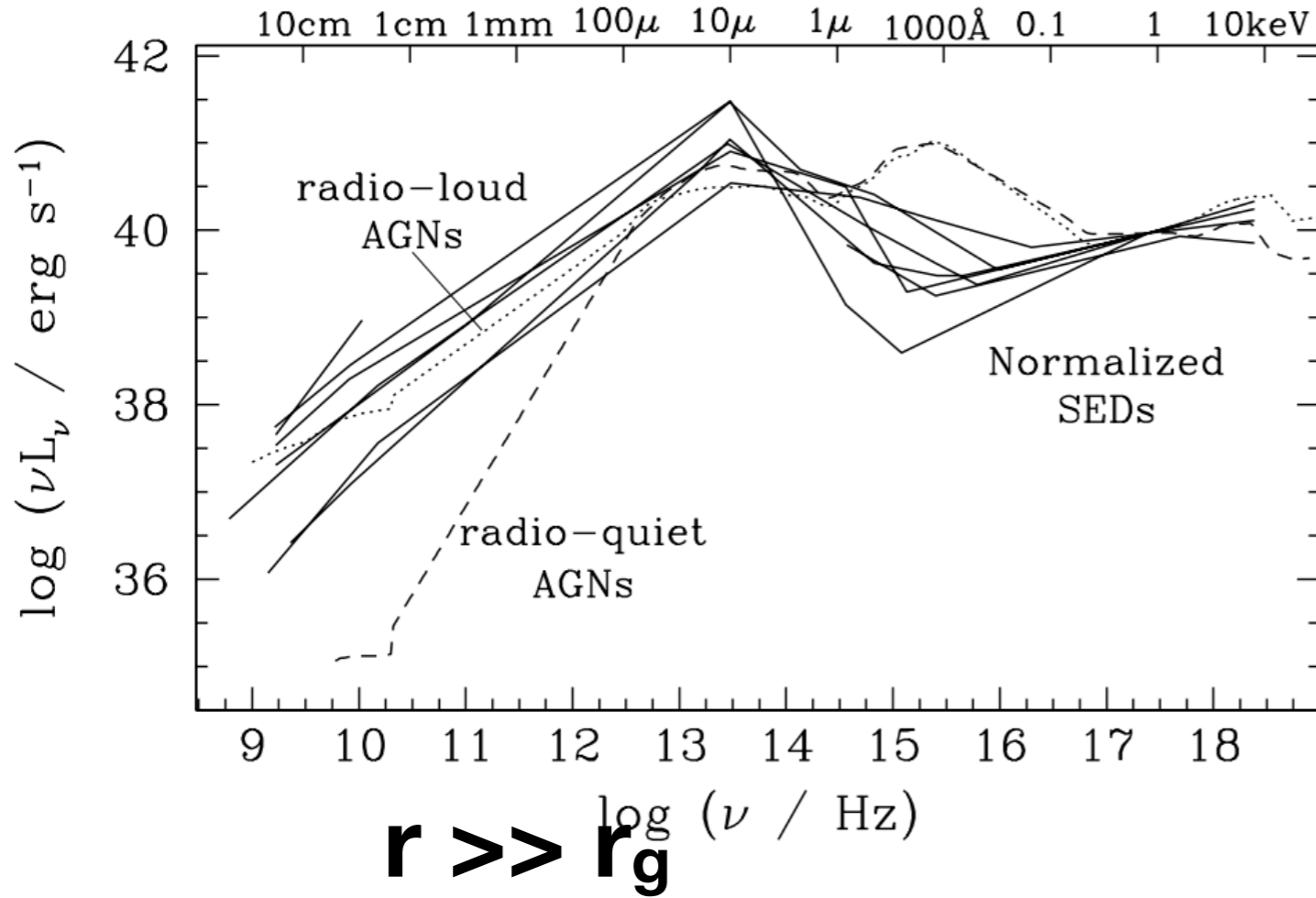
GR effect \uparrow

syn radiation from e- with thermal & non-thermal energy distribution!

LLAGN

Ho 1999

Li, Yuan, & Xie 2016



thanks to low accretion rate:
optically thin

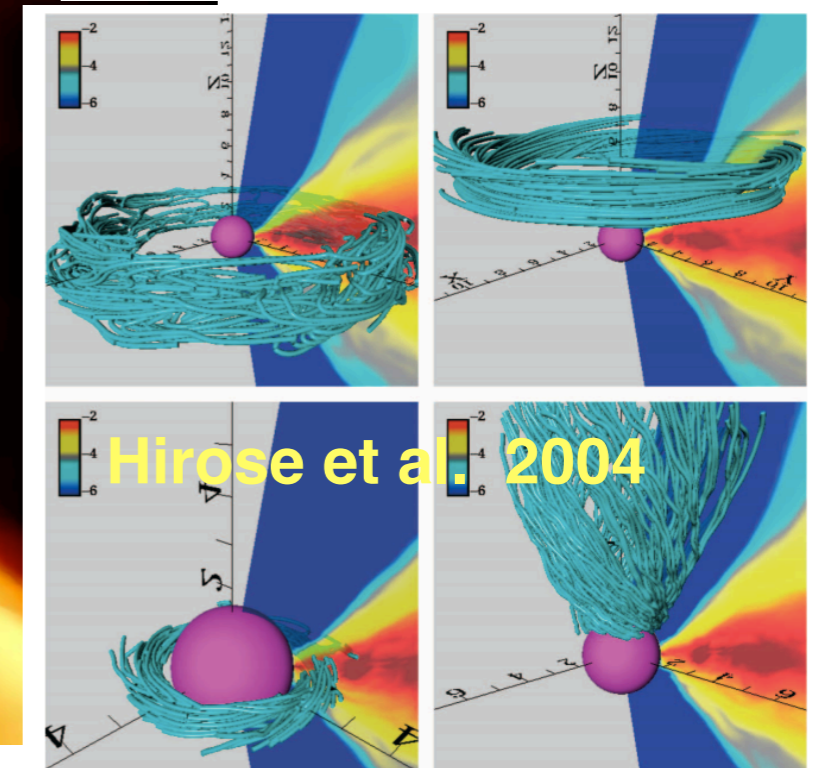
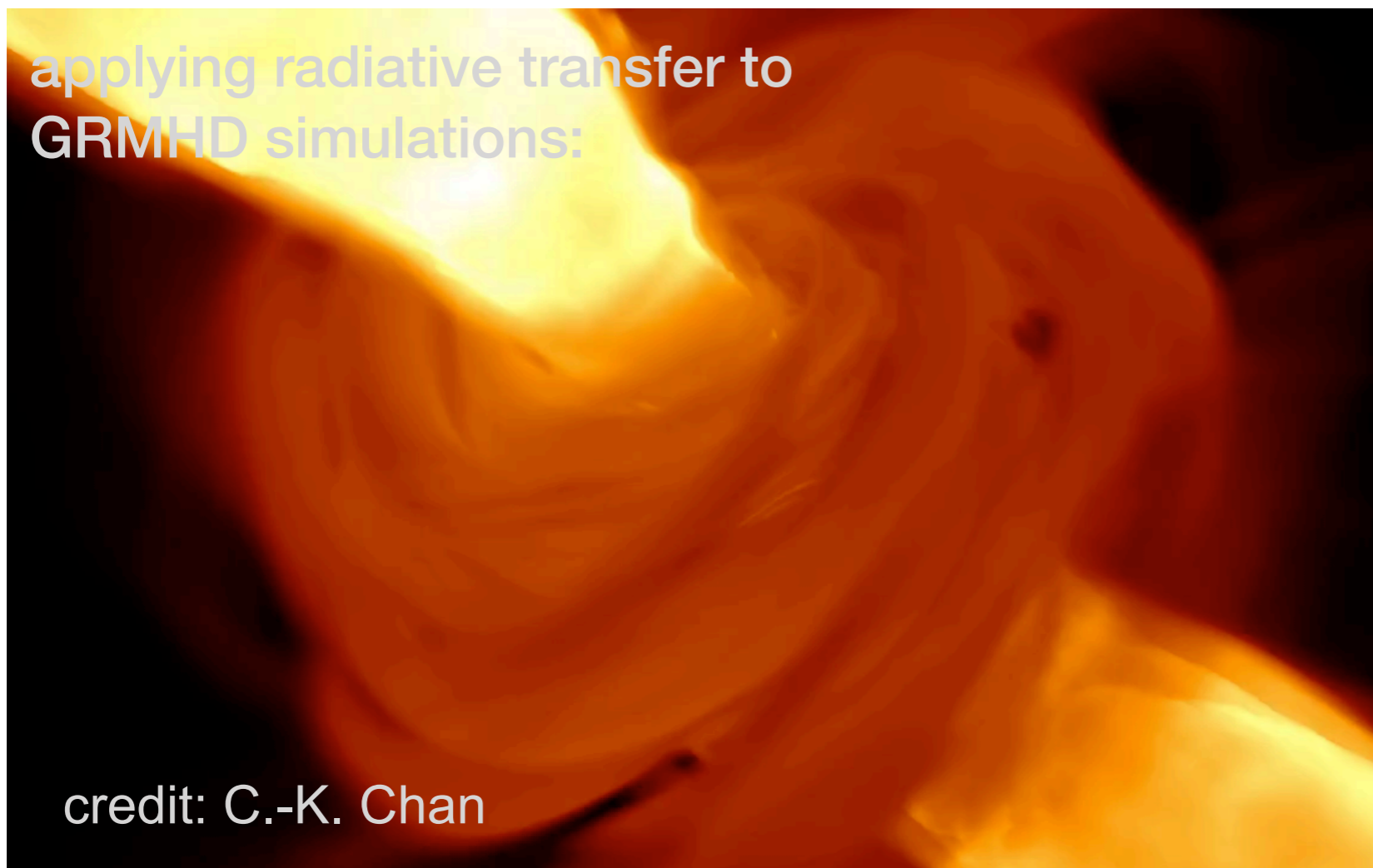
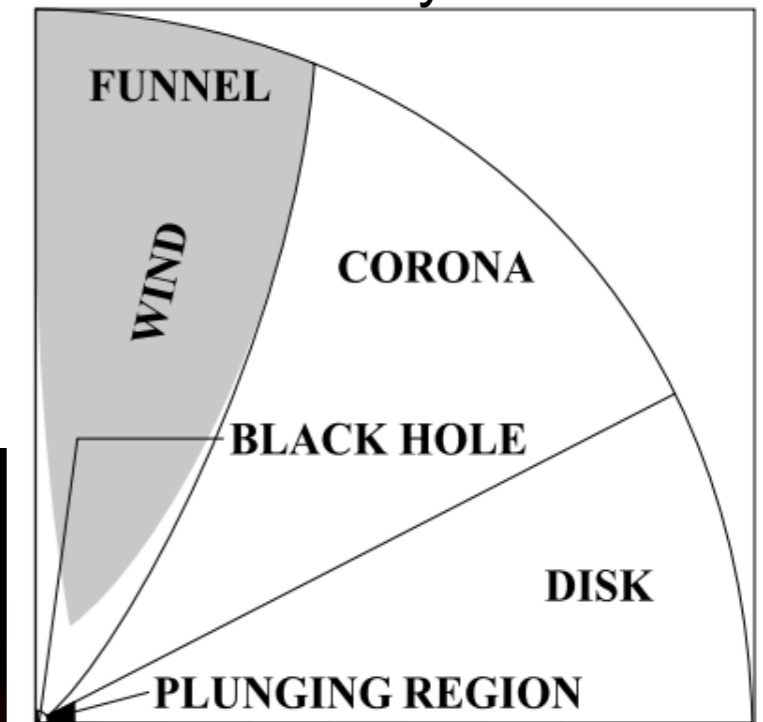


general relativistic magnetohydrodynamics

the need of **GRMHD** simulations

- to simulate 3D structure of jet
- to simulate dynamical feature

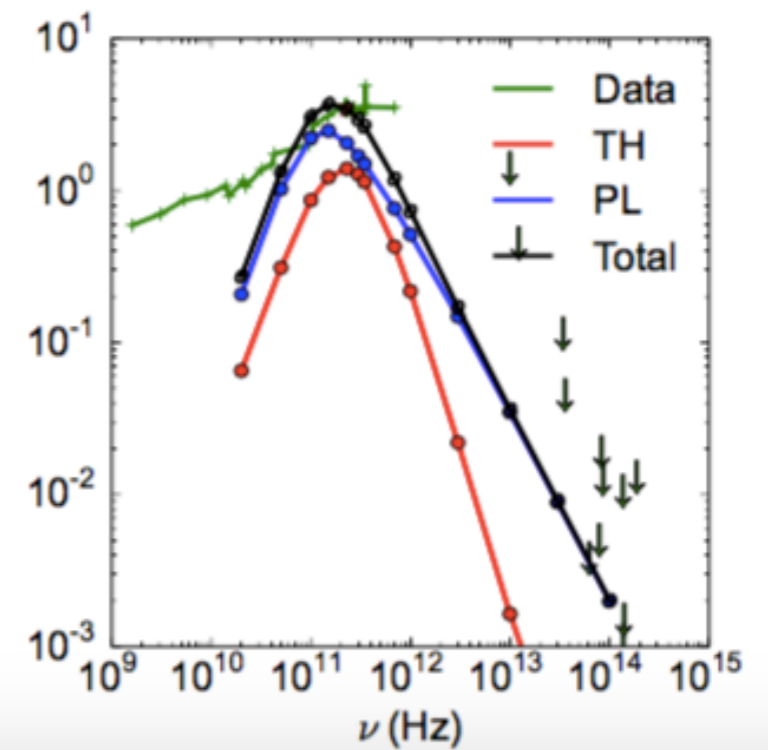
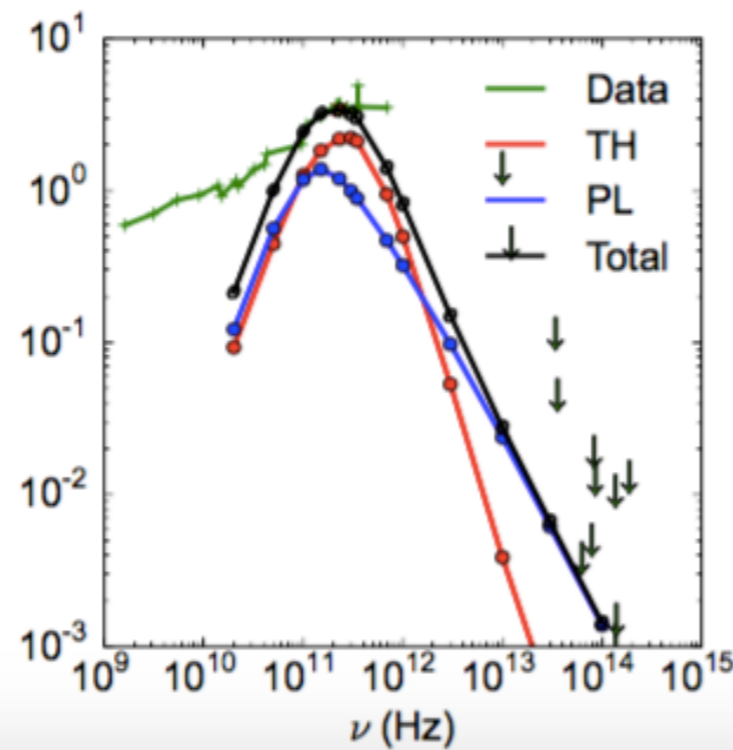
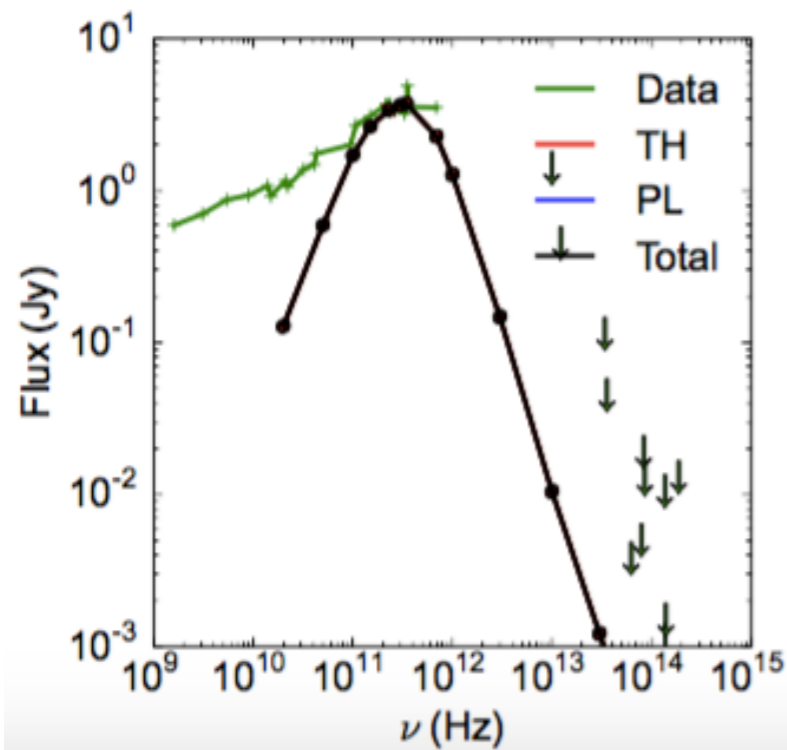
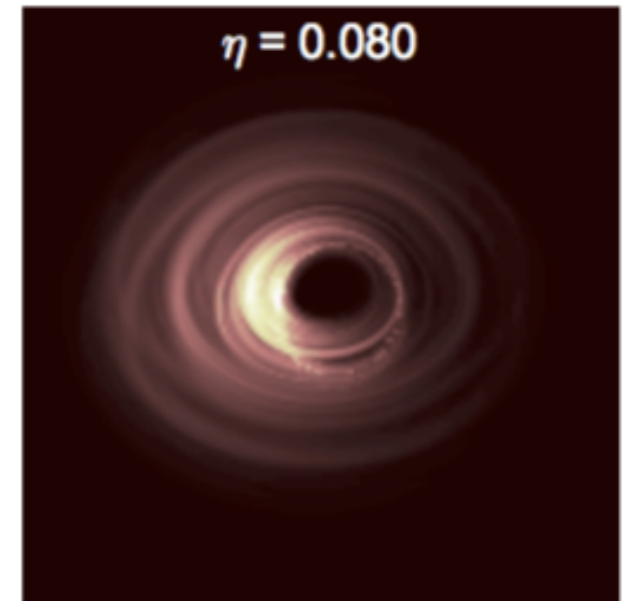
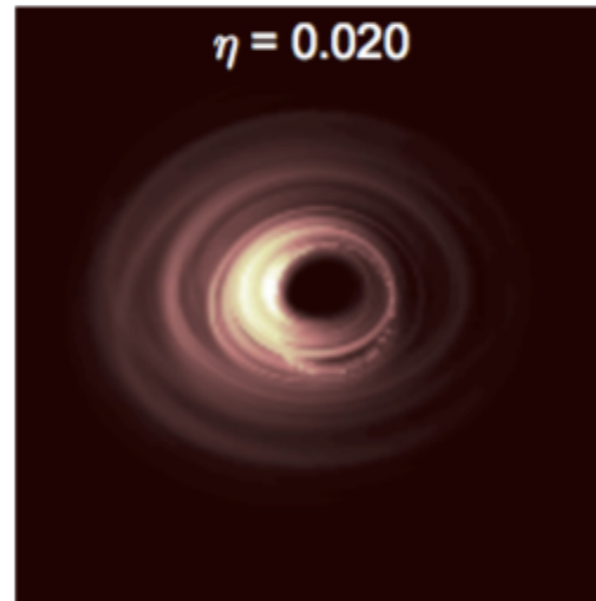
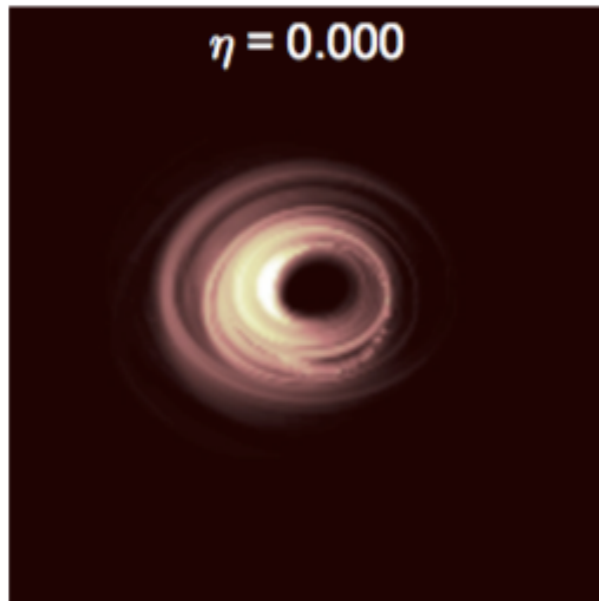
Mckineey & Gammie 04



the need of GRMHD simulations

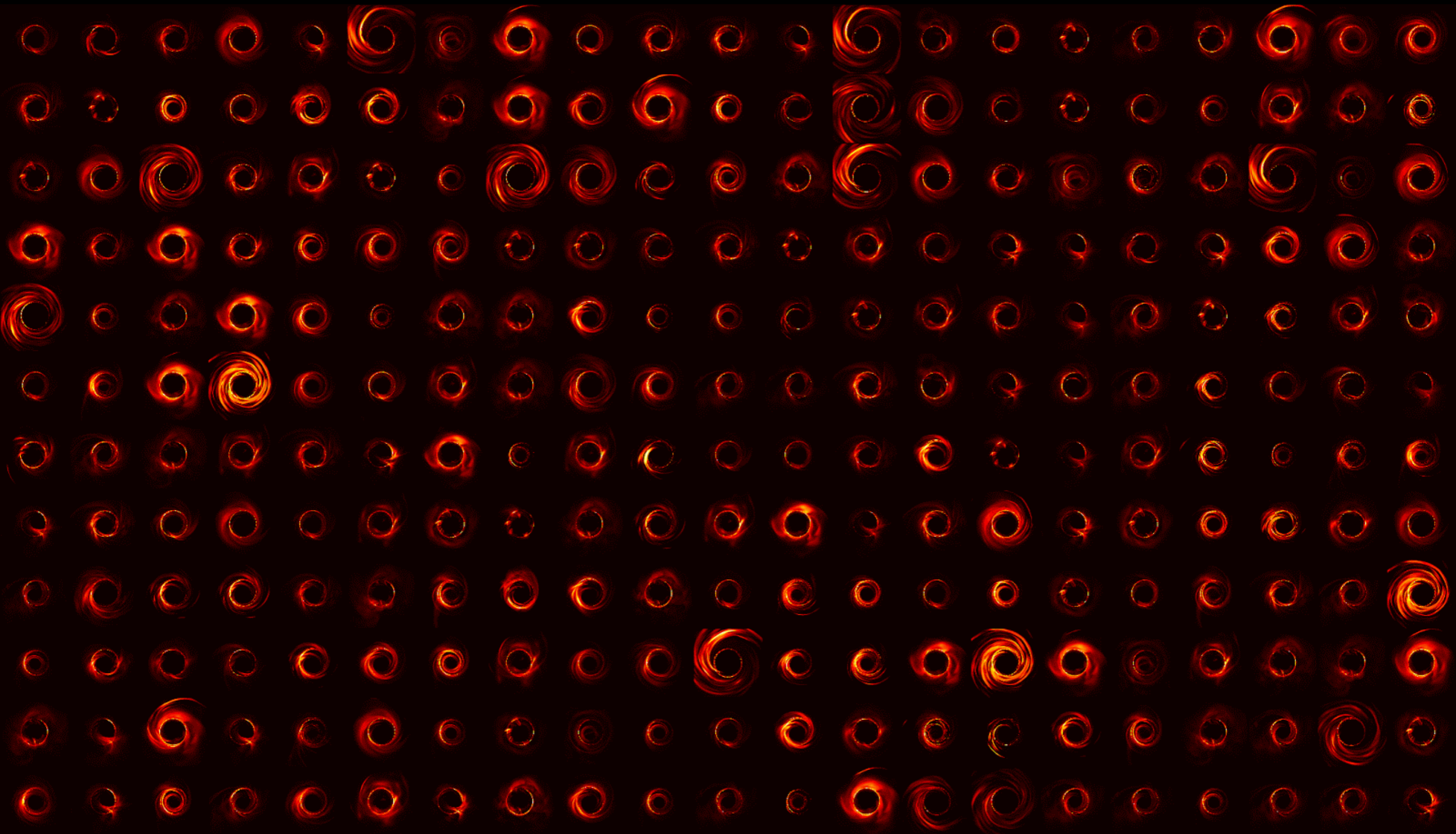
- to simulate 3D structure of jet
- to simulate dynamical feature
- for RIAF, radiation contribution to the **dynamics** can be ignored
- one-fluid simulation has been actively developed since ~2000
- uncertainty for simulating radiation feature of two temperature flow + non-thermal syn. jet

Mao et al. 2017

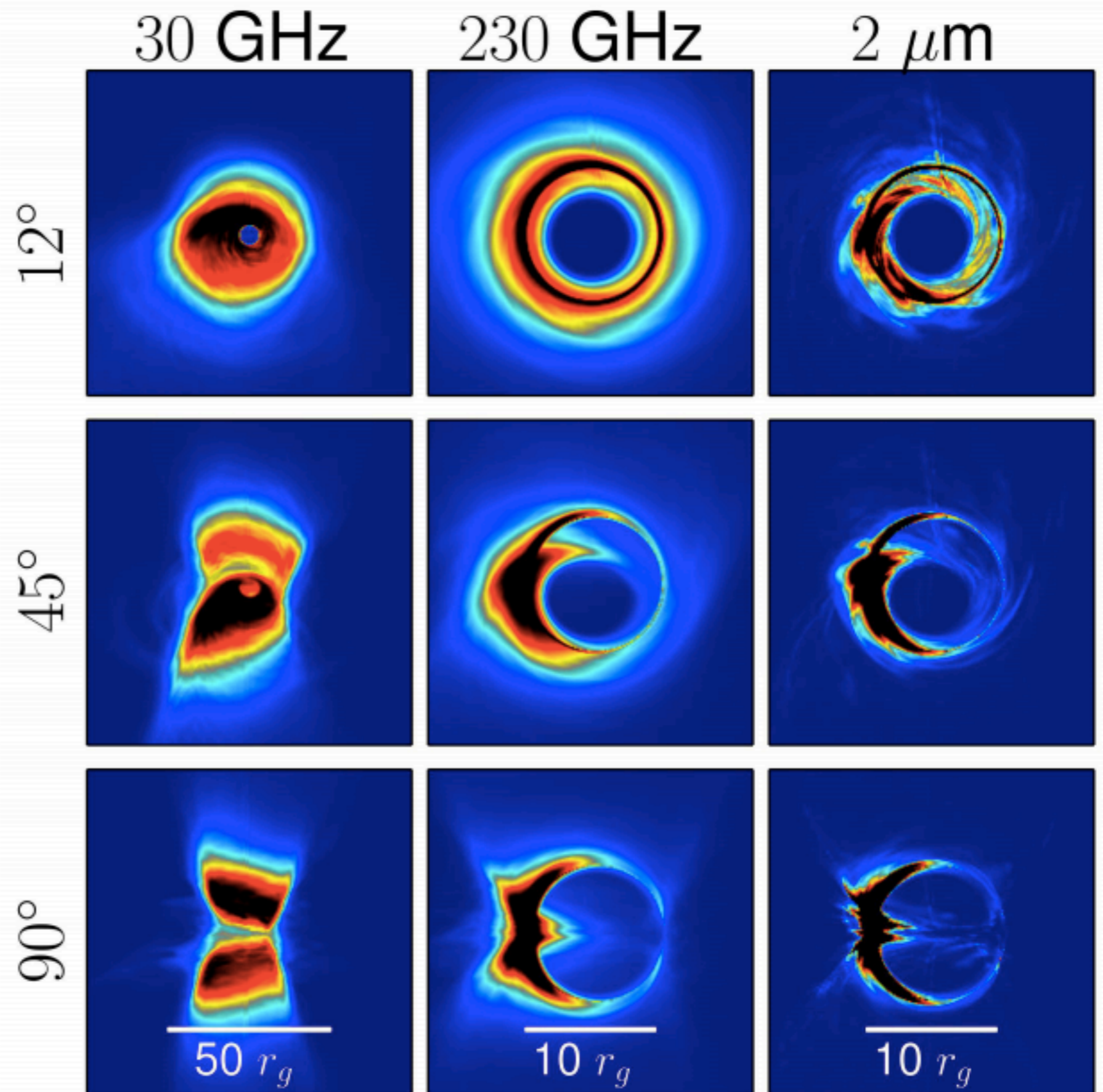


EHT image library

(Ti/Te parametrization according to plasma beta)



Ressler+ 2016
post-processing
“two-fluid” GRMHD
simulation of Sgr A*;
thermal synchrotron
images



how can phenomenological models help?

- usually **semi-analytical**: prompt, flexible, and heuristic (useful for demonstration and parameter search)
- a complementary way to study global properties of the jet (funnel) region
- wide applications: from **horizon to large scale**

why phenomenological models?

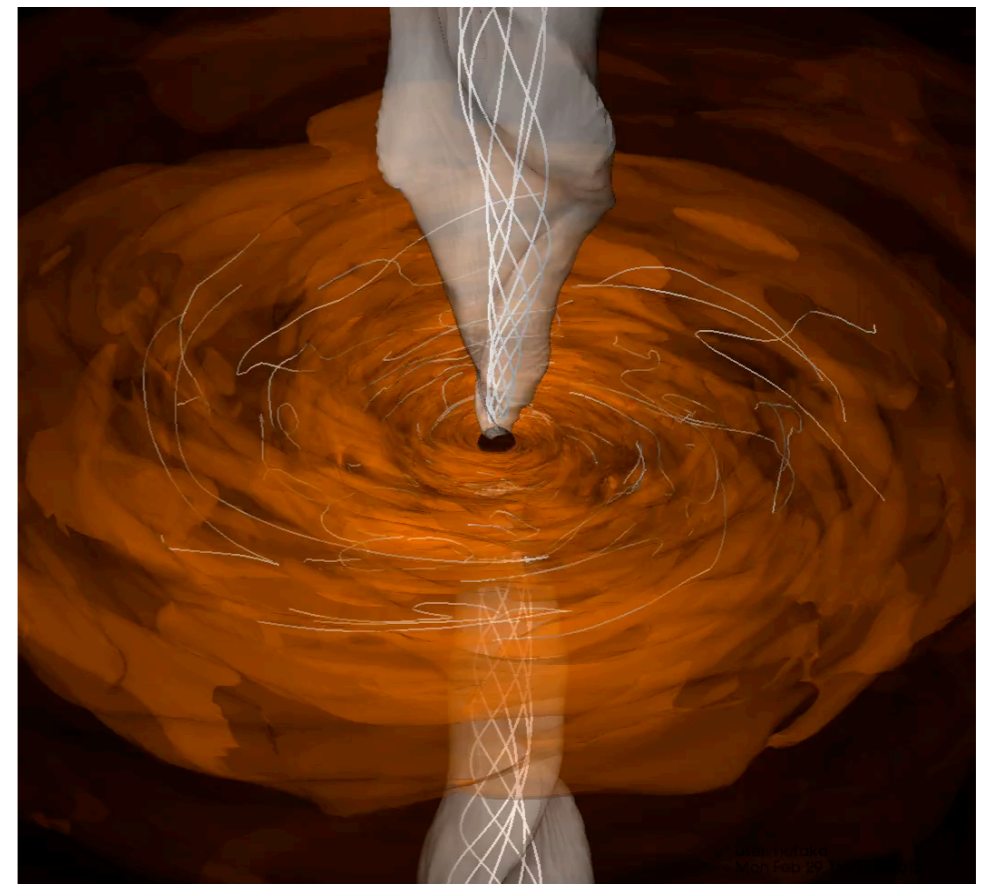
- Here we are talking about **GRMHD** models includes all GR features, model become **SRMHD** automatically far away from the source and gravity effect is negligible

why phenomenological models?

- Here we are talking about **GRMHD** models includes all GR features, model become **SRMHD** automatically far away from the source and gravity effect is negligible
- Not necessary works well if **key features** in not included, or the simplicity does not exist.
- What are the **key feature** for BH accretion/jet?

outline

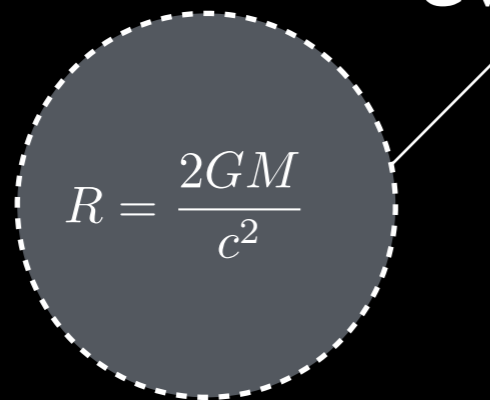
- Black hole structure/magnetosphere with RIAF environment
- accretion
 - numerical simulation
 - semi-analytical model
- jet
 - numerical simulation
 - semi-analytical model



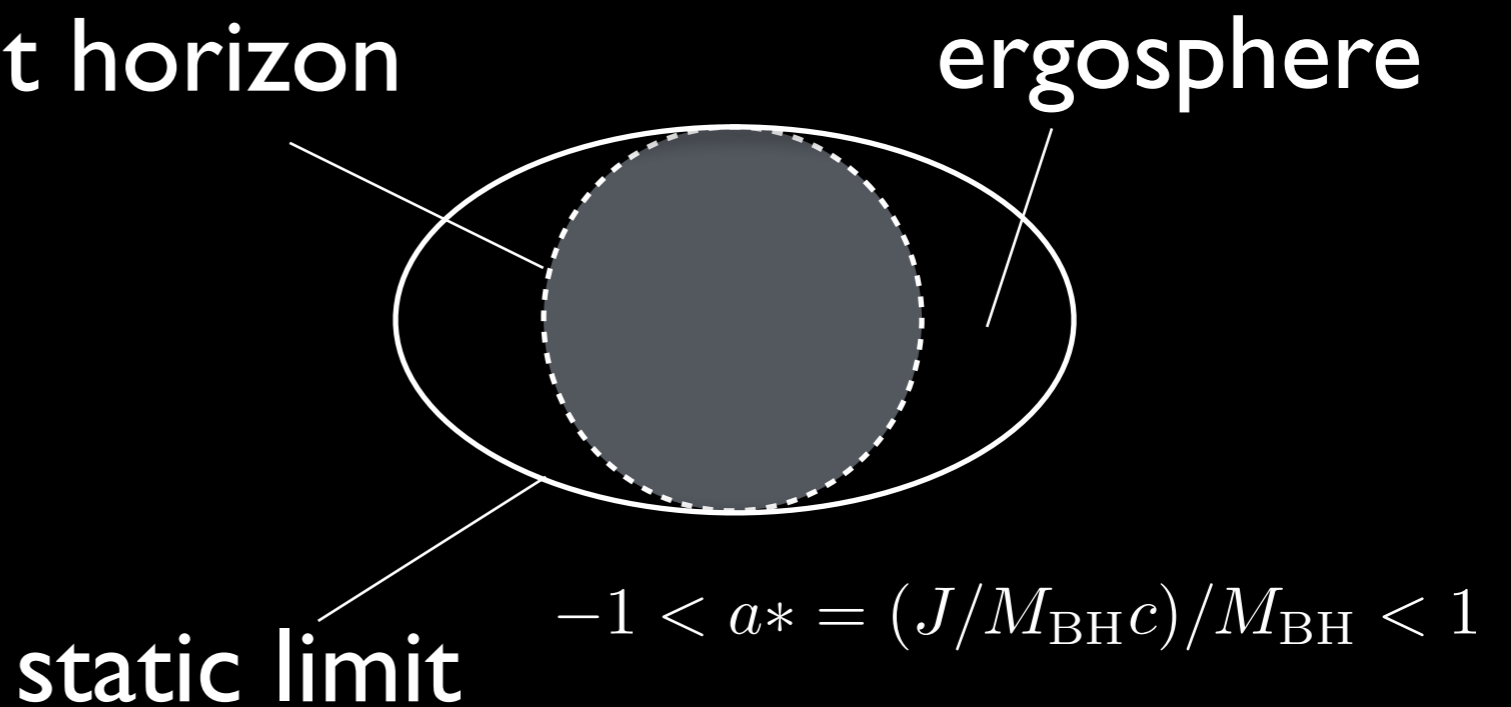
credit: Shiokawa

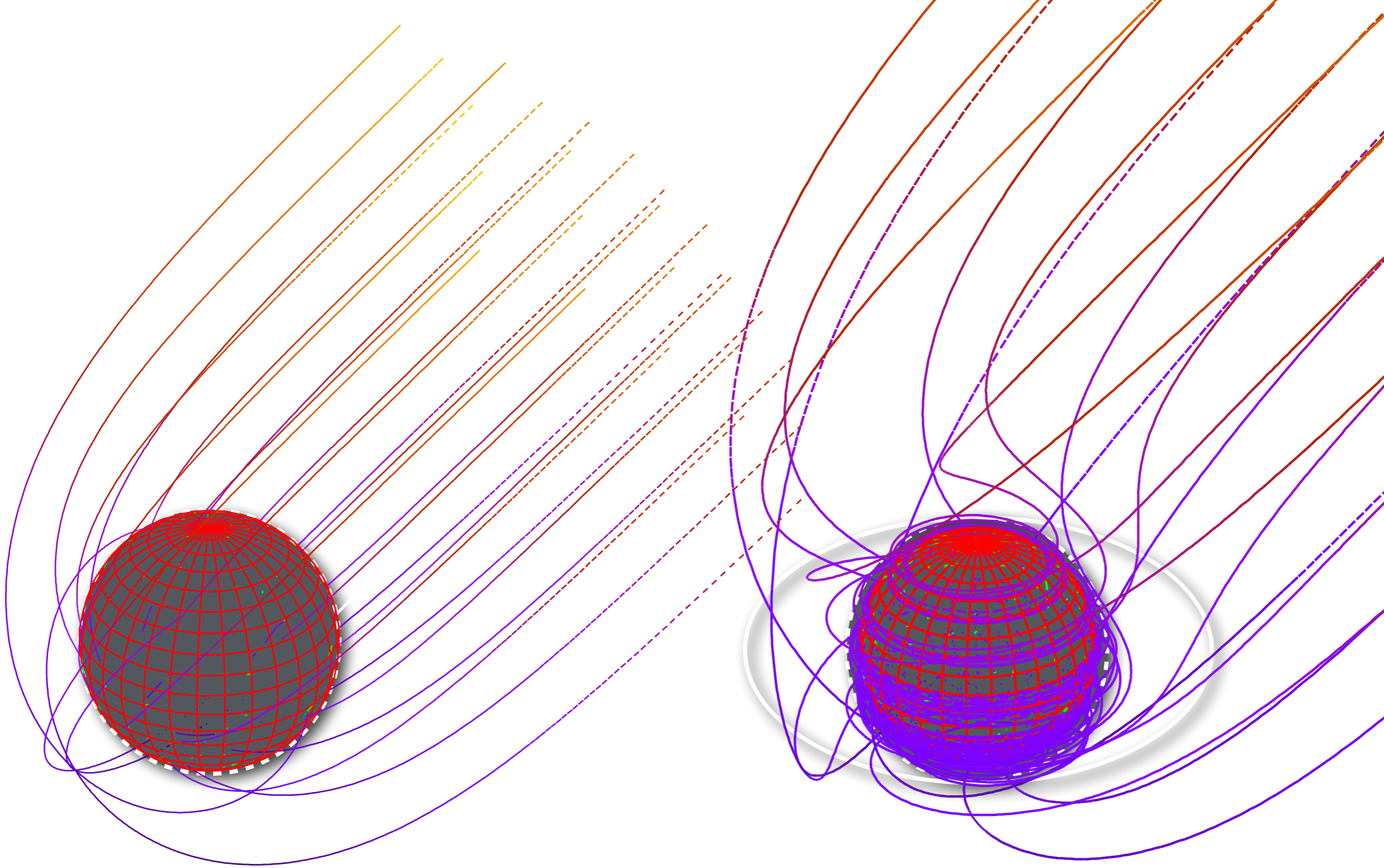
black hole: spacetime structure

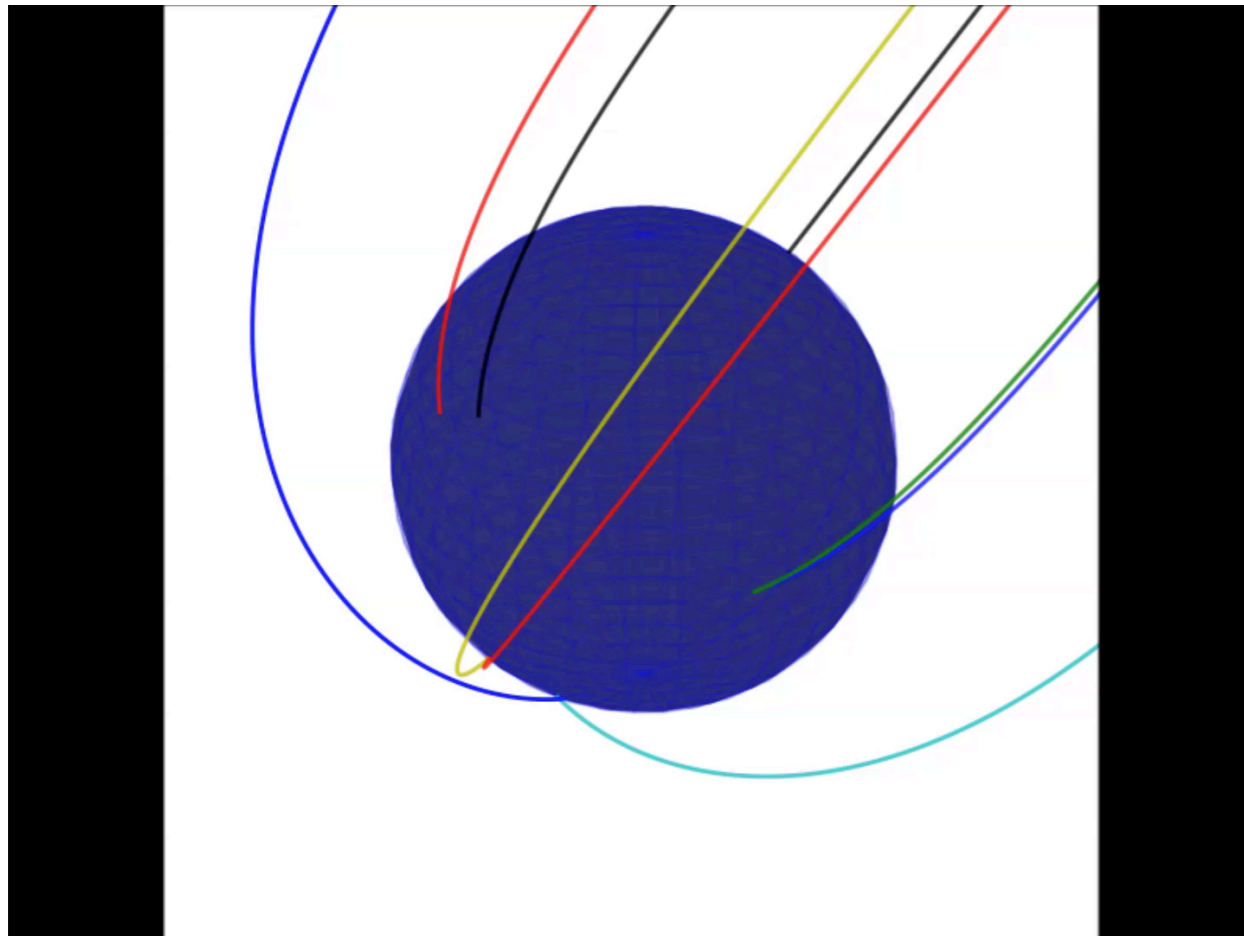
Non-rotating BH:
described by **Schwarzschild geometry (1916)**



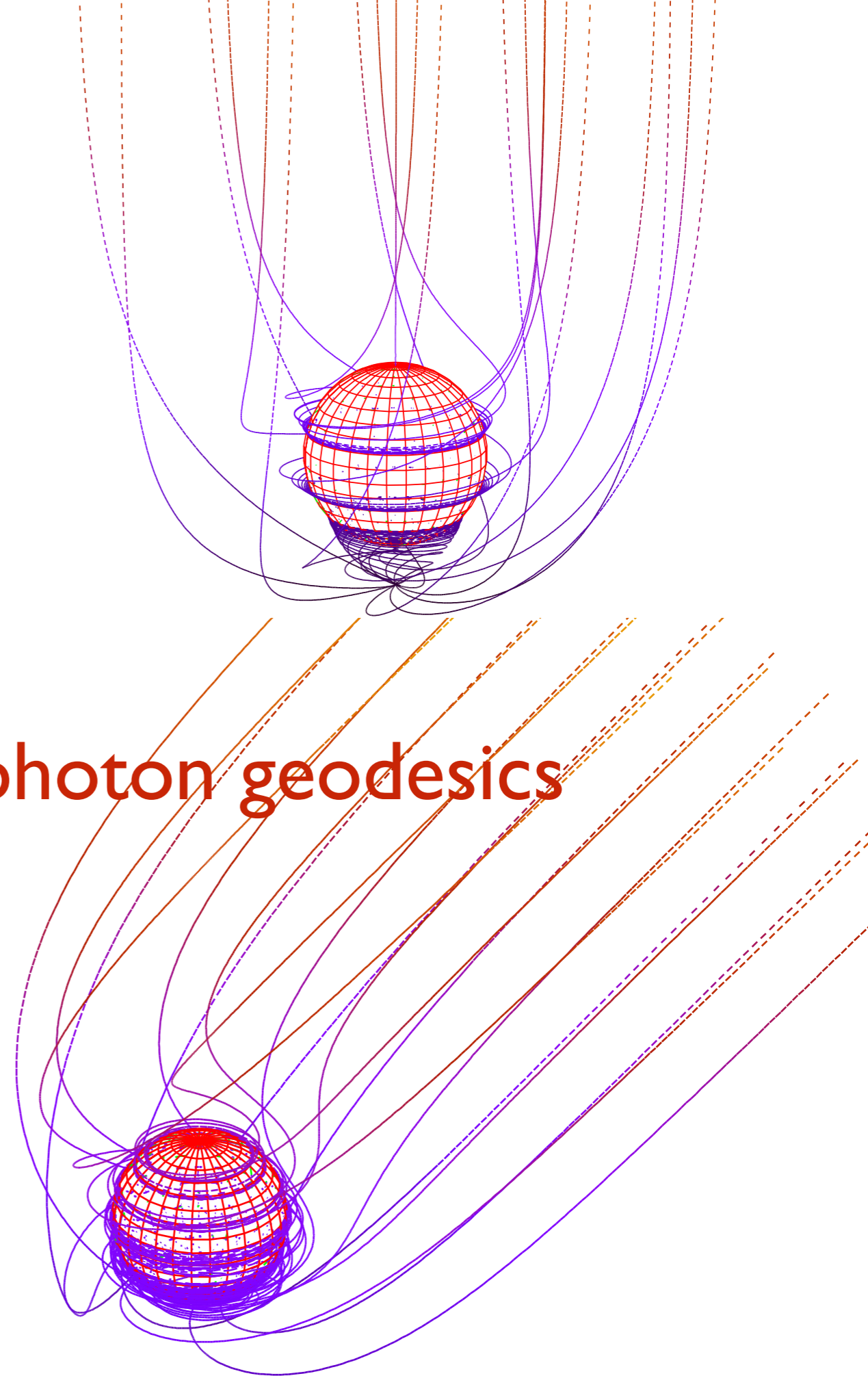
Rotating BH:
described by **Kerr geometry (1963)**





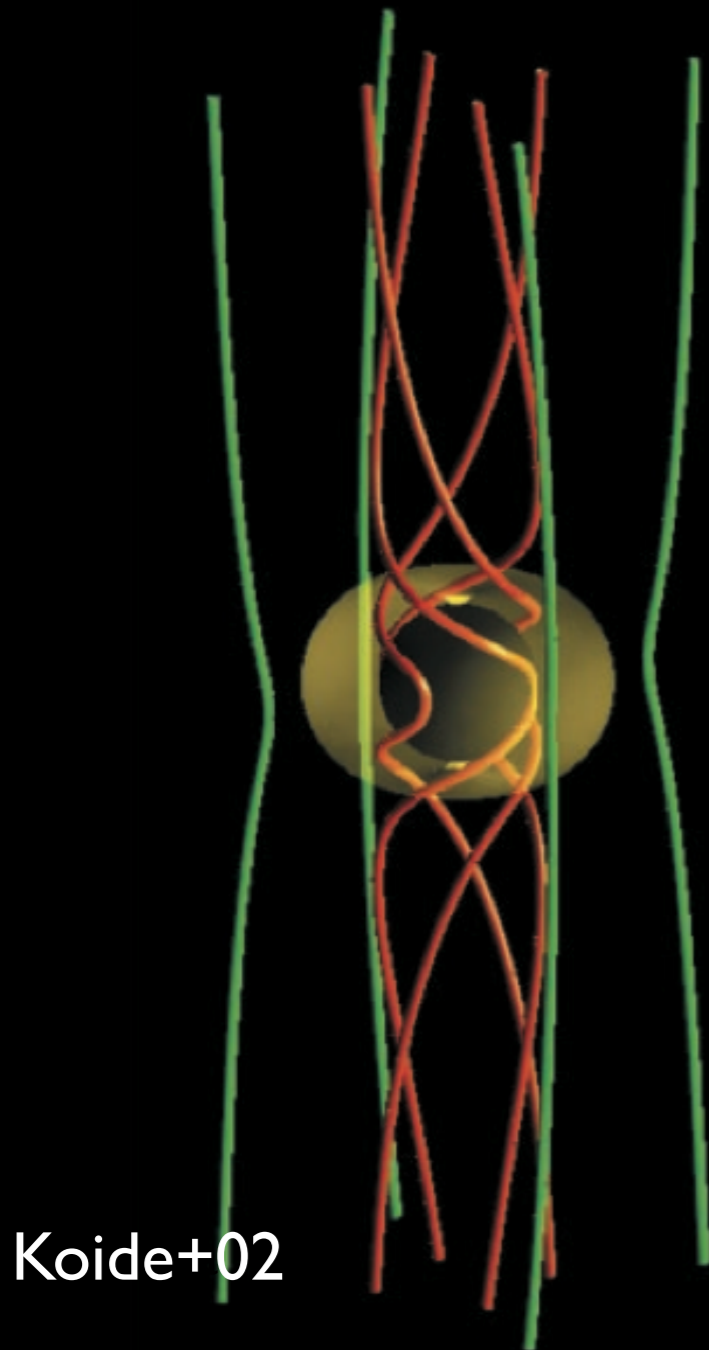


photon geodesics



extraction of BH rotational energy by magnetic fields

Penrose&Ford 71,
Blanford&Znajec 77,
Takahashi+90



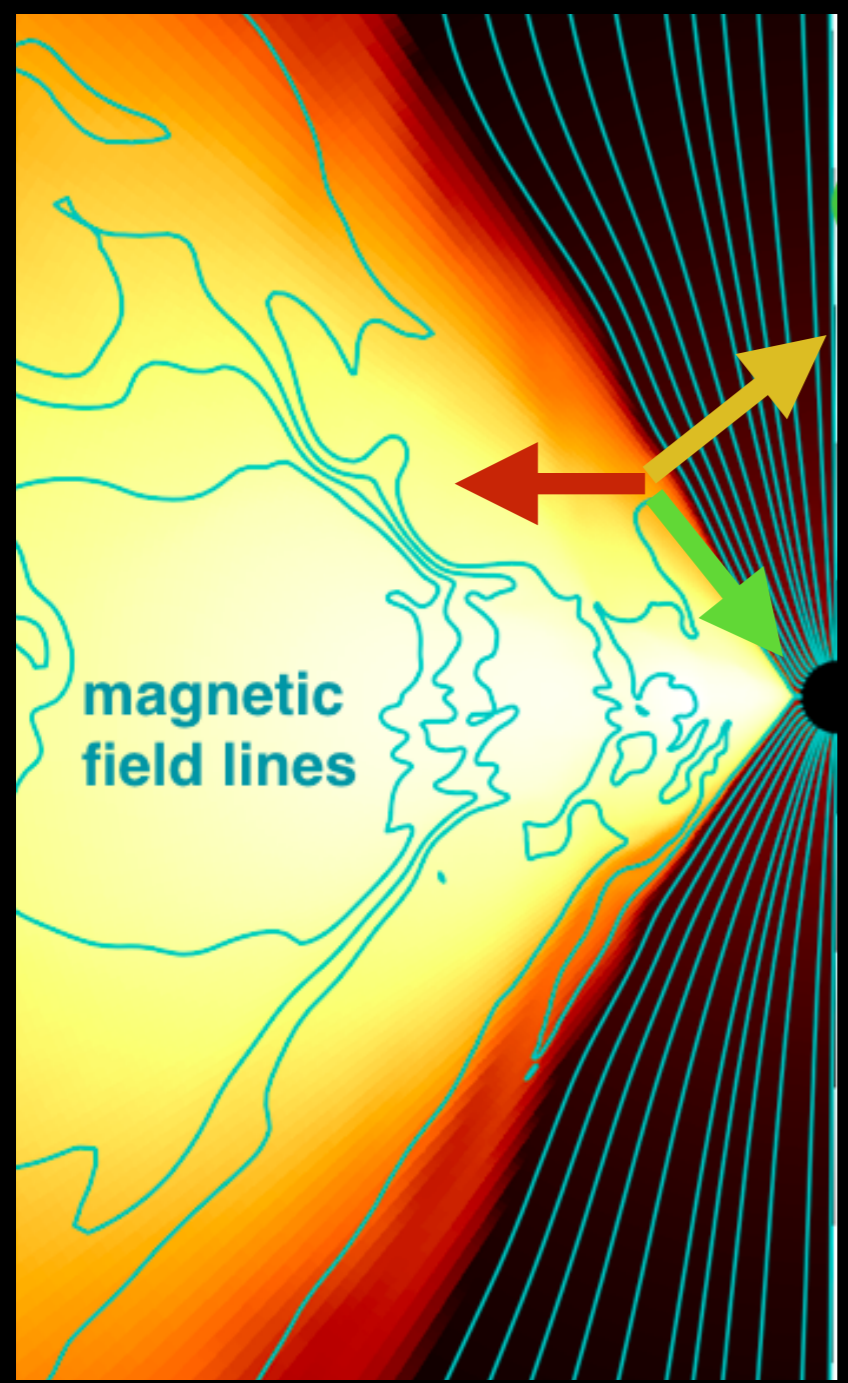
Koide+02

few plasma (nearly vacuum)

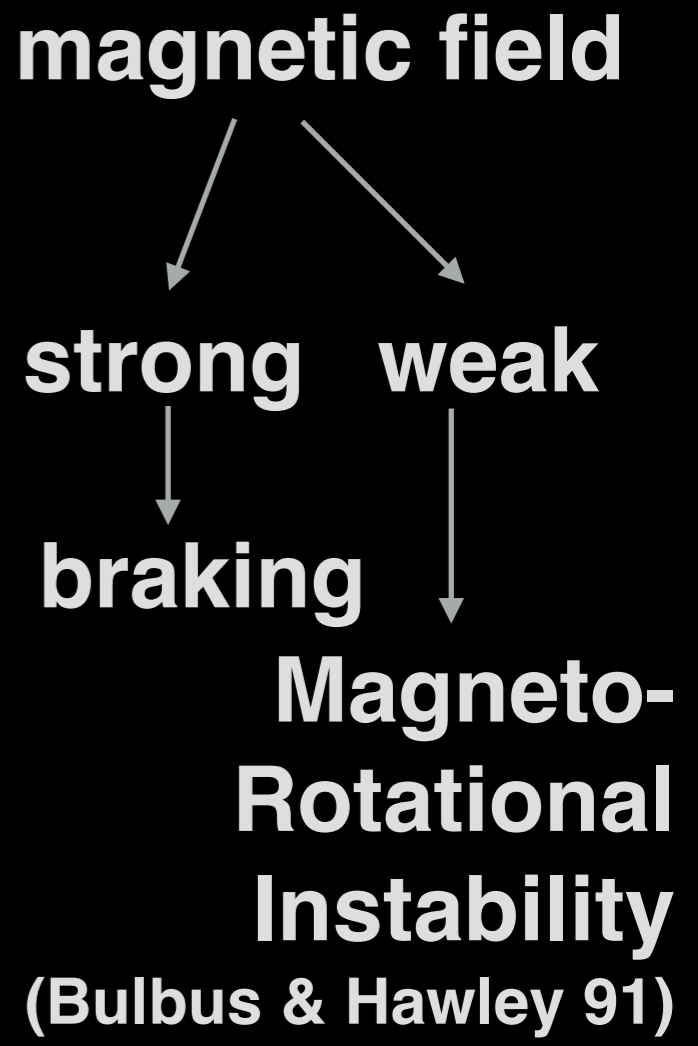
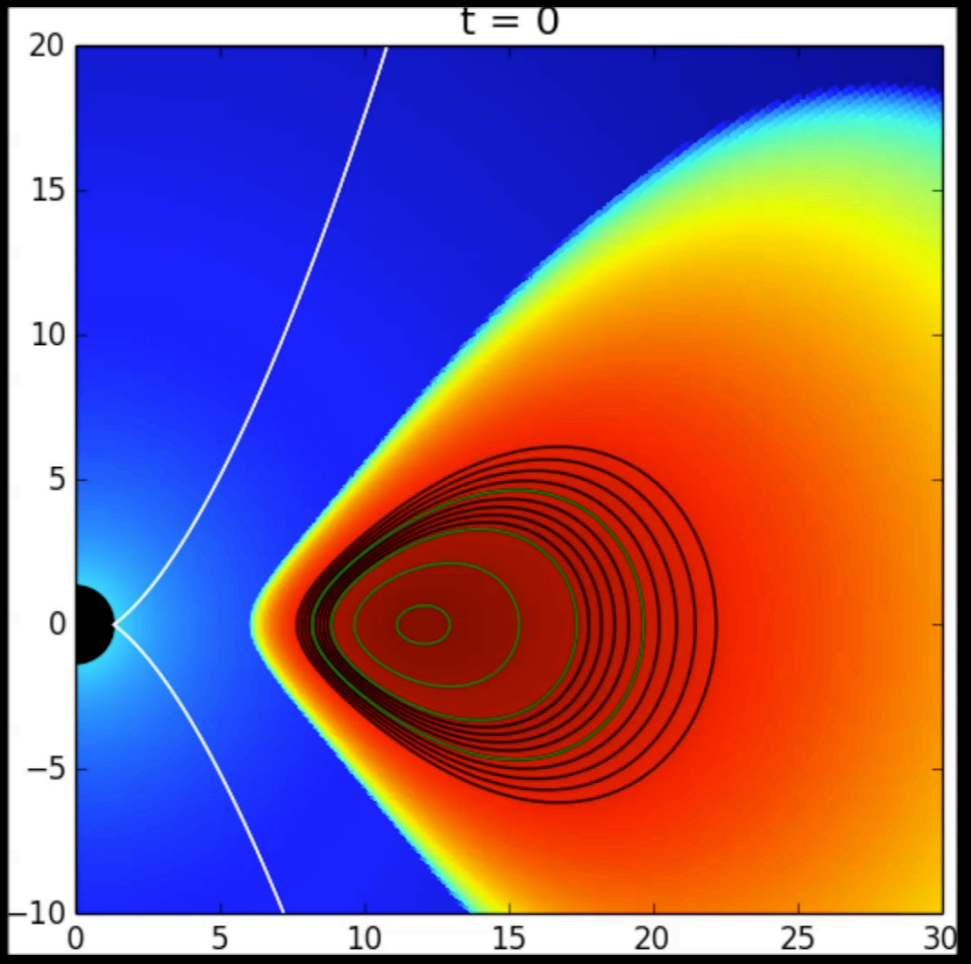
Semenov+04



“stationary state”

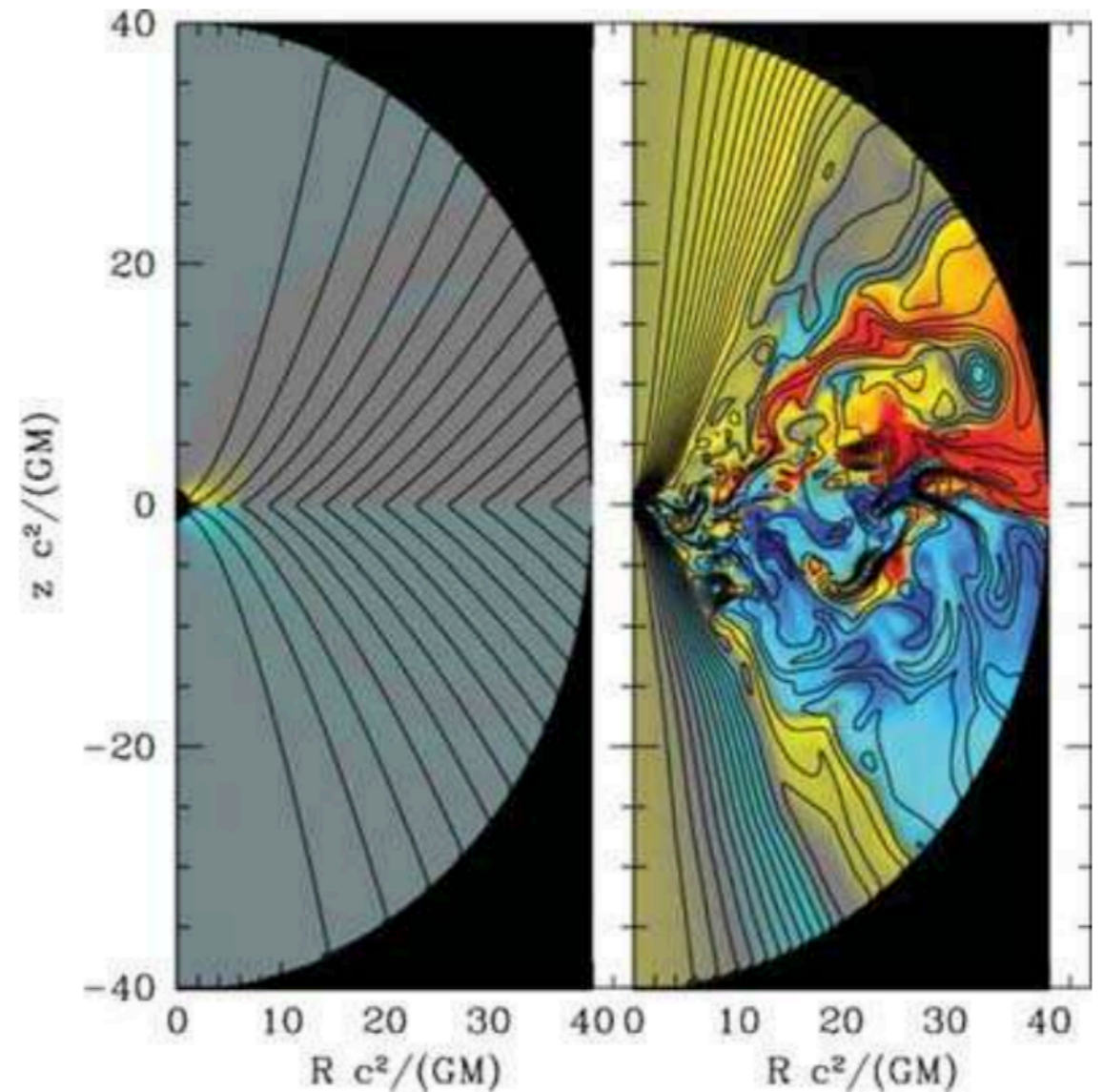
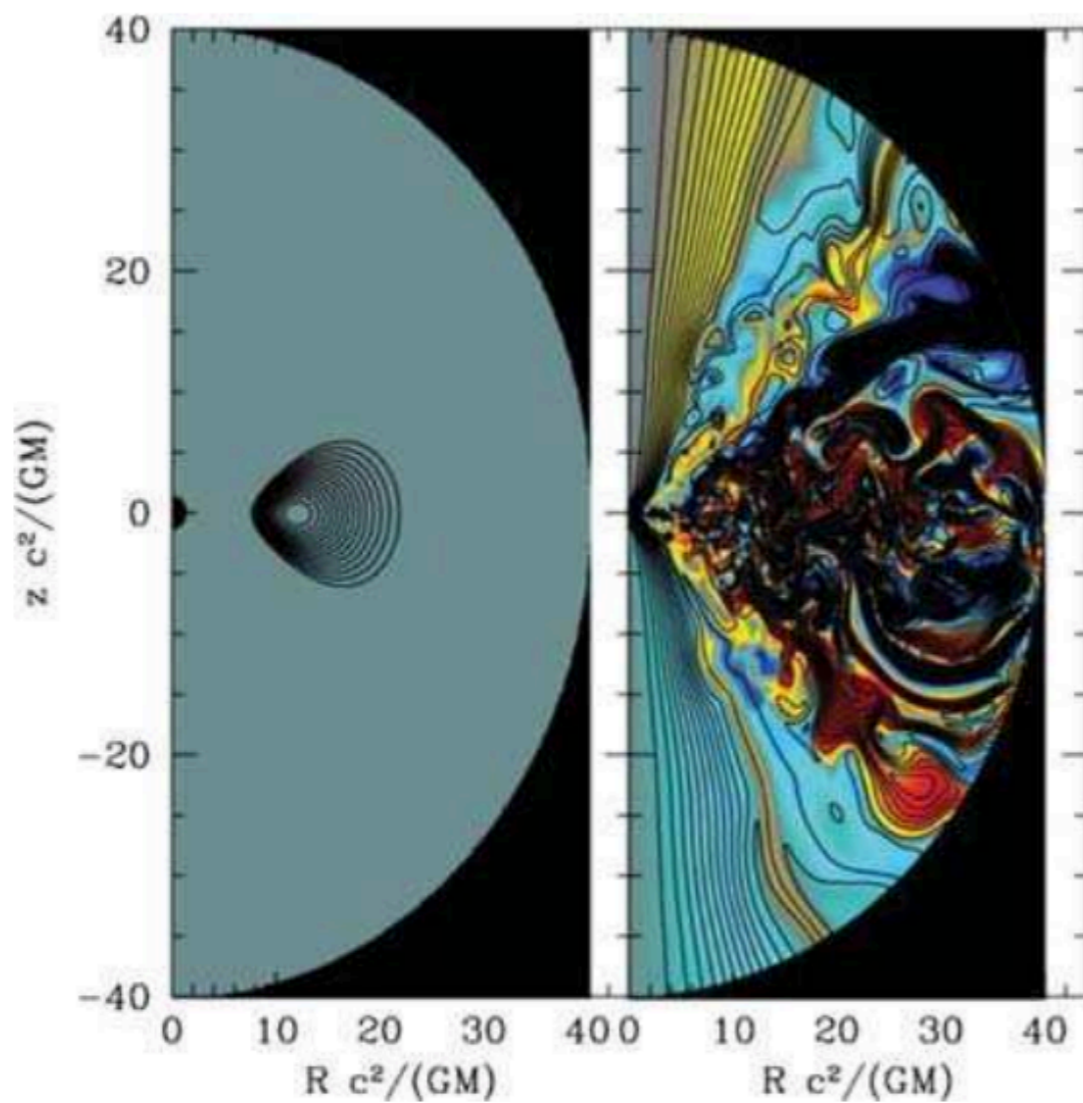


more plasma \rightarrow accretion flow



accretion

initial magnetic field configuration does matter



magnetic energy
= mass energy

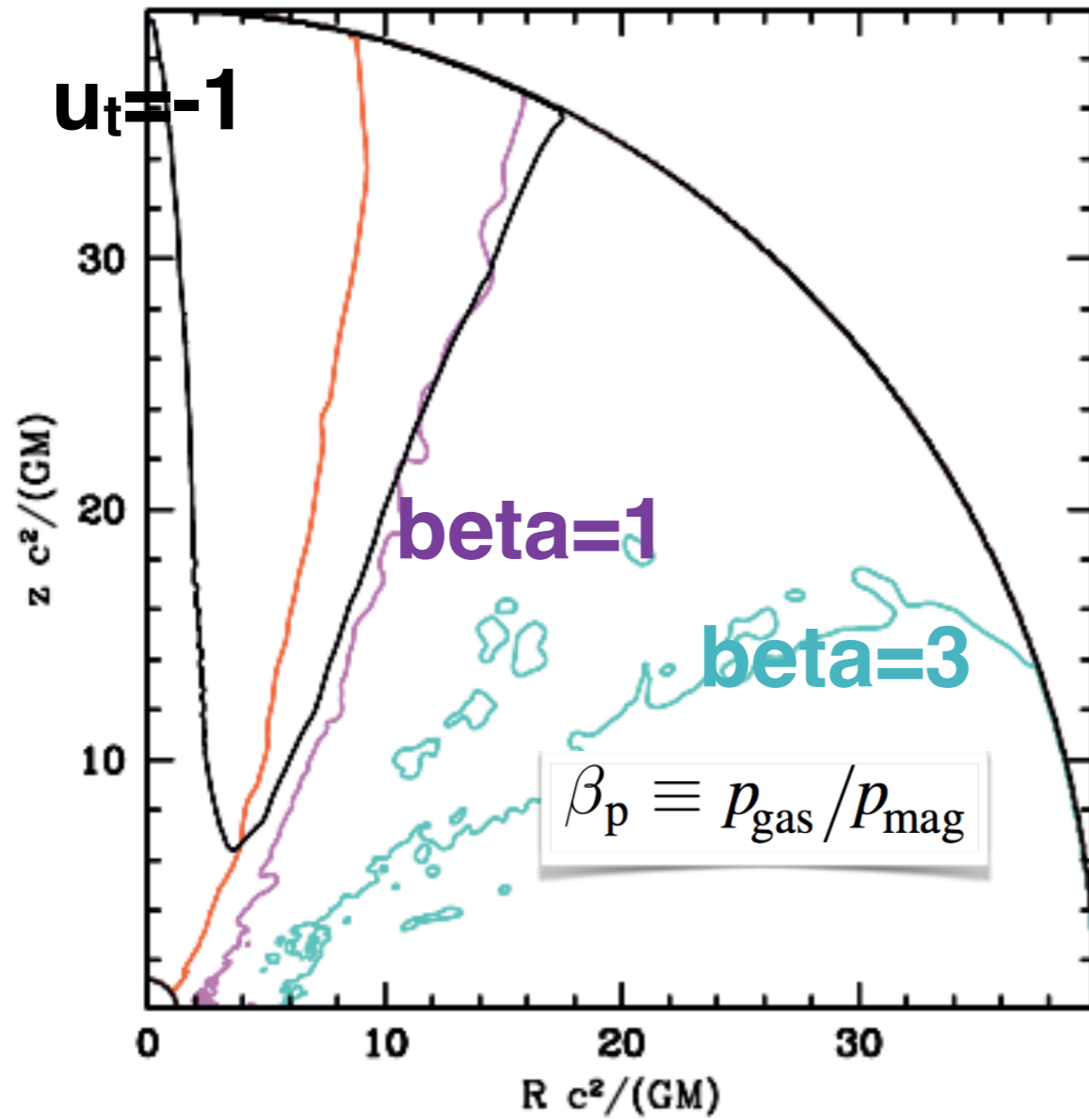


FIG. 2a

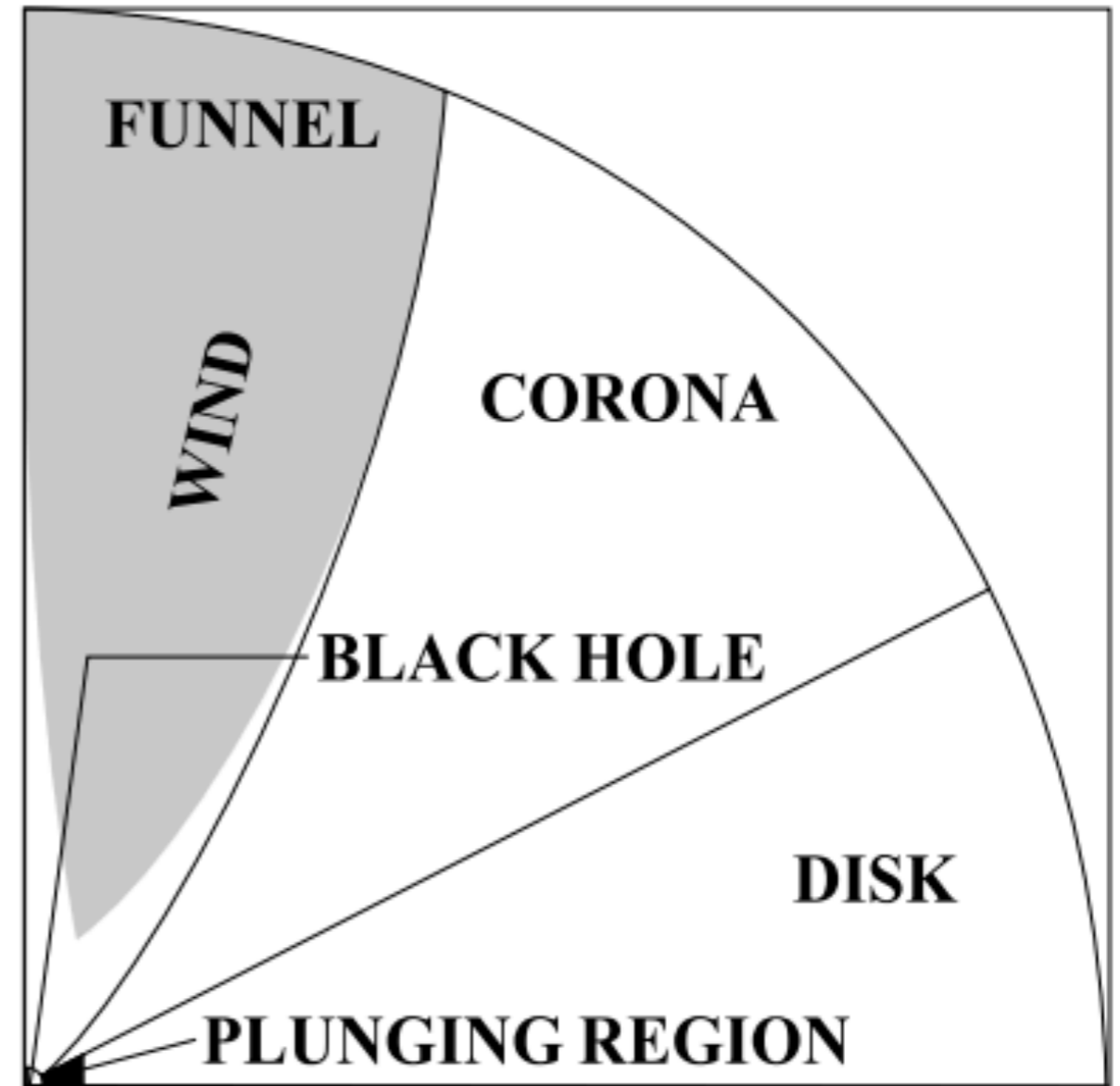
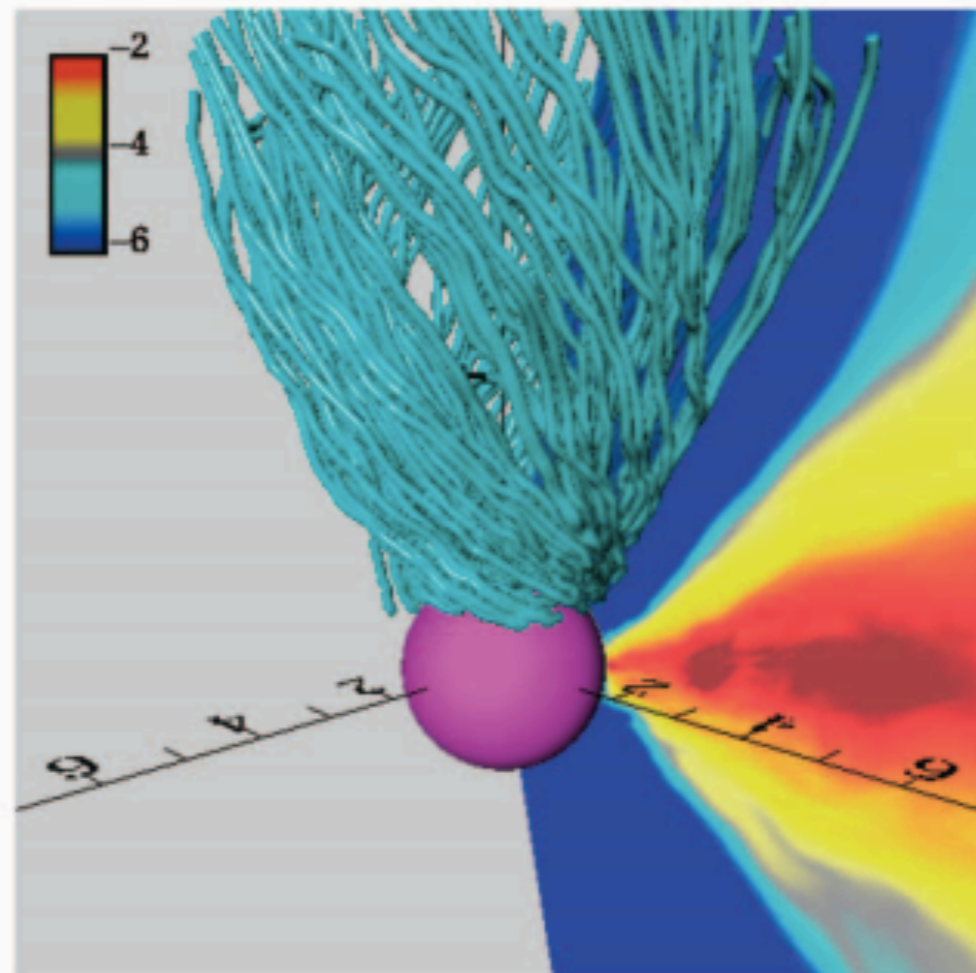
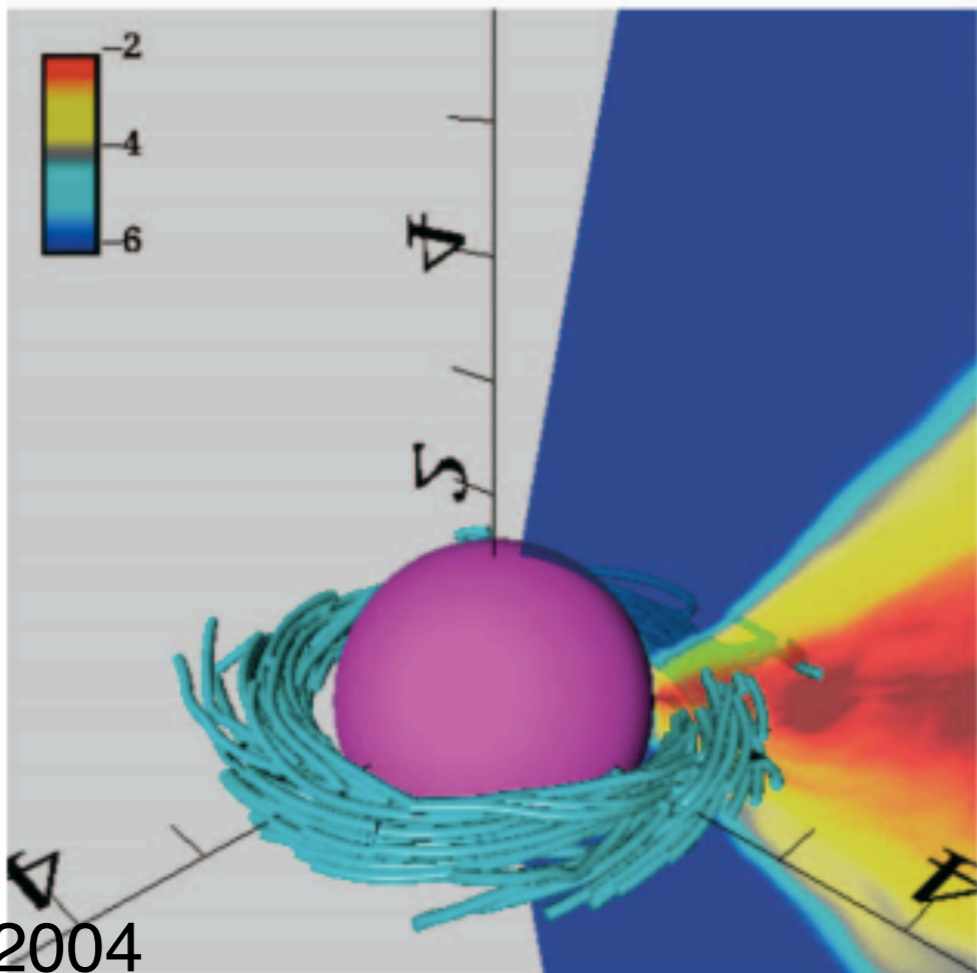
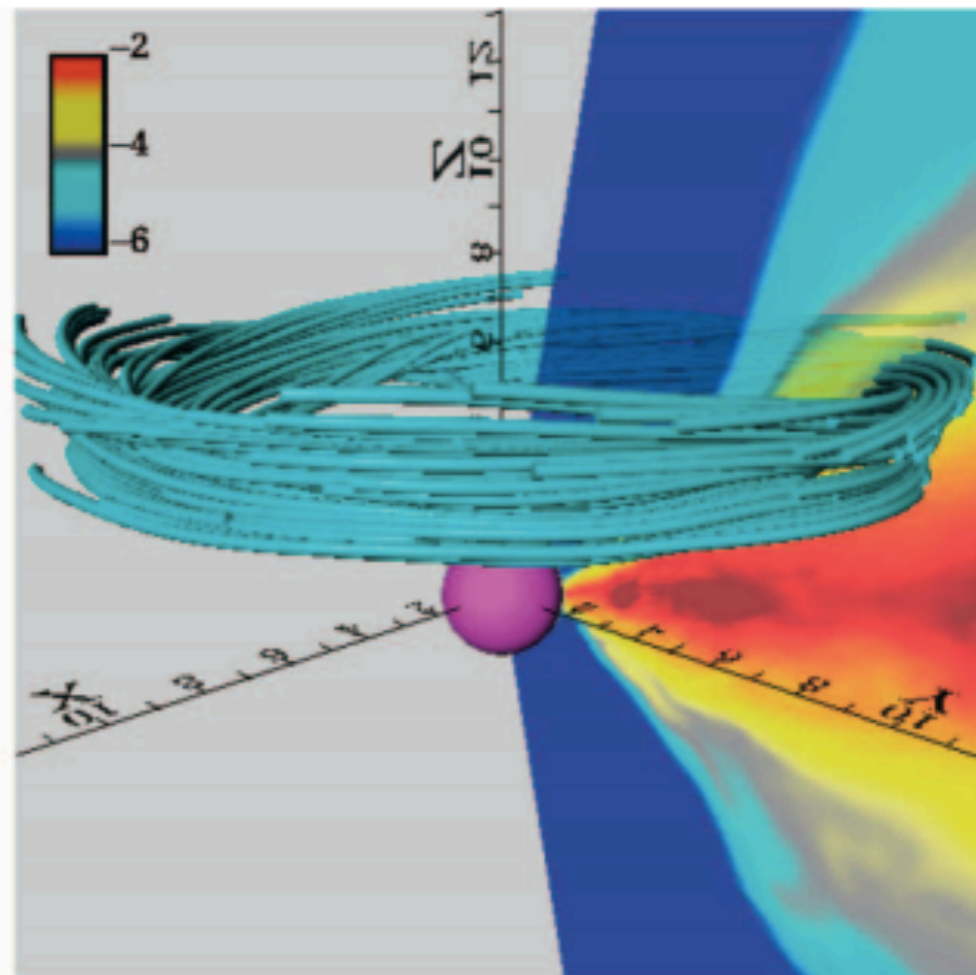
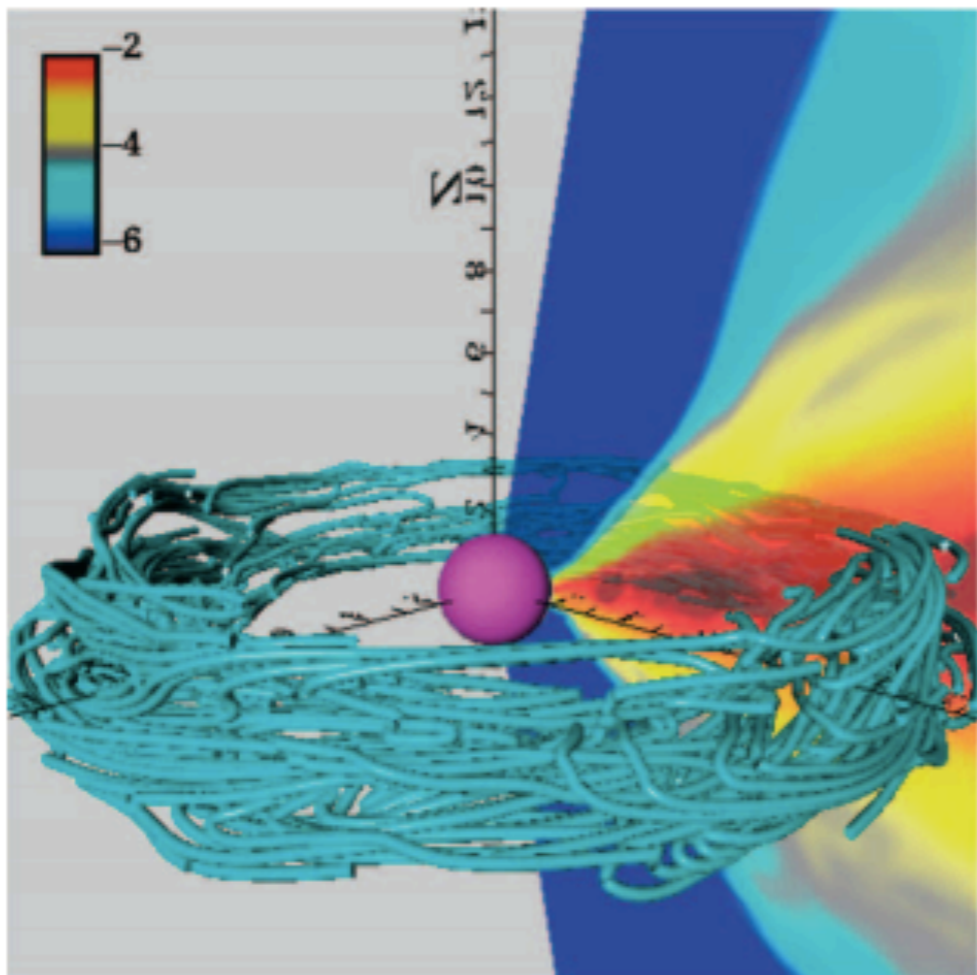
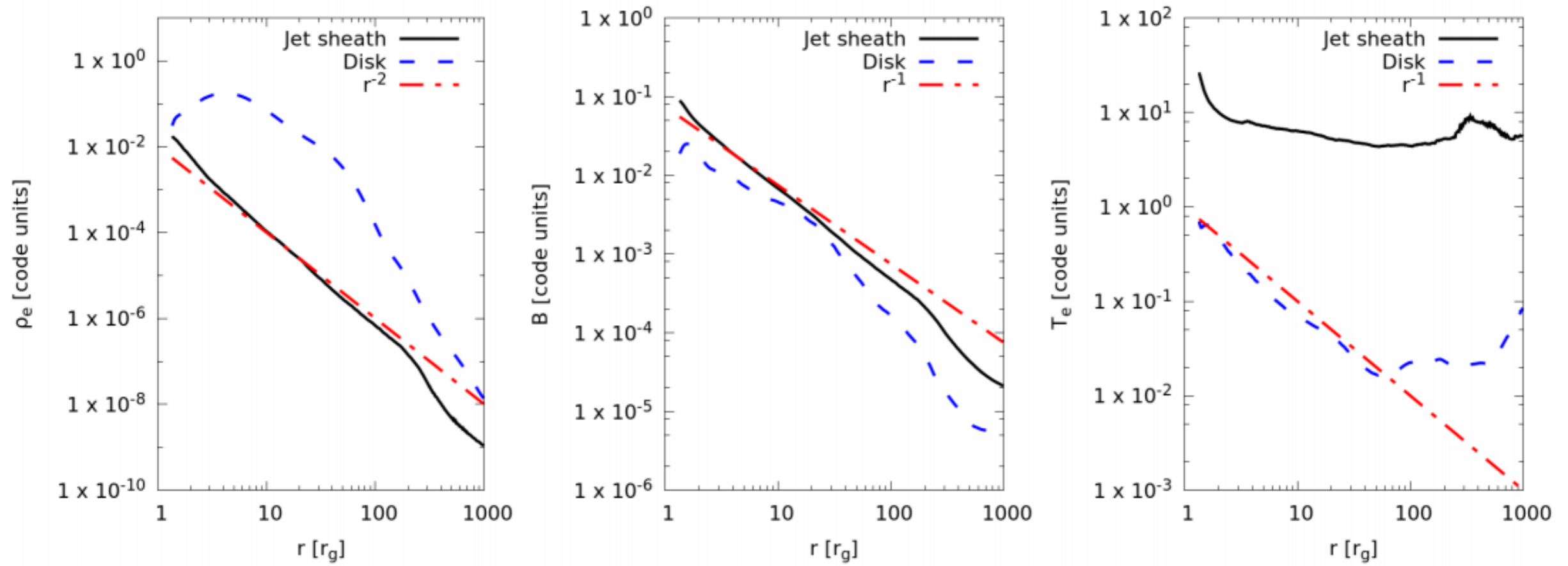


FIG. 2b



Davelaar+ 2020(see, e.g. Moscibrodzka+2013,2014)



GR Radiative Transfer

*Photon

$$p_\alpha$$

*GR + HD/ MHD

$$u^\alpha$$

$$\mathcal{I} = I_\nu / \nu^3 = \text{invariant}$$

*energy shift

$$\frac{E_{\text{comoving}}}{E_{\text{obs}}} = \frac{p_\alpha u^\alpha|_0}{p_\alpha u^\alpha|_\infty}$$

*radiative transfer

$$\frac{d\mathcal{I}}{d\tau_\nu} = -\mathcal{I} + \frac{\eta}{\chi}$$

$$\chi = \nu \alpha_\nu$$

(invariant)

$$\eta = j_\nu / \nu^2$$

(invariant)

$$\gamma^{-1} \equiv \frac{E_{\text{comoving}}}{E_{\text{obs}}} = \frac{p_\alpha u^\alpha|_0}{p_\alpha u^\alpha|_\infty}$$

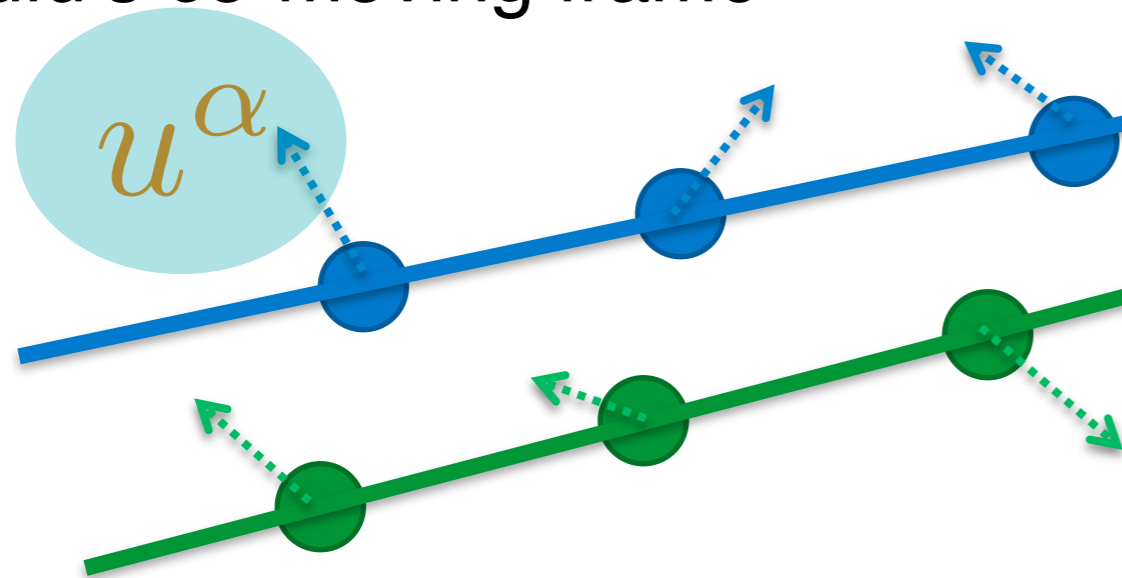
$$\mathcal{I} = I_\nu / \nu^3 = \text{invariant}$$

$$p_t = -E$$

$$p_\phi = L_z$$

“observer-to-source approach”

fluid's co-moving frame



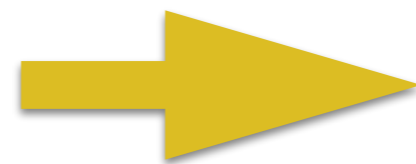
observer's image frame

$$\left(\frac{dt}{d\lambda}, \frac{dr}{d\lambda}, \frac{d\theta}{d\lambda}, \frac{d\phi}{d\lambda}, \frac{dp_r}{d\lambda}, \frac{dp_\theta}{d\lambda} \right)$$

Fuerst & Wu 2004

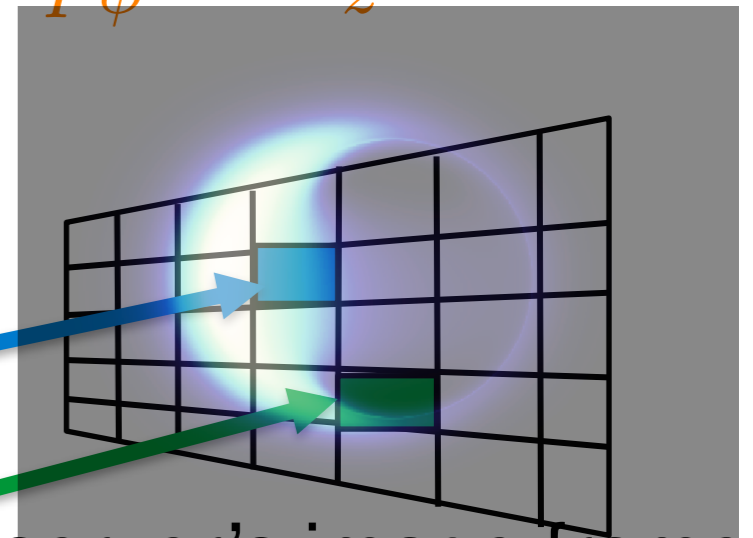
$$\frac{d\mathcal{I}}{d\tau_\nu} = -\mathcal{I} + \frac{\eta}{\chi}$$

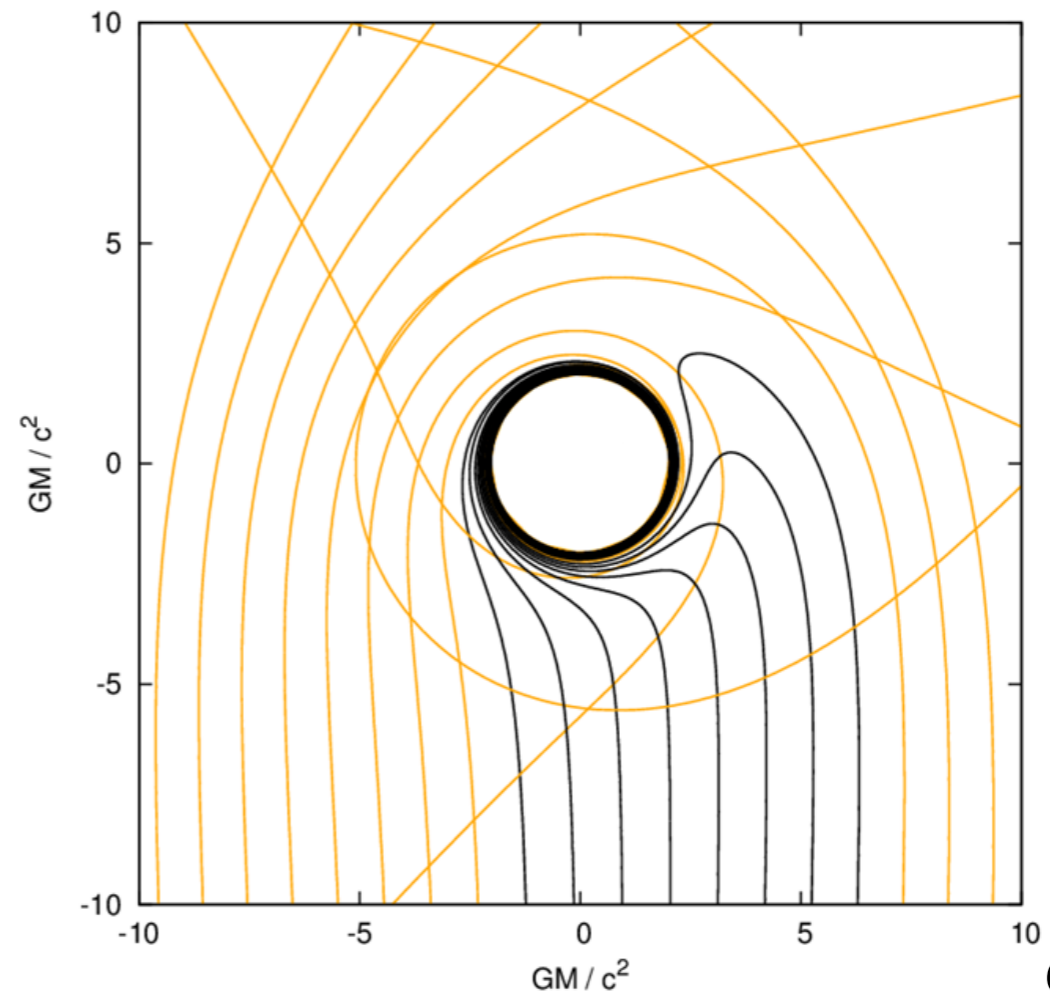
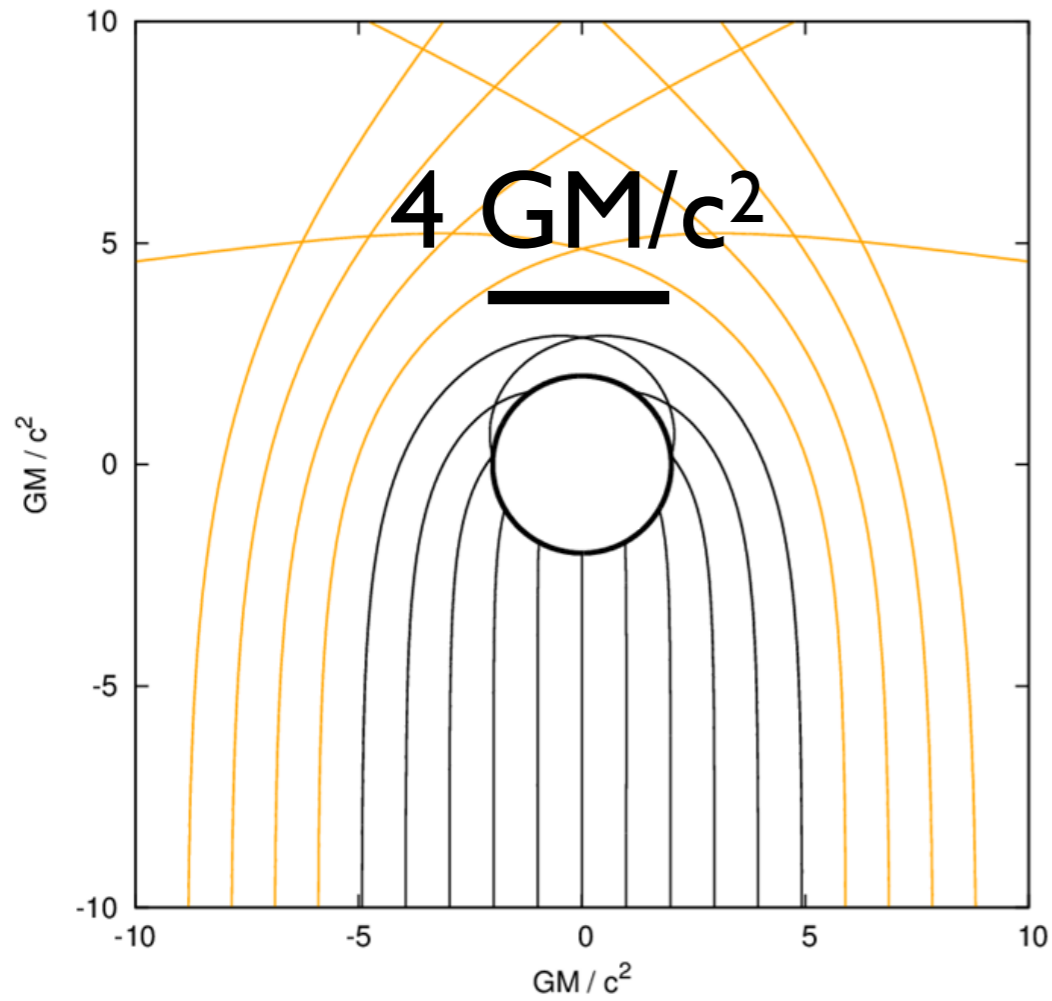
Younsi et al. 2012



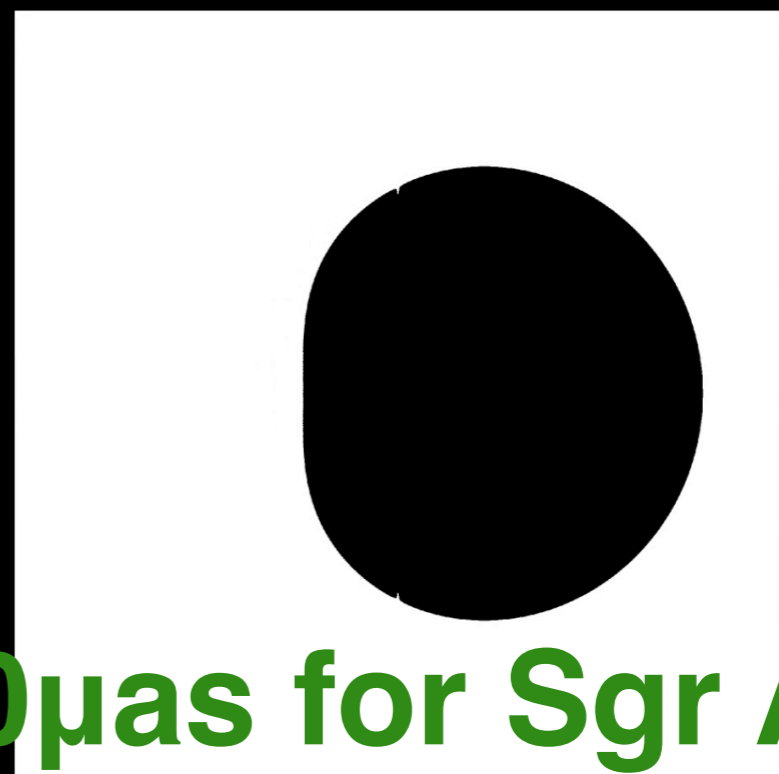
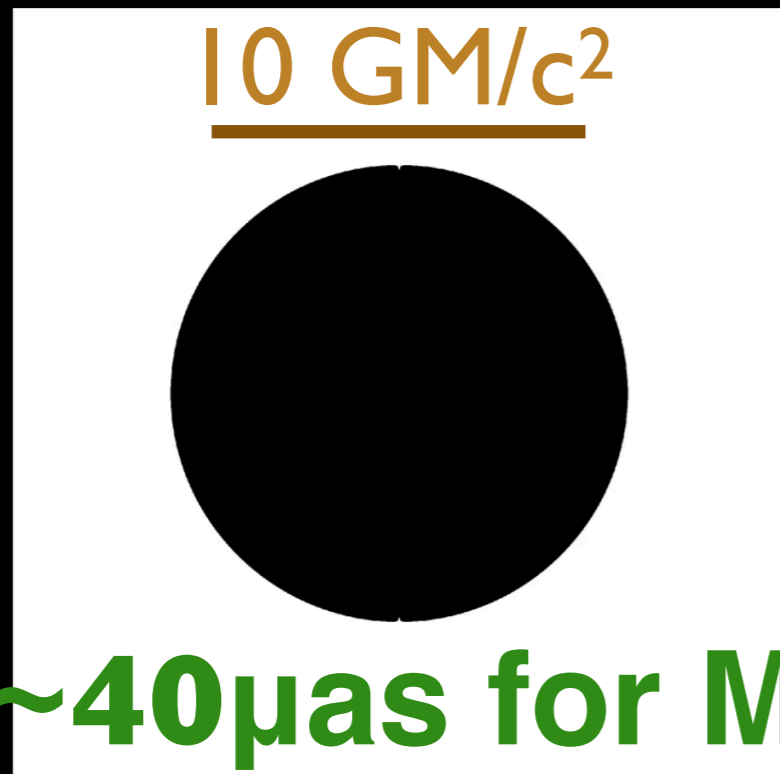
$$\frac{d\tau}{d\lambda} = \gamma^{-1} \alpha_{0,\nu}$$

$$\frac{d\mathcal{I}}{d\lambda} = \gamma^{-1} \left(\frac{j_{0,\nu}}{\nu^3} \right)$$



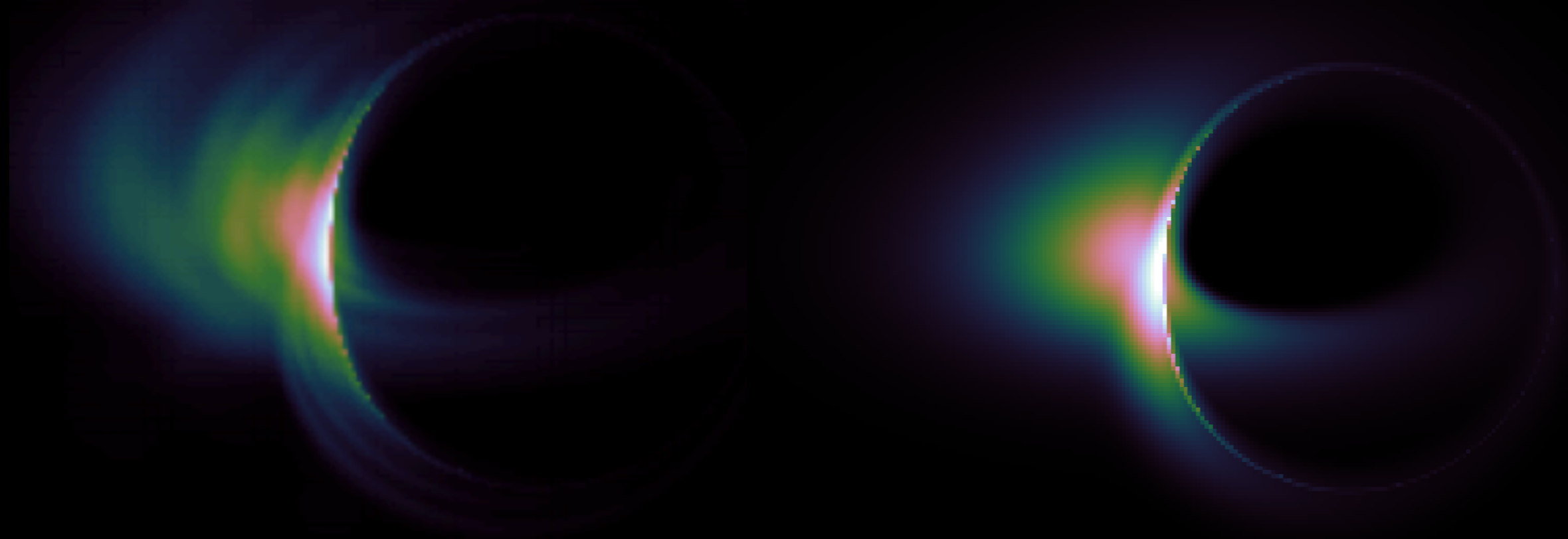


credit: Pu



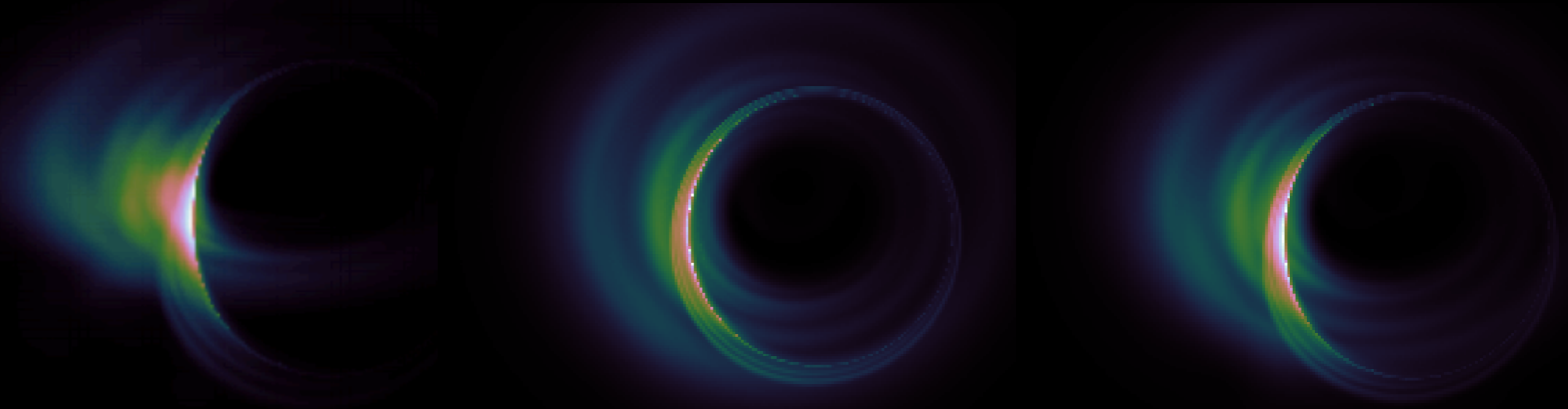
$\sim 40 \mu\text{as}$ for M87; $\sim 50 \mu\text{as}$ for Sgr A*

GRMHD numerical simulation + GRRT

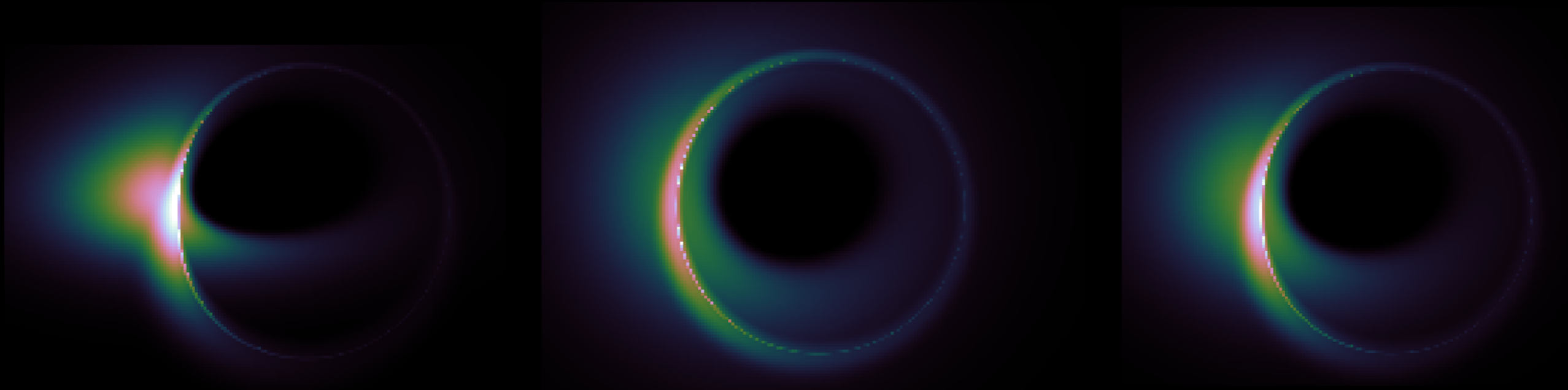


phenomenological model + GRRT

GRMHD numerical simulation + GRRT



phenomenological model + GRRT



$$Z = r \cos \theta$$

$$R = r \sin \theta$$

$$n_e = n_e^0 \exp\left(\frac{Z^2}{2(HR)^2}\right) r^{-1.1}$$

$$T_e = T_e^0 r^{-0.84}$$

$$\frac{B^2}{8\pi} = 0.1 \frac{n_e m_p c^2}{6r}$$

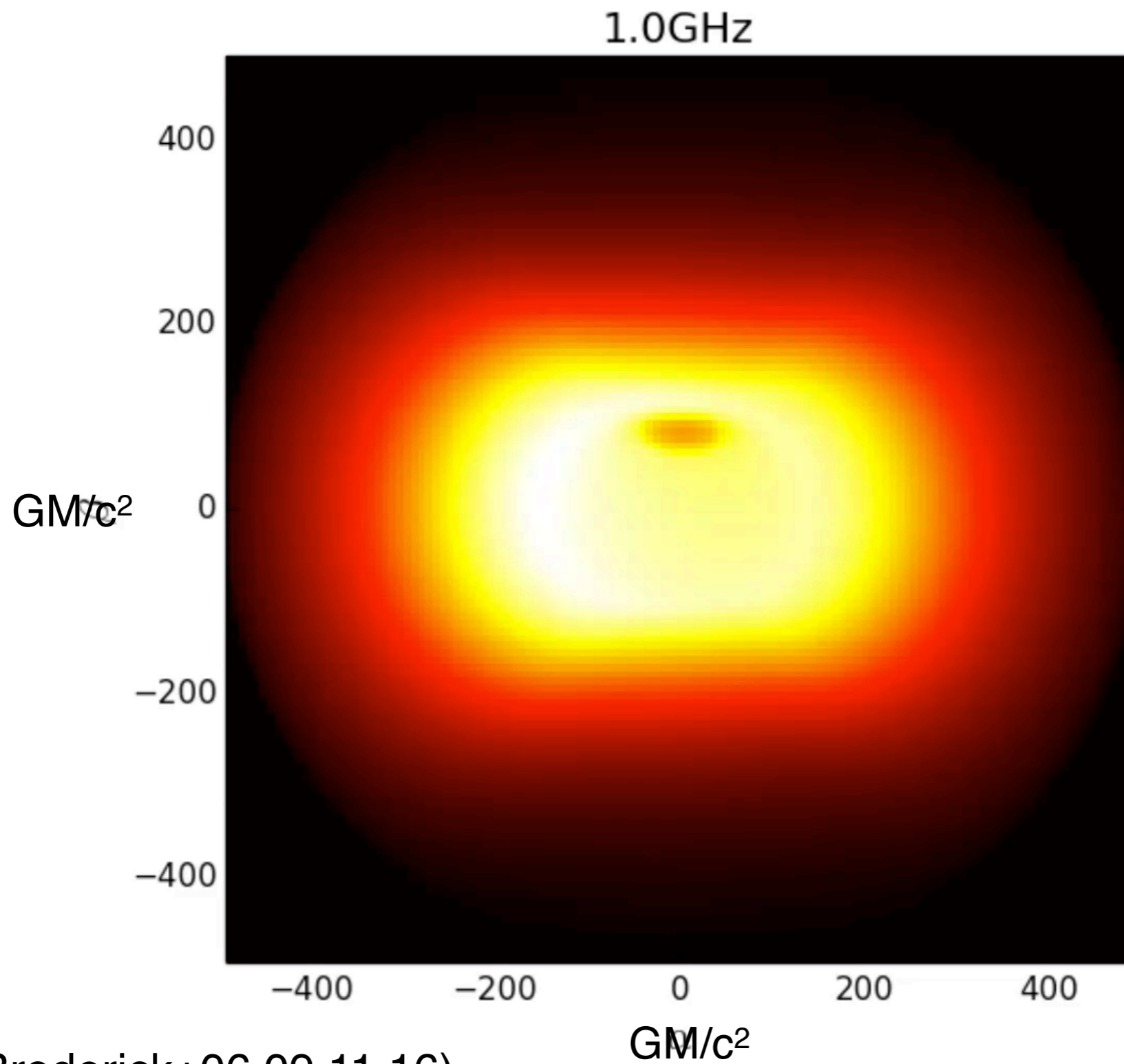
$$n_e^0 = 5e6(\text{cm}^{-3})$$

$$T_e^0 = 5.3e11(\text{K})$$

$$H = 0.1$$

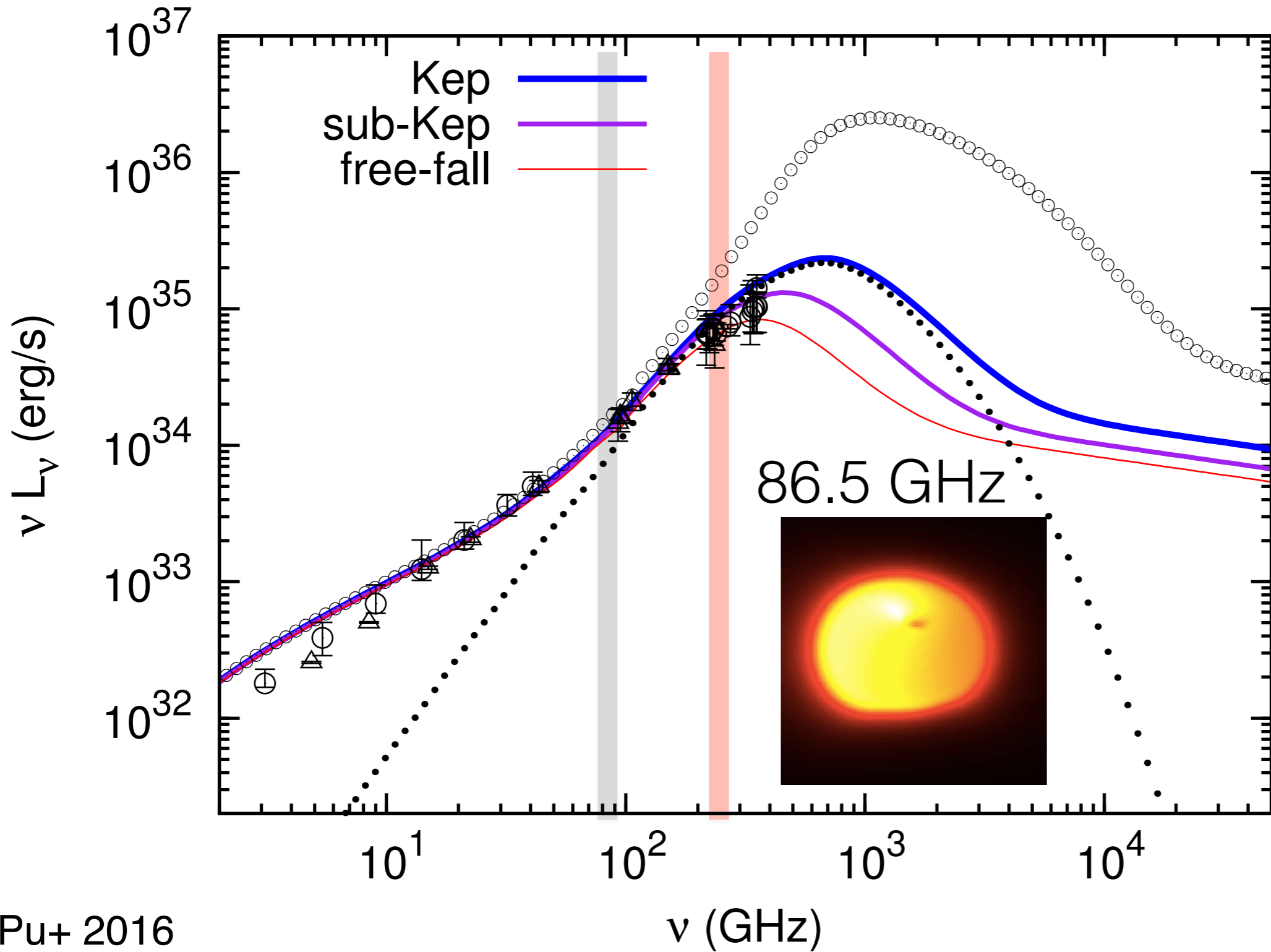
Keplerian rotation

Modeling Sgr A*

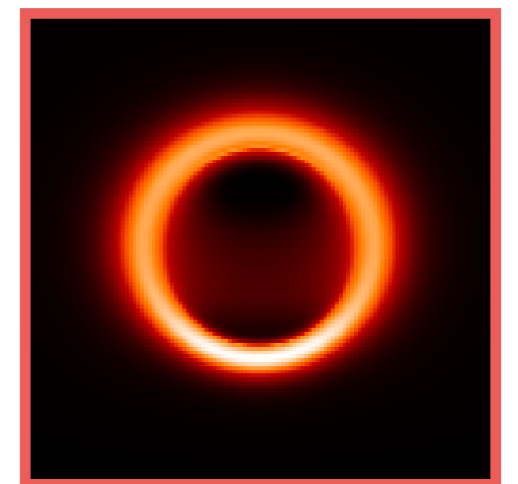
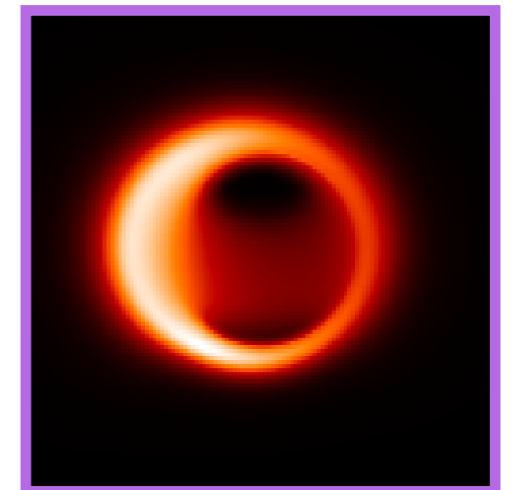
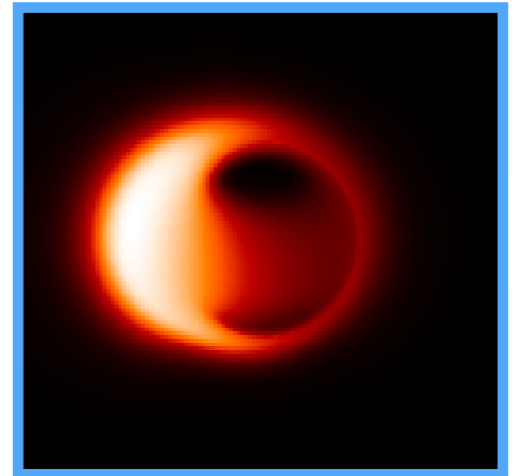


Pu+ 2016
(see also Broderick+06,09,11,16)

*a=0, viewing angle =68° (Broderick et al.2011)



230GHz



Dynamical dependency

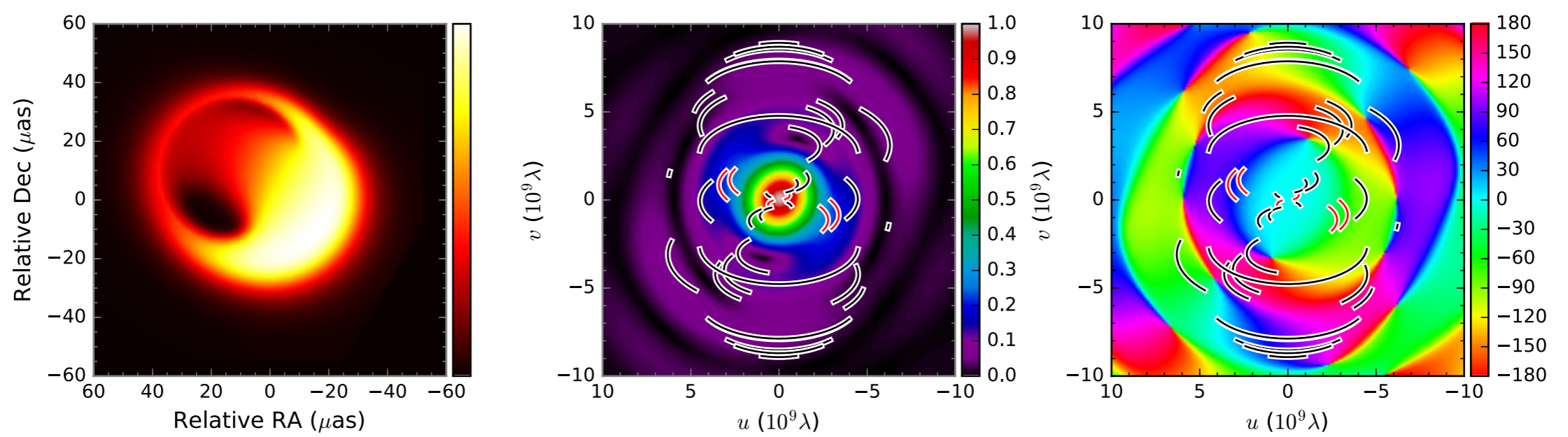
*effect of flow dynamics **only**



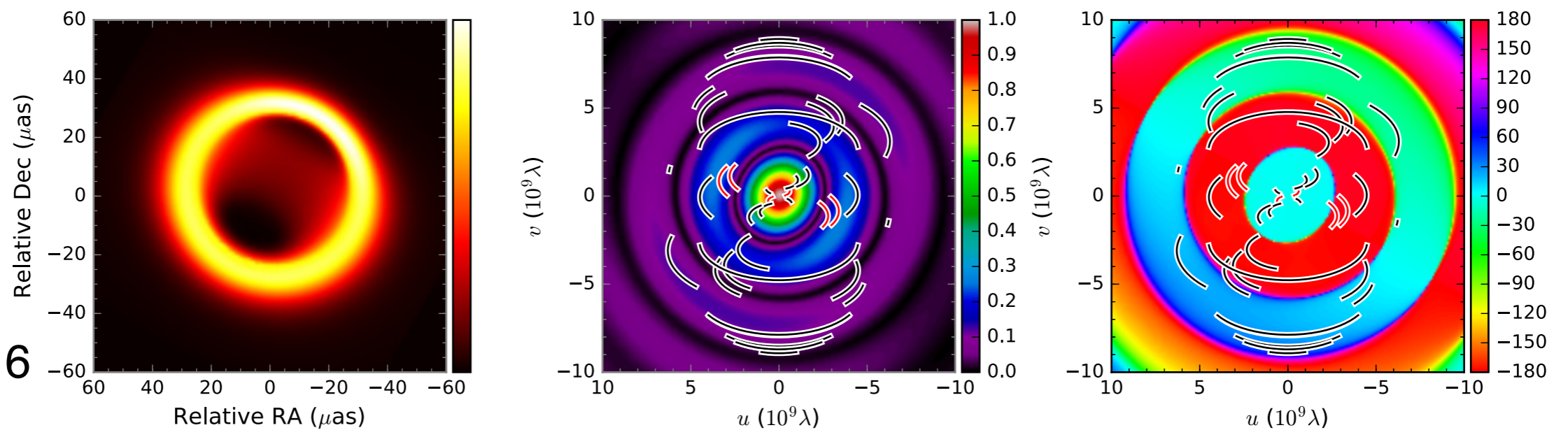
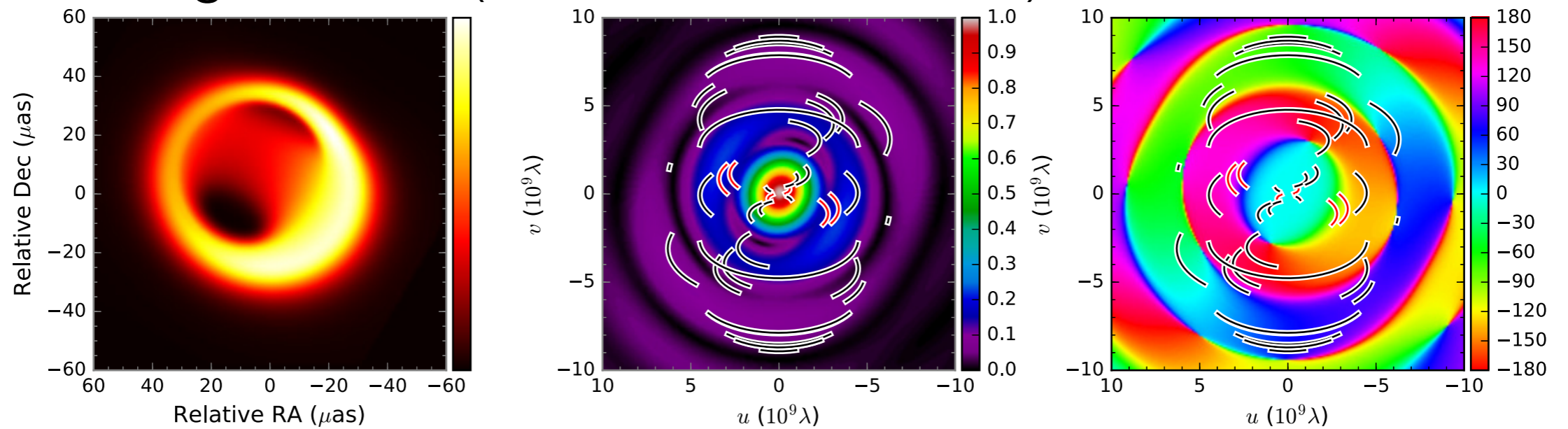
free-fall

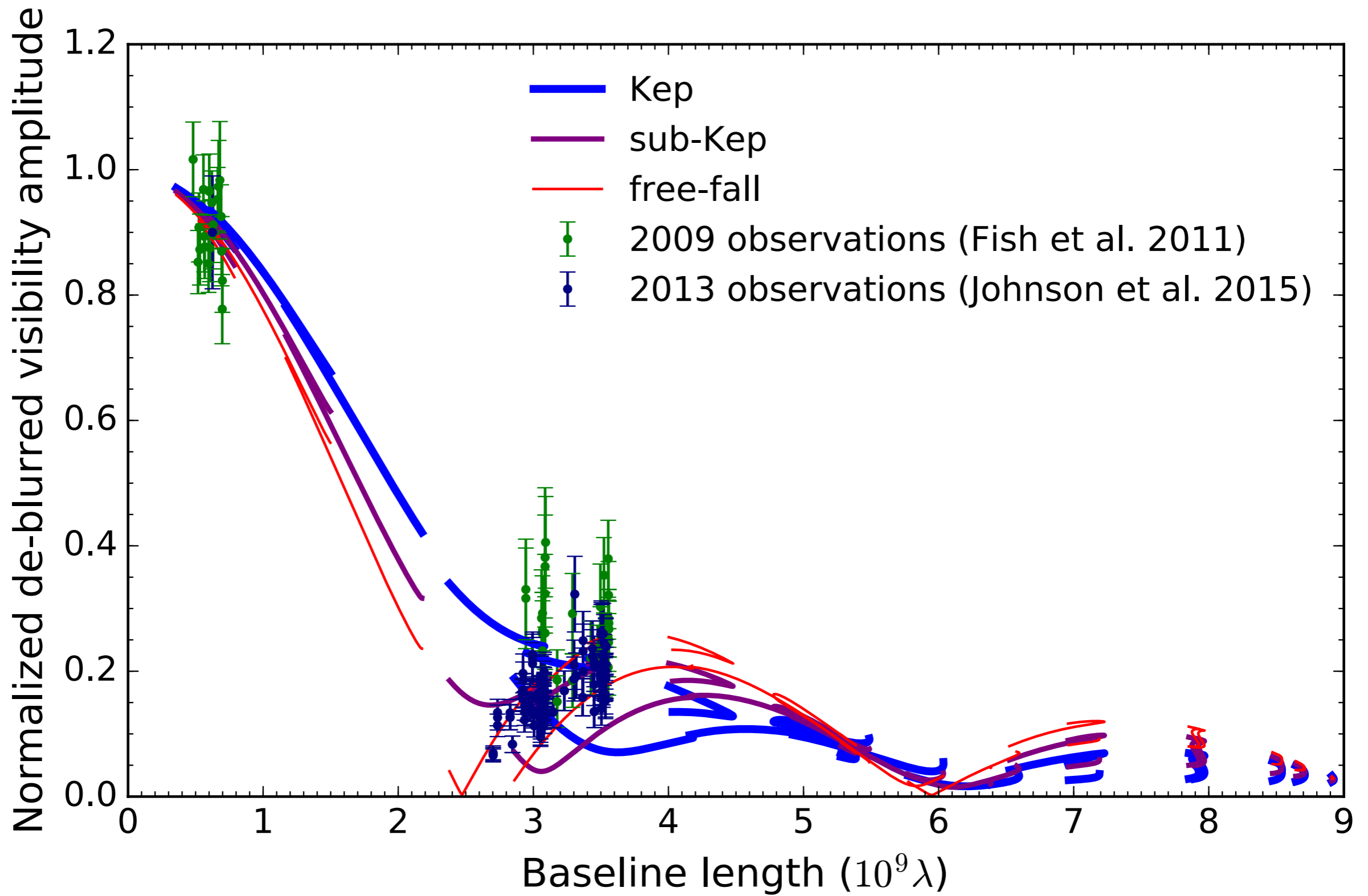
rotation
sub-Keplerian

Keplerian
rotating



*position angle = 150° (Broderick et al.2016)

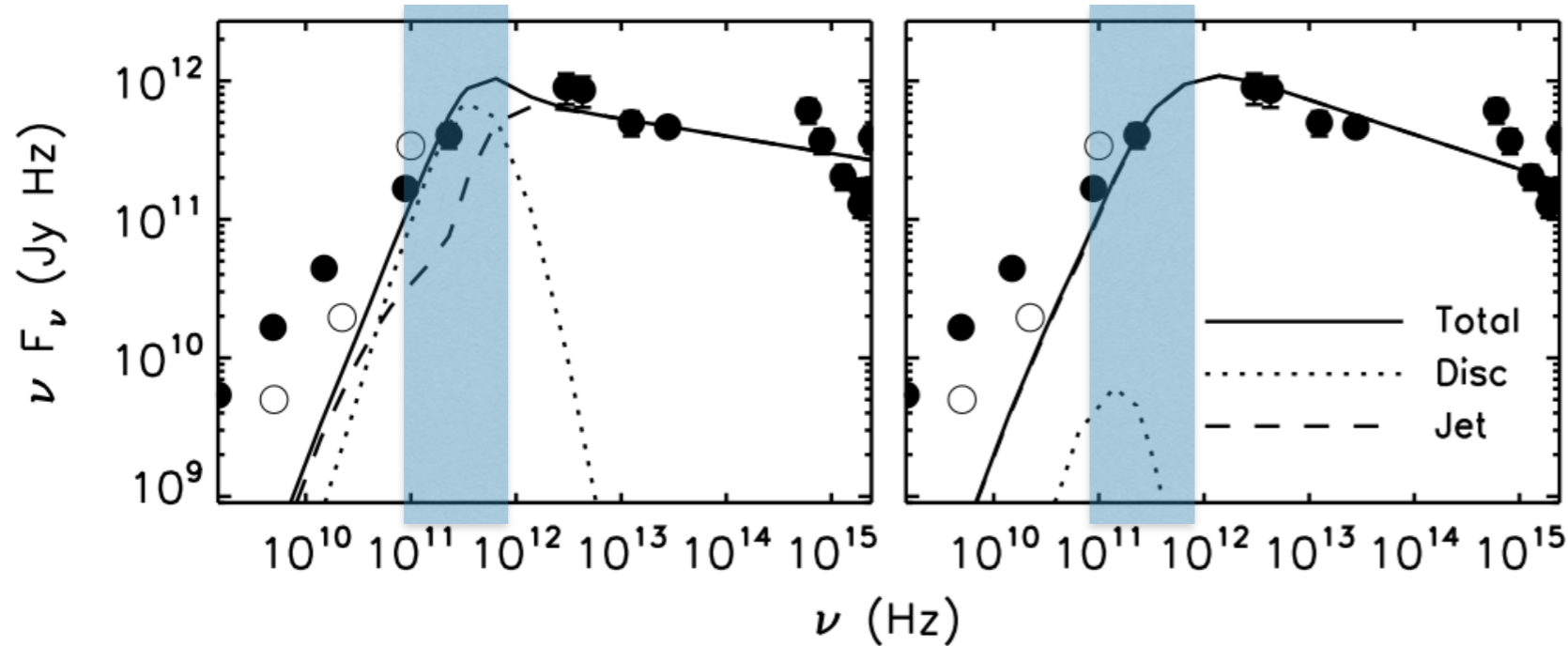




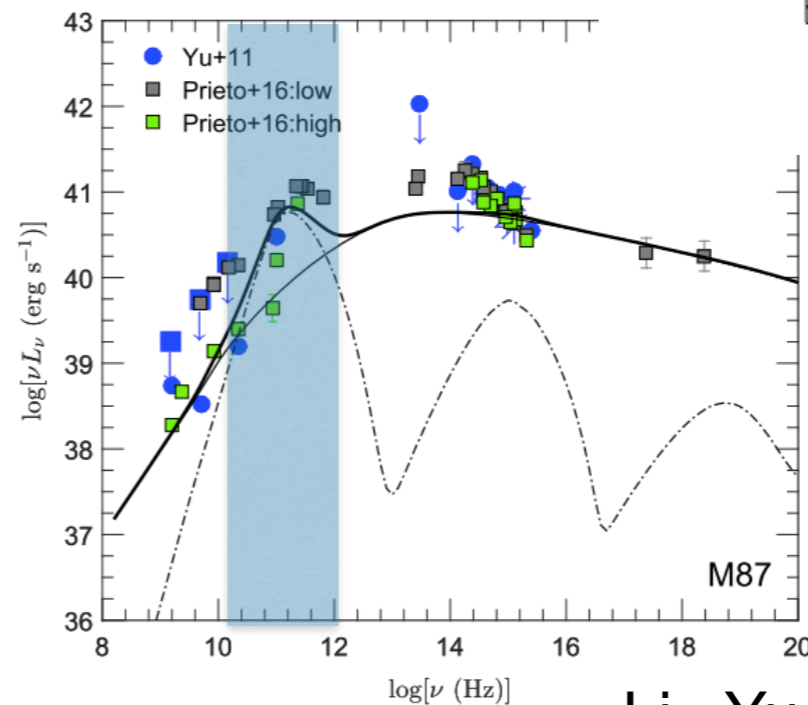
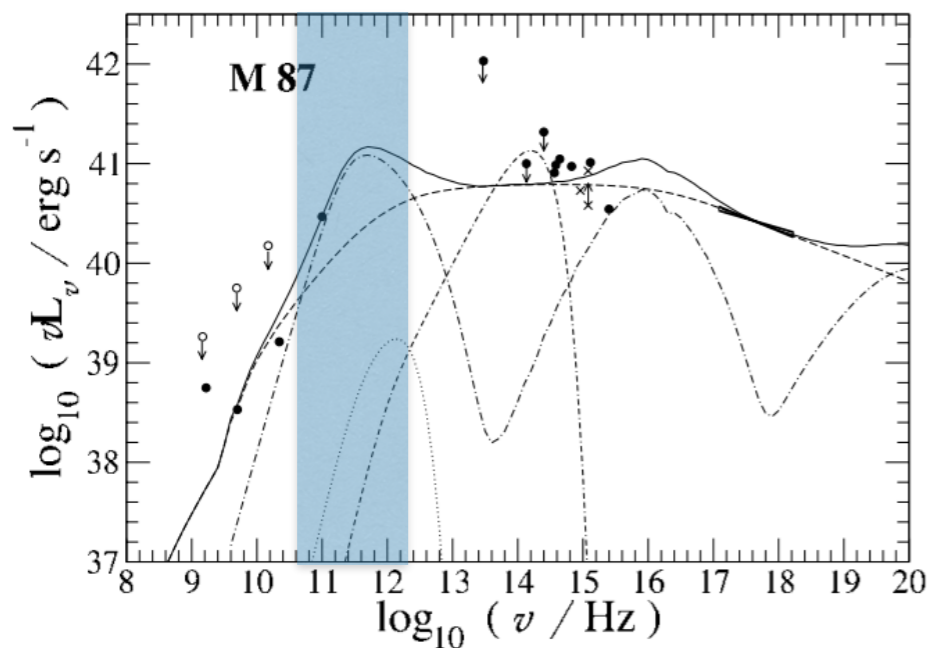
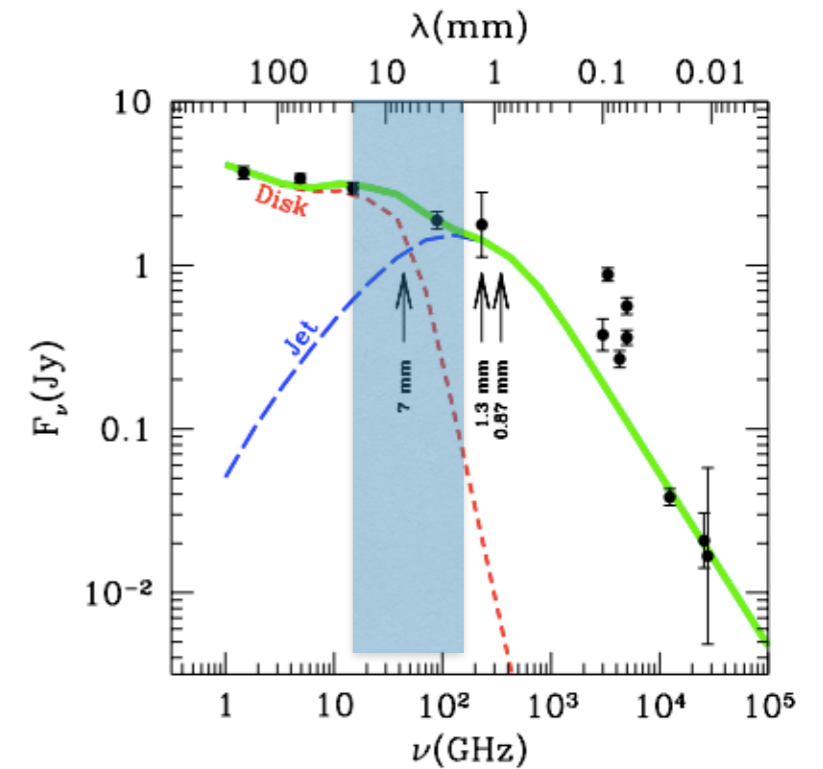
Jet

accretion or jet ? compact or extended?

Dexter, McKinney, & Agol 2012



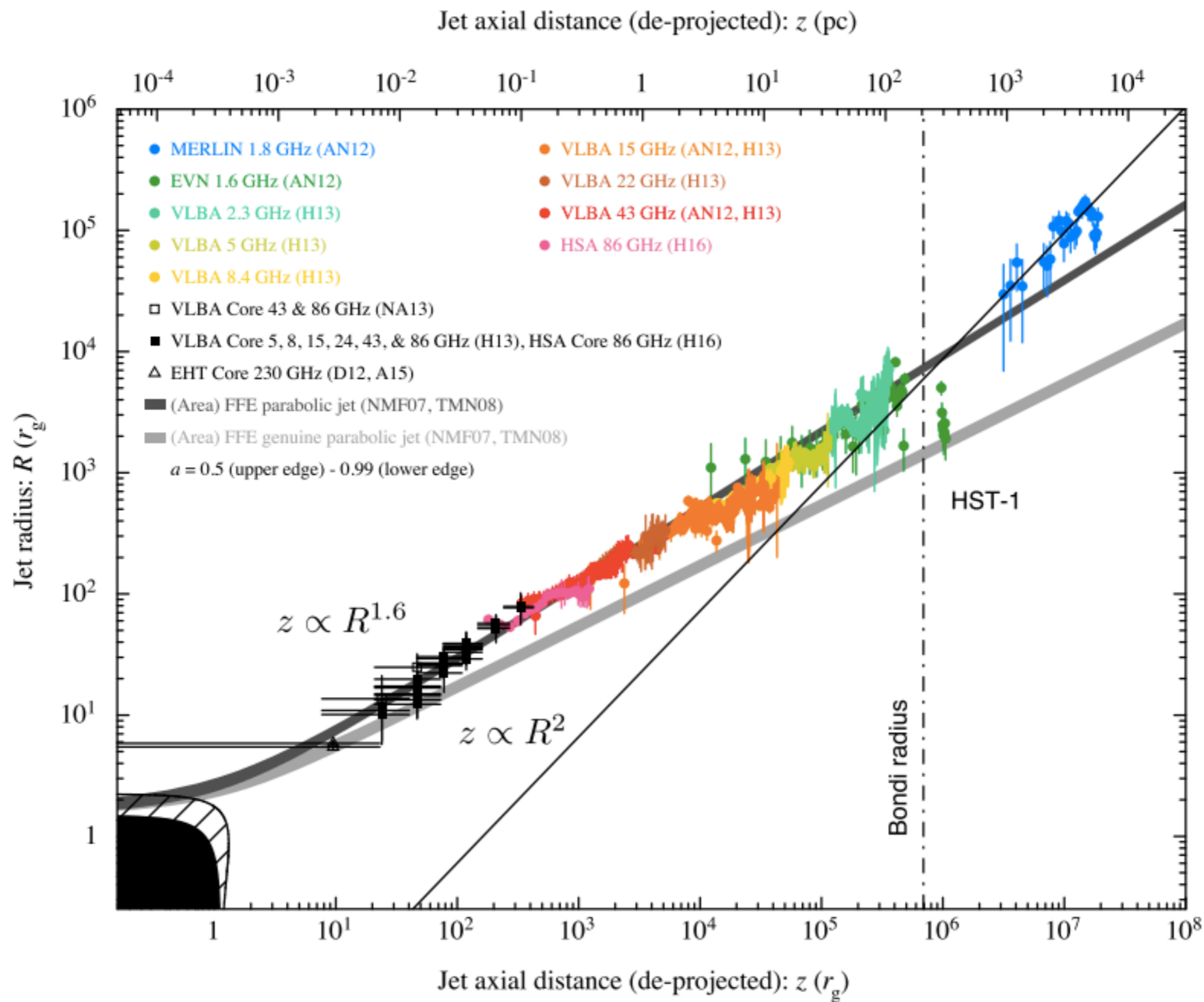
Broderick & Loeb 2009



M 87

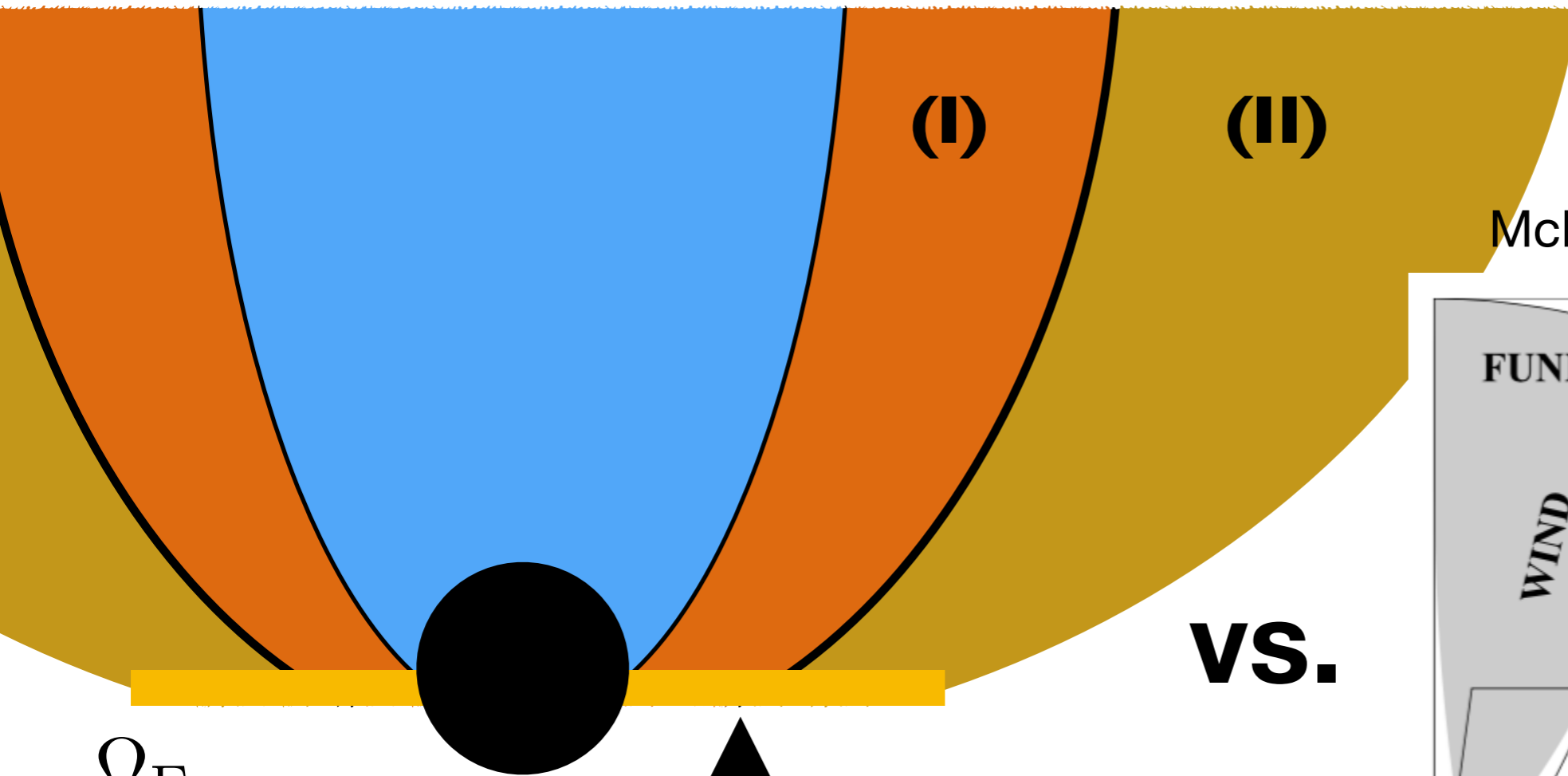
Yu, Yuan, & Ho 2016

Li, Yuan, & Xie 2016



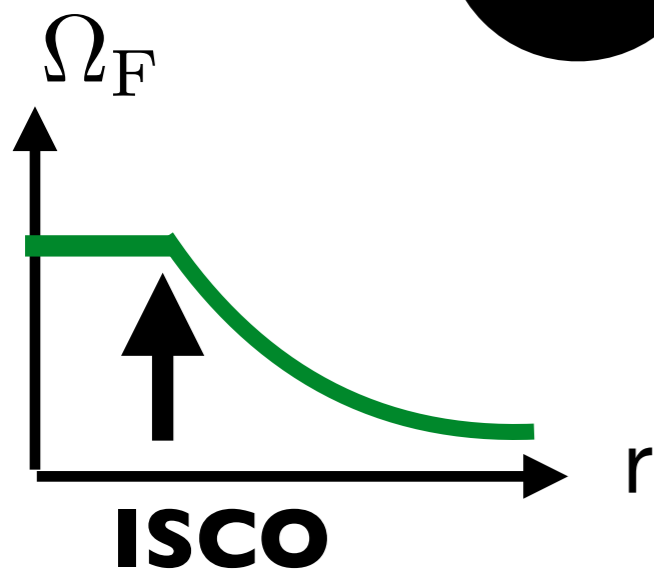
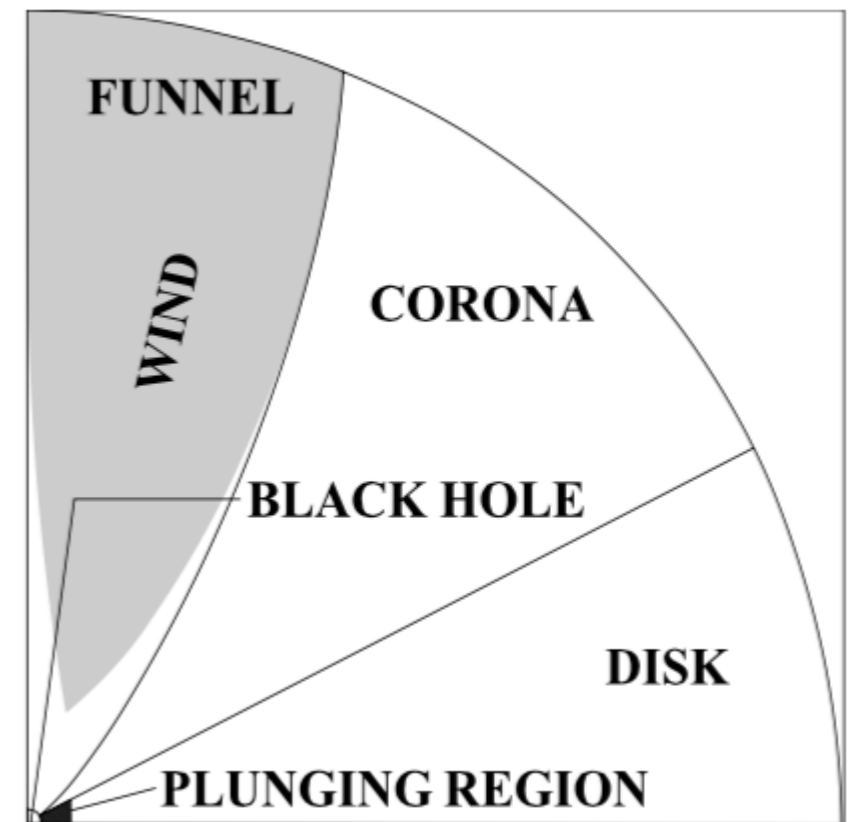
Notes on the jet region

funnel

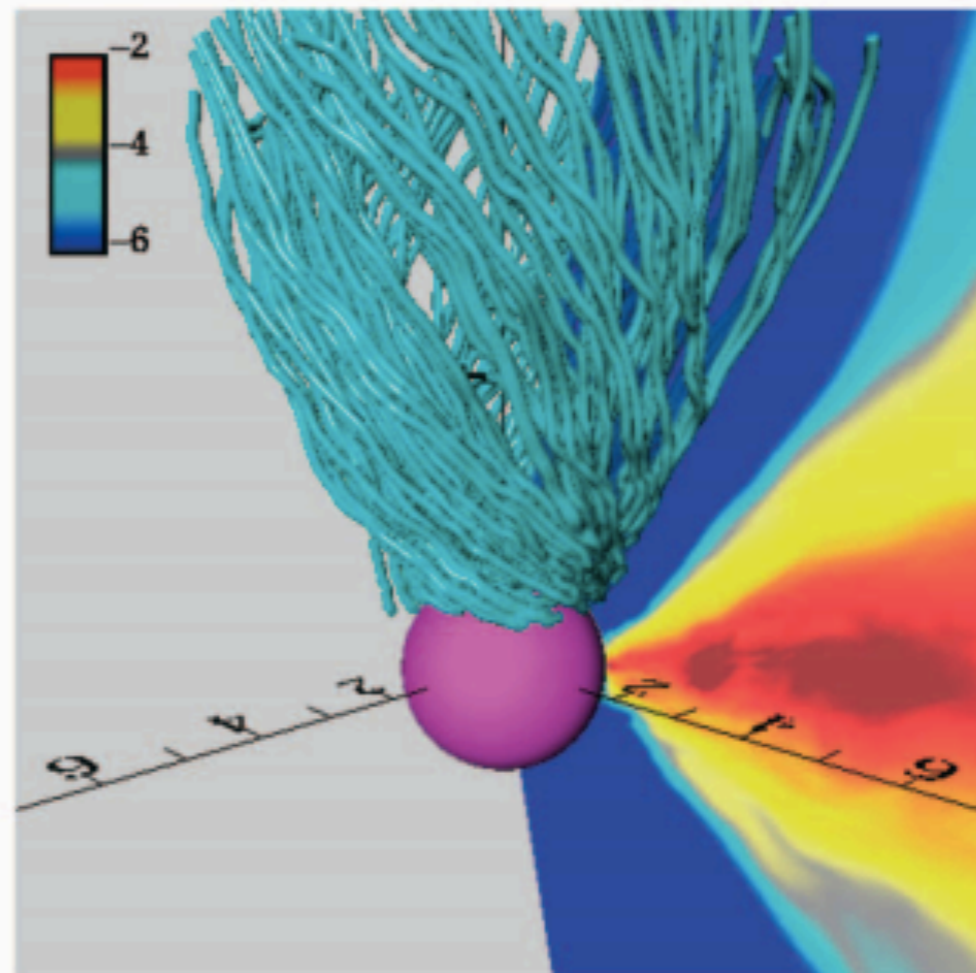
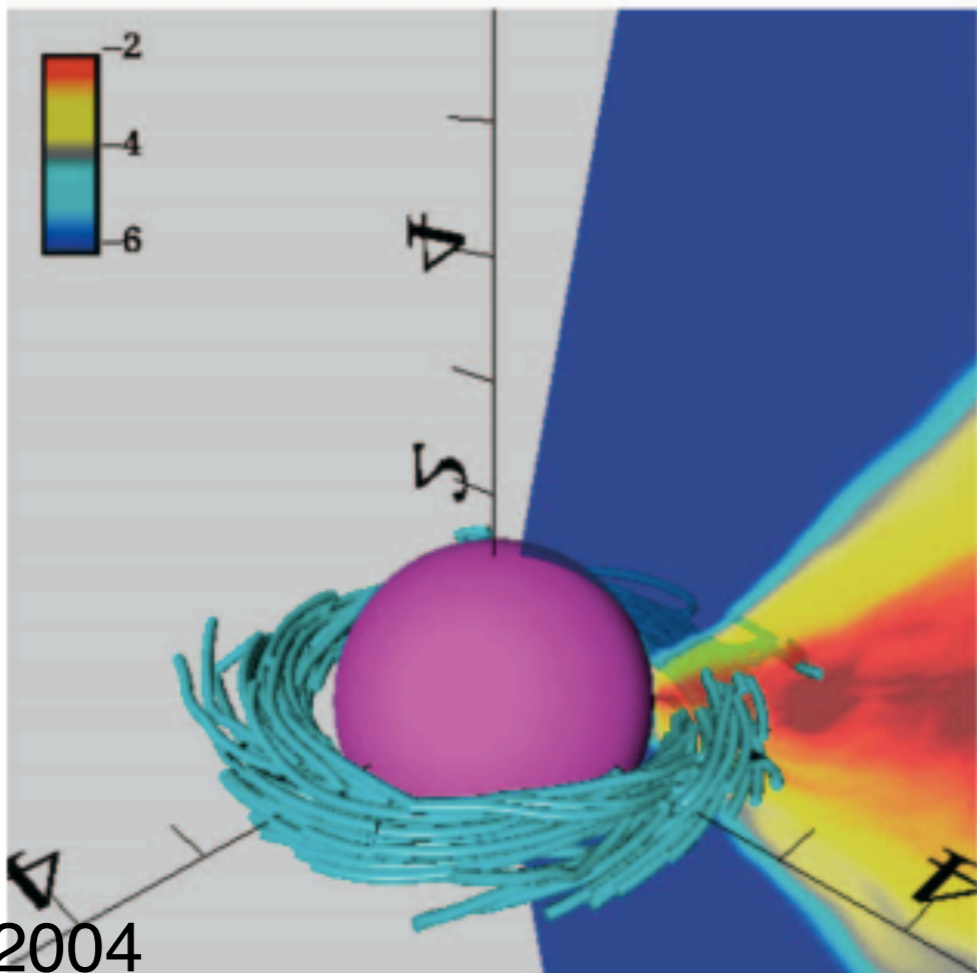
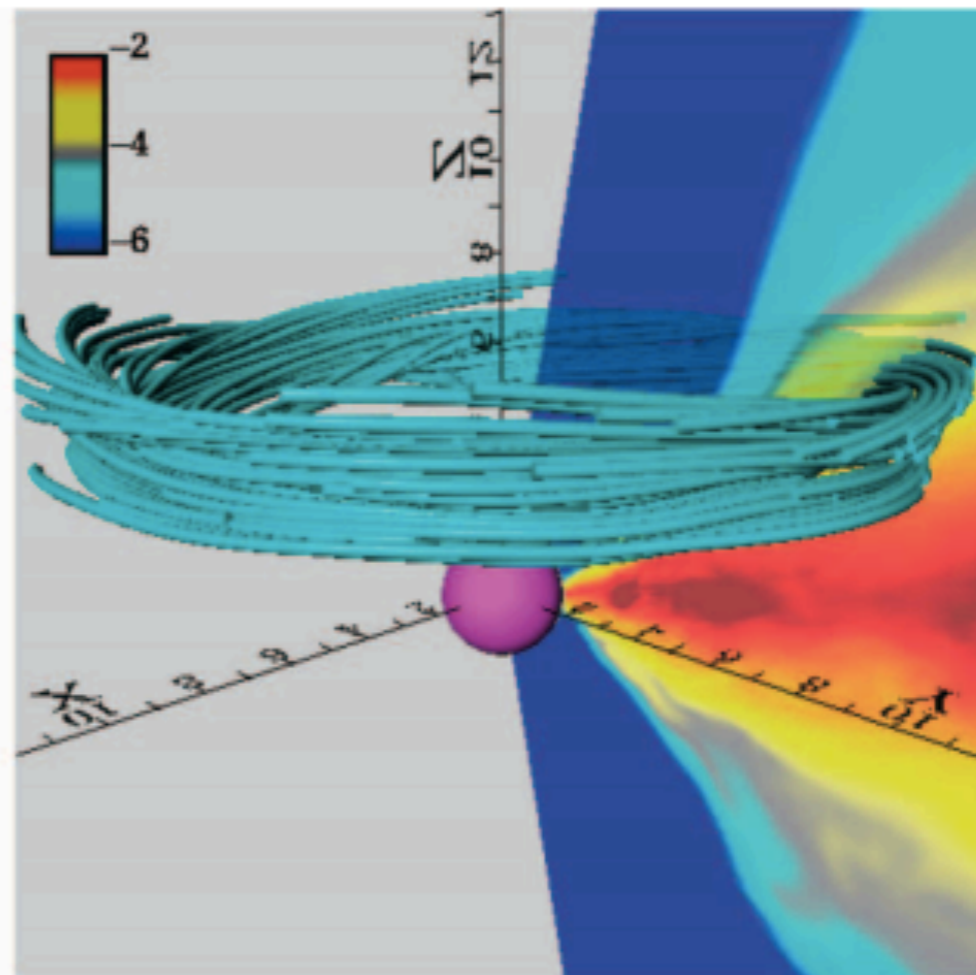
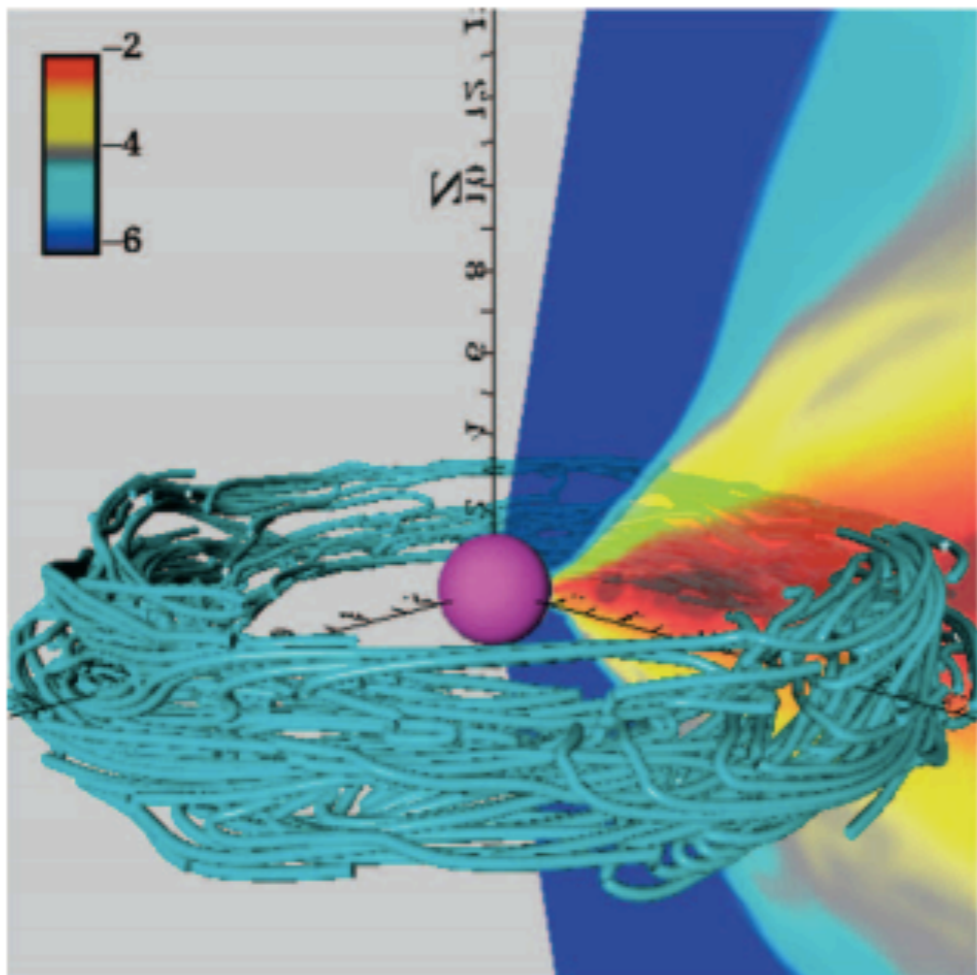


McKinney & Gammie (2004)

VS.



contribution from regions (I) (II) for LLAGN?



Flow acceleration (I)

Hydrodynamics:



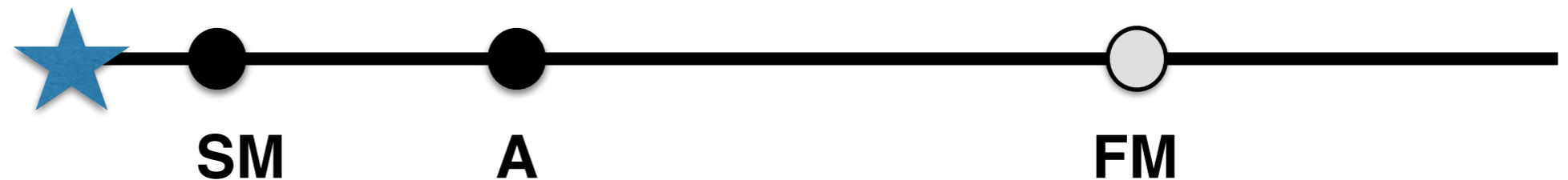
Flow acceleration (II)

A = Alfvén
SM = slow magnetosonic
FM = fast magnetosonic

Hydrodynamics:



MagnetoHydrodynamics (MHD)



Flow acceleration (II)

A = Alfvén

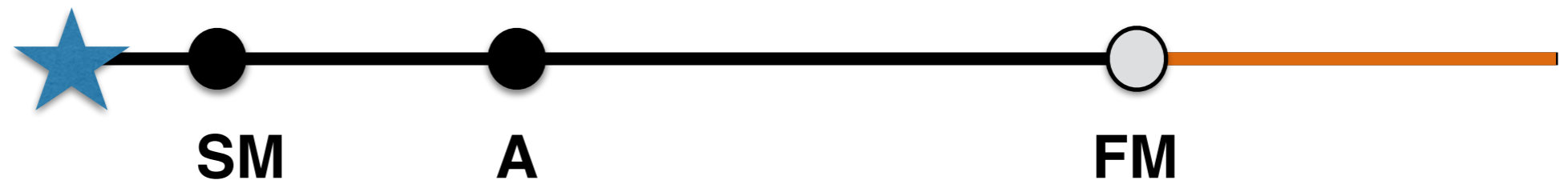
SM = slow magnetosonic

FM = fast magnetosonic

Hydrodynamics:



MagnetoHydrodynamics (MHD)



(acceleration = conversion from EM energy to kinetic energy)

super-fast MHD

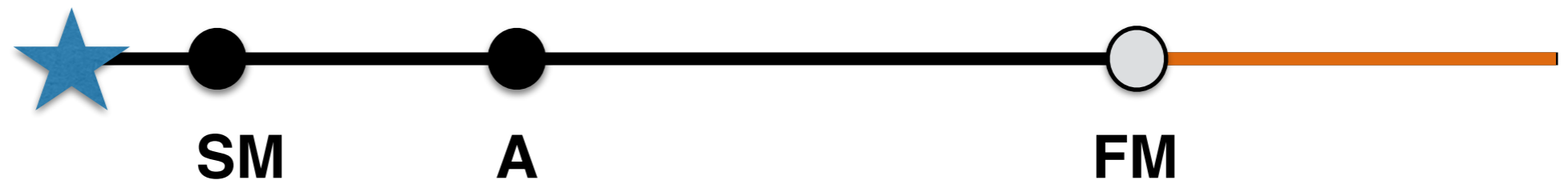
Flow acceleration (III)

A = Alfvén
SM = slow magnetosonic
FM = fast magnetosonic

Hydrodynamics:



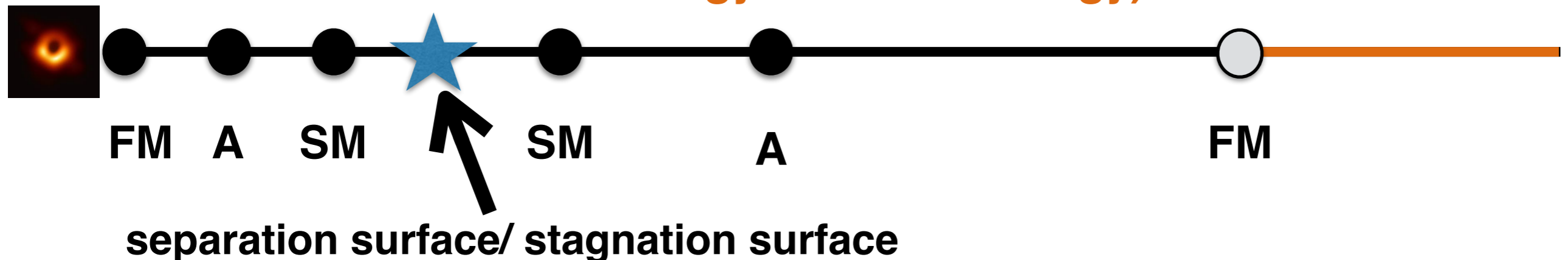
MagnetoHydrodynamics (MHD)



(acceleration = conversion from EM energy to kinetic energy)

super-fast MHD

GRMHD



Flow acceleration (III)

A = Alfvén

SM = slow magnetosonic

FM = fast magnetosonic

$$R\Omega_F \approx 1$$

(inner) light surface

(outer) light surface

$$R\Omega_F < 1$$

$$R\Omega_F > 1$$

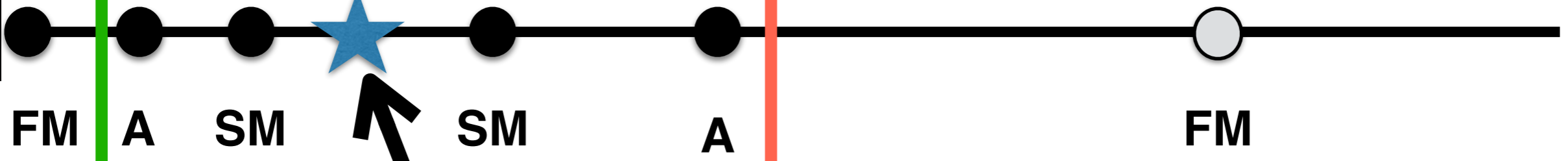
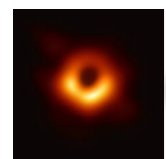
$$B_p > B_\phi$$

$$B_p < B_\phi$$

$$V_p < V_\phi$$

$$V_p > V_\phi$$

GRMHD



FM

A

SM

SM

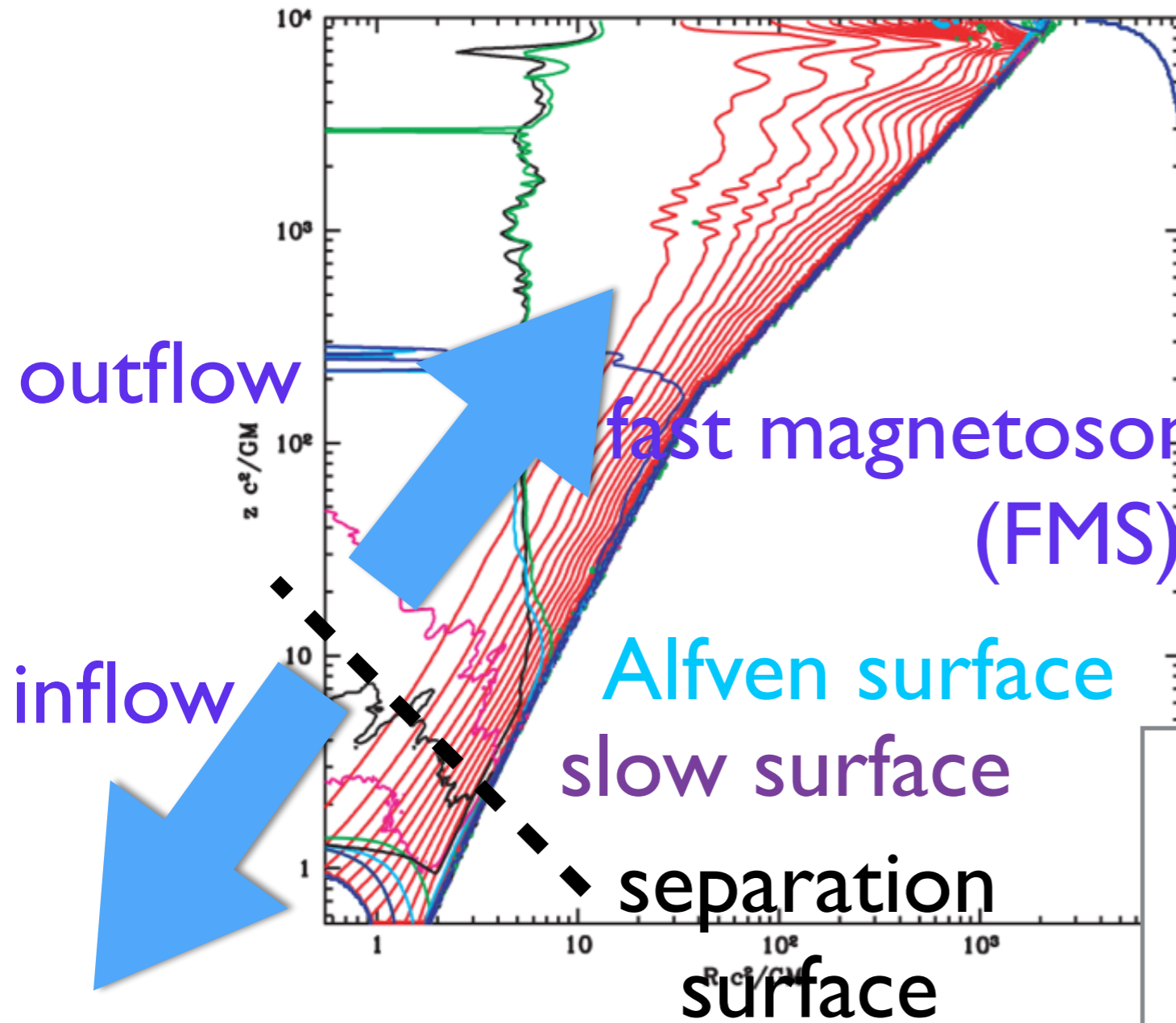
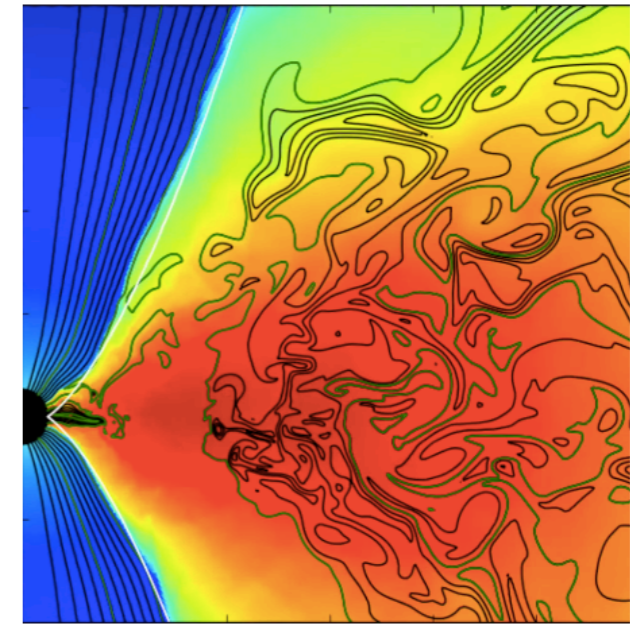
A

FM

separation surface/ stagnation surface

form MHD jet to GRMHD jet

Light Surface



fast magnetosonic surface
(FMS)

Alfven surface

slow surface

separation
surface

$$T^{\mu\nu}_{;\nu} = 0$$

$$T^{\mu\nu} = T^{\mu\nu}_{EM} + T^{\mu\nu}_{Plasma}$$

trans-field:

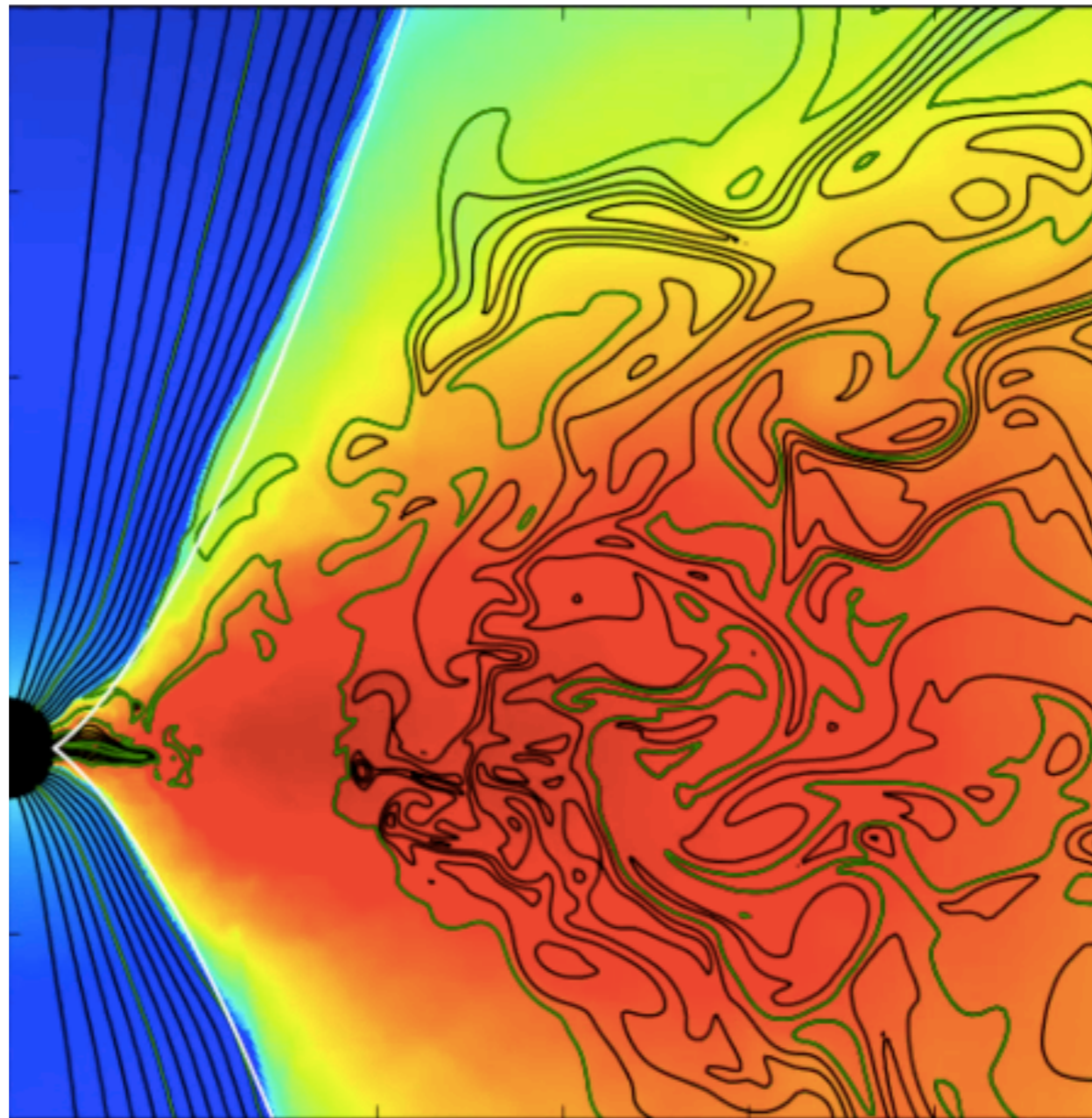
Grad-Shafranov (GS) eqn.

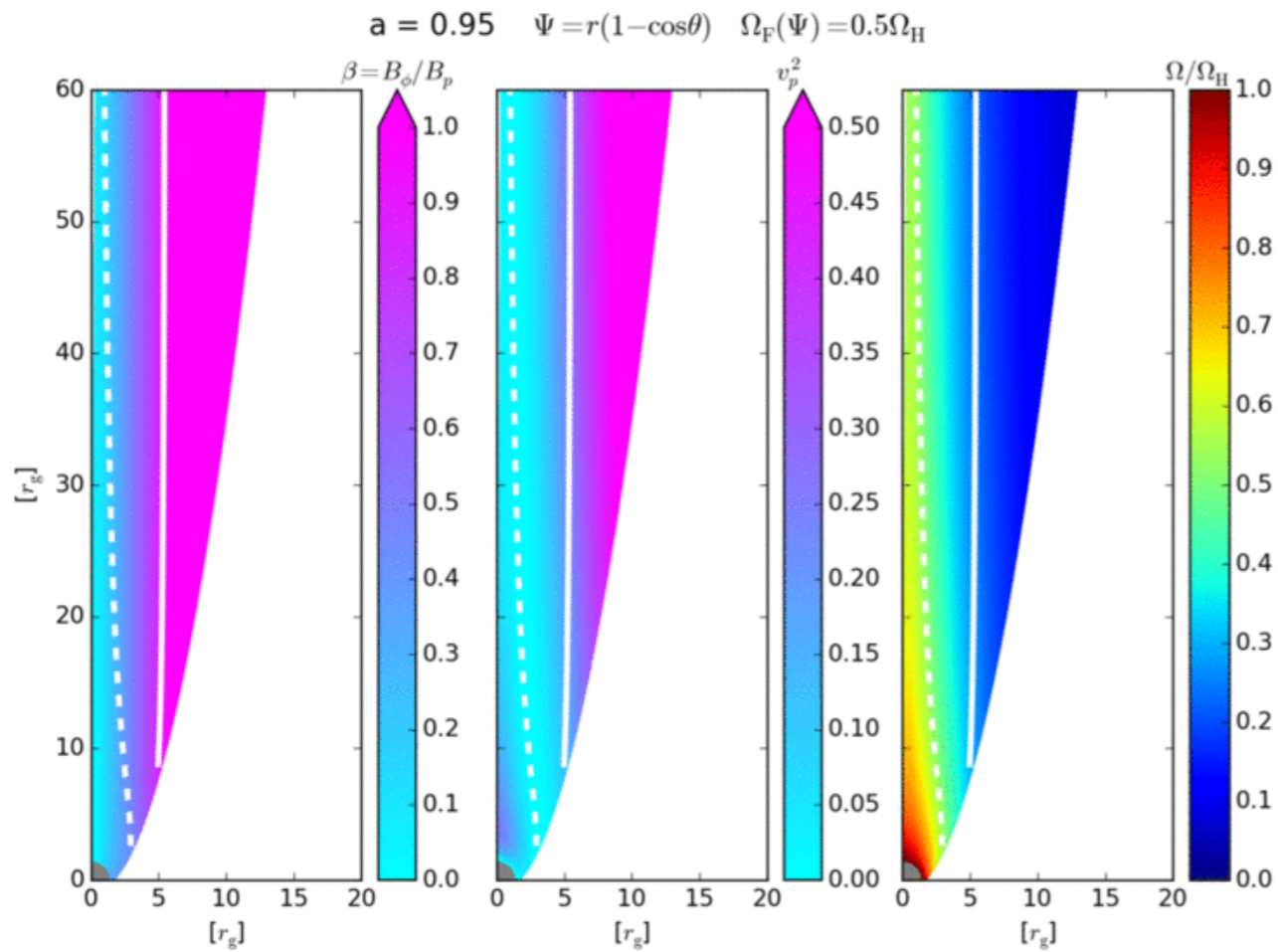
parallel-field:

wind (Bernoulli) eqn.

time-averaged GRMHD simulation result
McKinney 2006

Q: Is it possible to quickly approximate GRMHD flow structure in the funnel region?

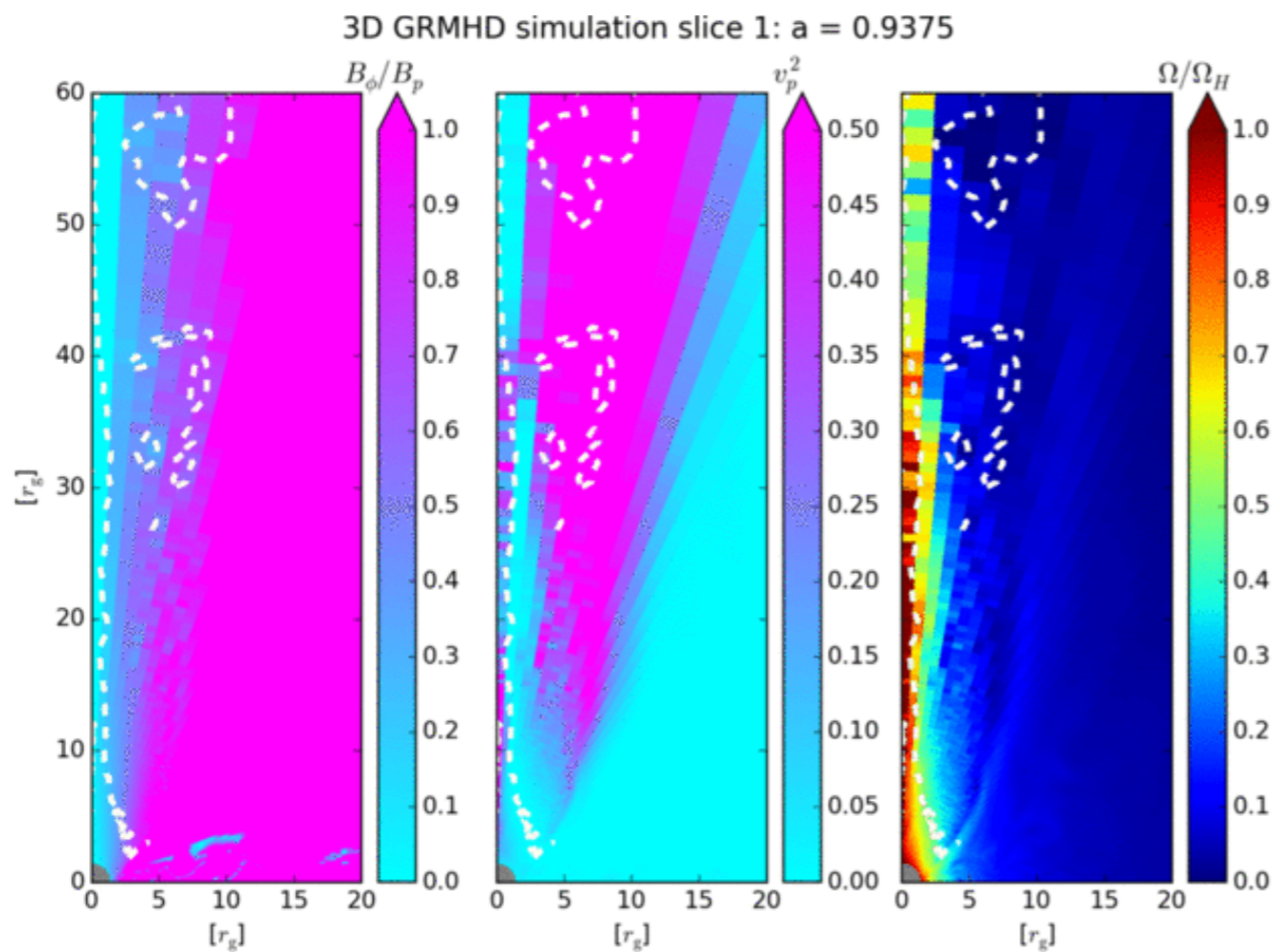




semi-analytical model



Yes!



phi-slides of 3D GRMHD simulation (one snapshot)

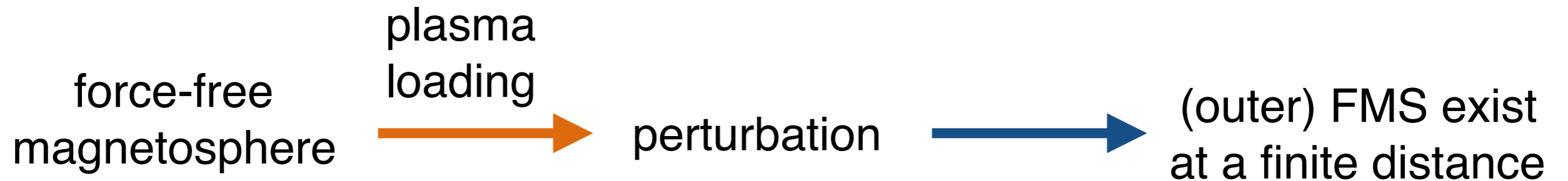


Known MHD features

- **fast-magnetosonic (FMS)** point exist only when the magnetic flux decreases with distance (Takahashi & Shibata 1998, Tchekhovskoy et al. 2009)
- **mass loading + force-free field = FMS surface exist** and efficient MHD flow acceleration take place (Beskin et al. 1998, Beskin & Nokhrina 2006)
- collimation \longleftrightarrow acceleration \longleftrightarrow FMS

solving
Grad-Shafranov (GS) eqn.:
tough or time-consuming

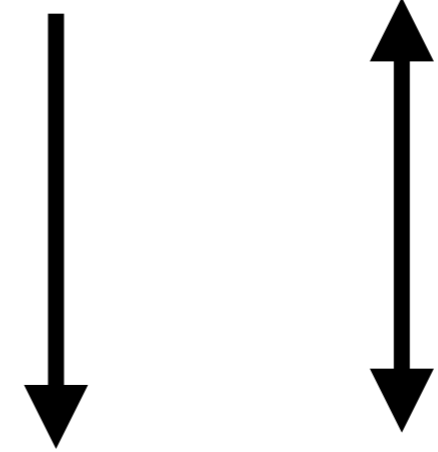
GIVEN a global field geometry, solving
wind eqn.:
easier to handle



GIVEN a global field geometry, solving **wind eqn.:**
easier to handle

$$AM^4 - 2BM^2 + C = 0$$

rewrite the wind eqn as
func of pitch angle, then
analysis the **condition for
the existence of FMS**
(Tomimatsu & Takahashi 2003,
Takahashi & Tomimatsu 2008)



magnetic pitch angle

approx.

force-free
magnetosphere

plasma
loading



perturbation



(outer) FMS exist
at a finite distance



apply constraint for the magnetic pitch angle

approx.

$$\beta(r; \Psi) \equiv \frac{B_\phi}{B_p}$$

Pu & Takahashi 20:
even without iteration, the approximation is good enough

iteration (in principle possible)



approx.

force-free
magnetosphere

plasma
loading



perturbation



(outer) FMS exist
at a finite distance



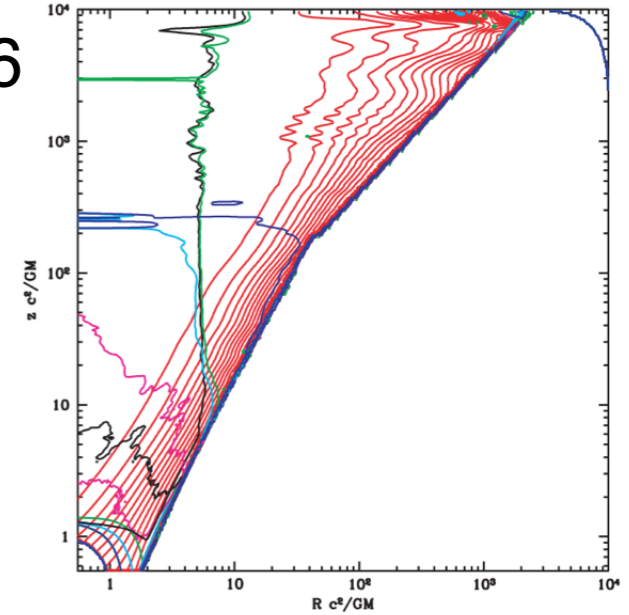
apply constraint for the magnetic pitch angle



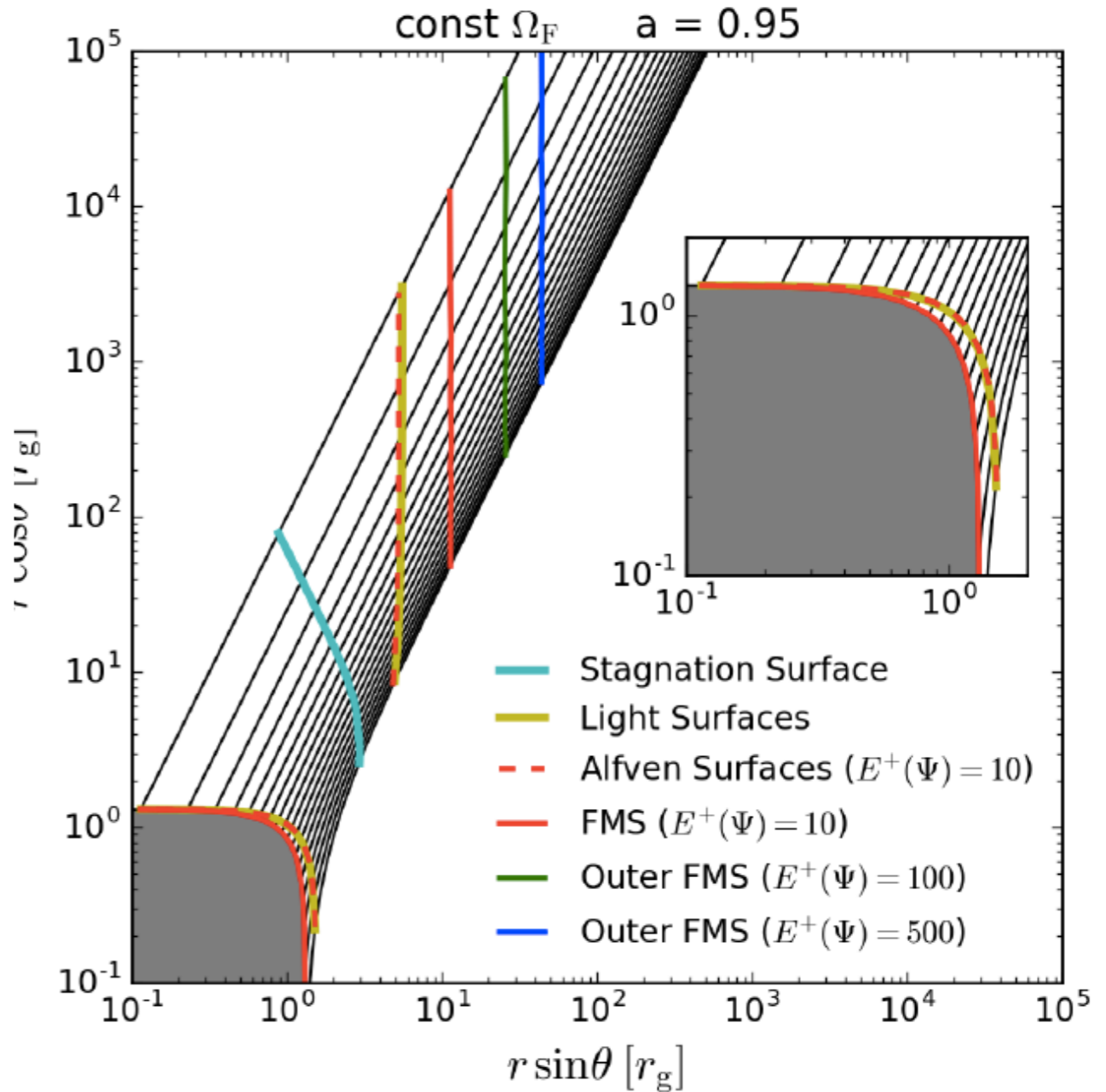
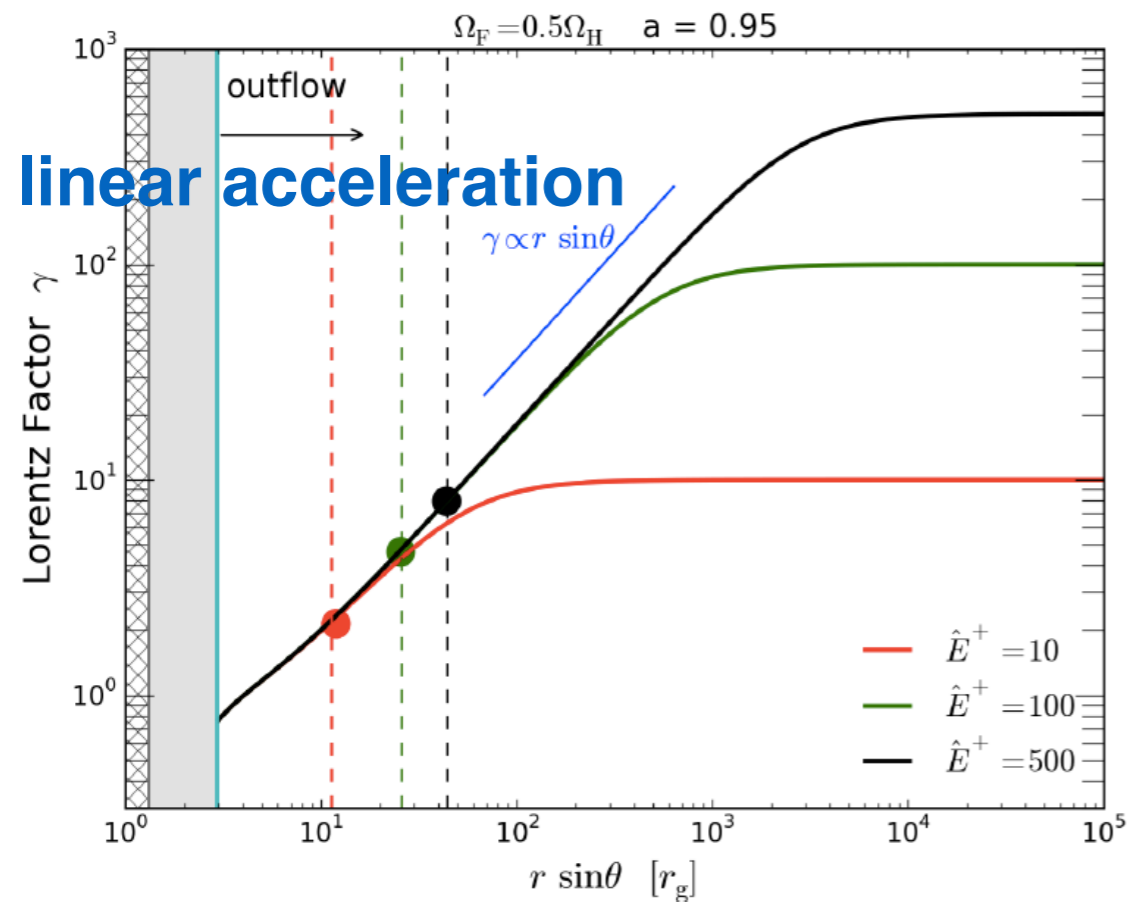
$$\beta(r; \Psi) \equiv \frac{B_\phi}{B_p}$$

approx.

model includes:
 extract BH energy by the inflow → outward
 Poynting flux is smoothly propagate to outflow
 region → jet acceleration



varying jet energy



Pu & Takahashi 20
 (see also Pu+ 13, 15)

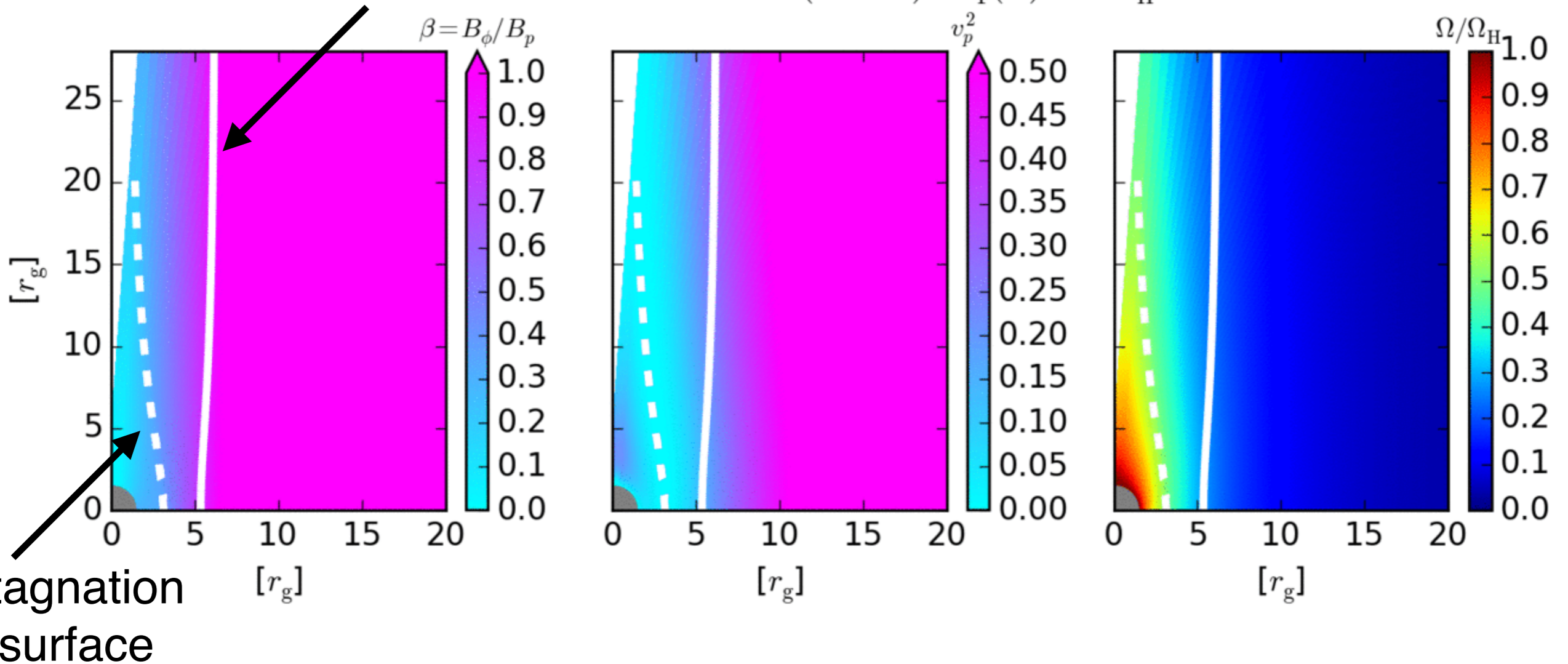
a good approximation of the force-free field (Tchekhovskoy et al. 2008)

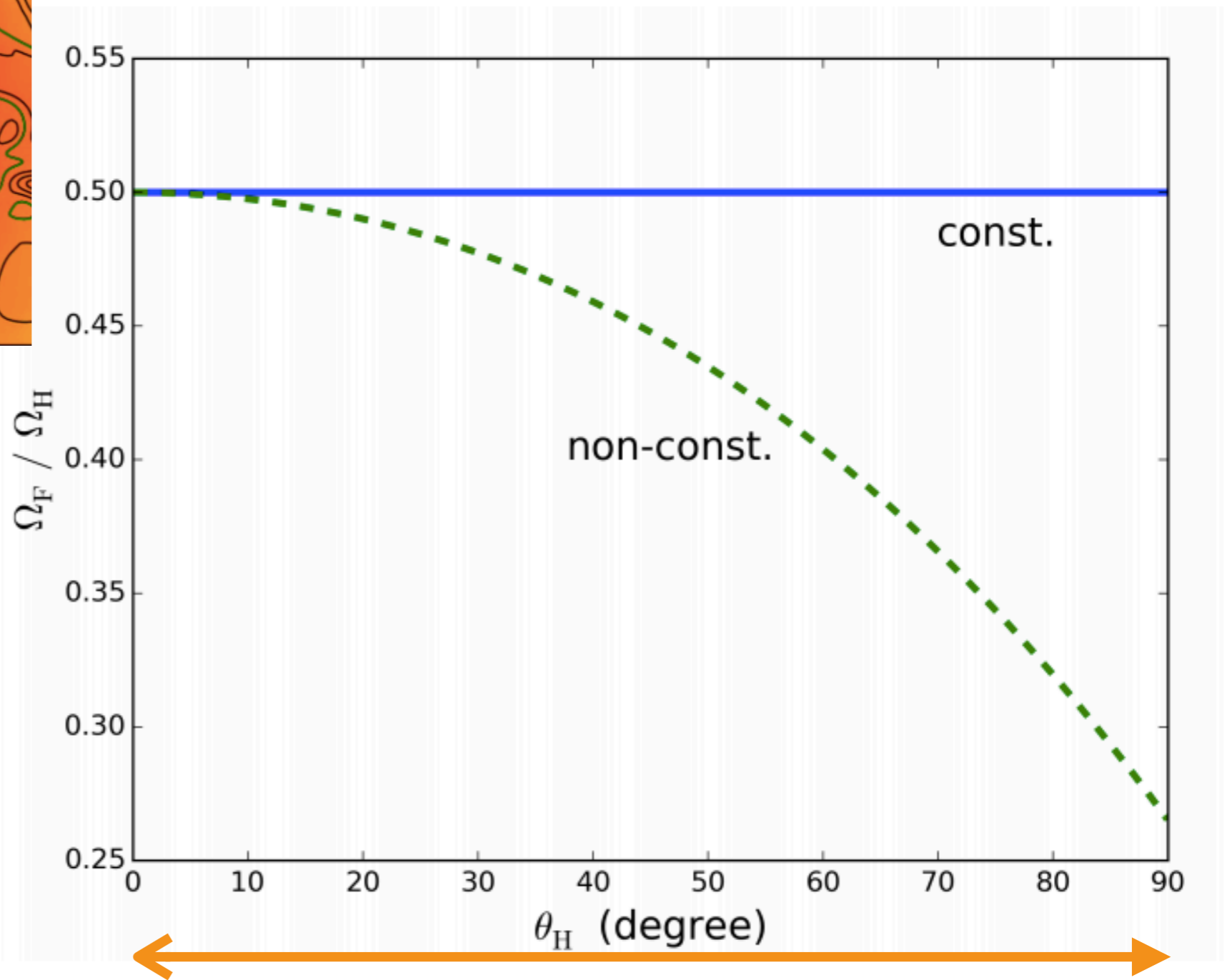
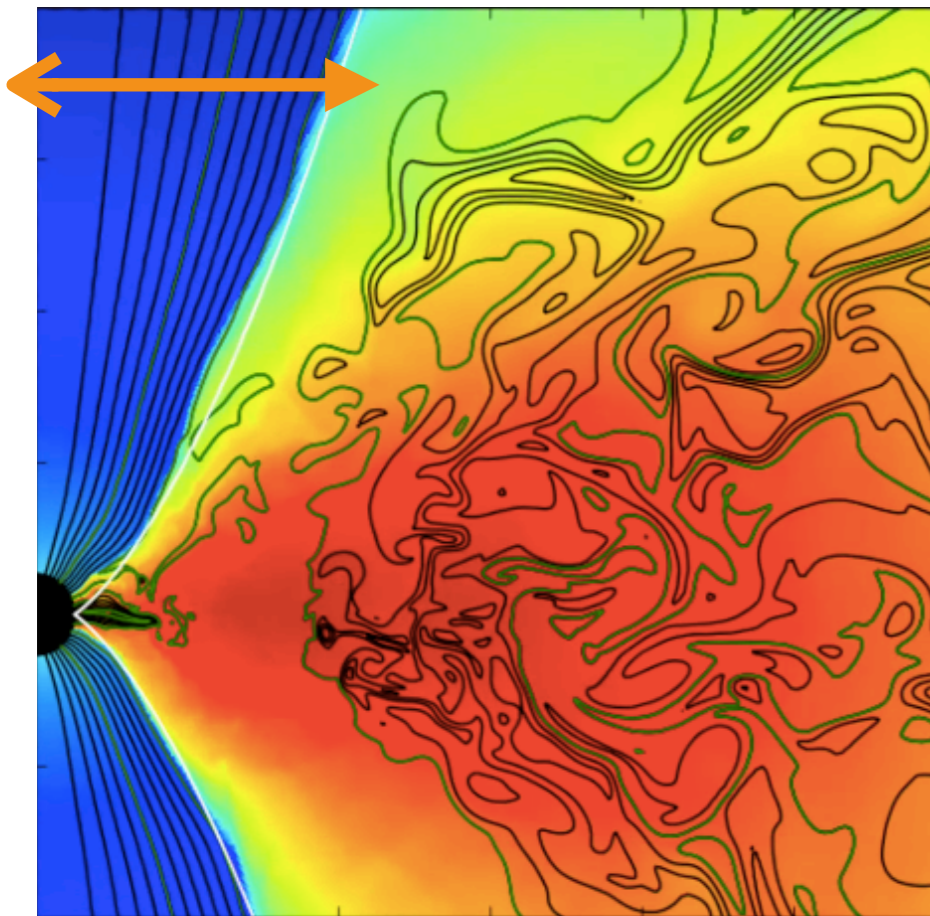
$$\Psi(r, \theta; p) = r^p (1 - \cos \theta)$$

varying field line configuration

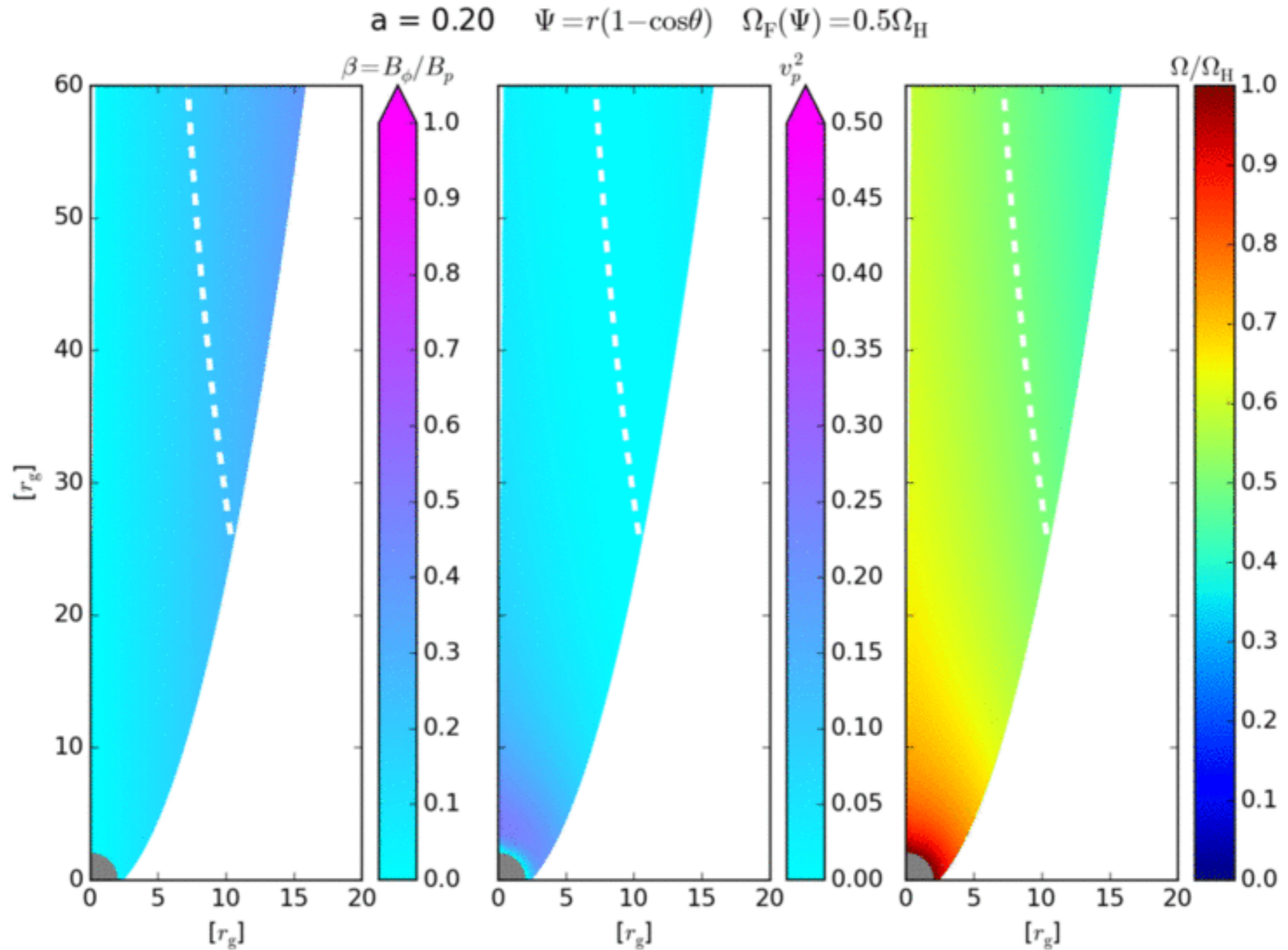
outer light surface

$a = 0.9 \quad \Psi = r^{0.0} (1 - \cos \theta) \quad \Omega_F(\Psi) = 0.5 \Omega_H$



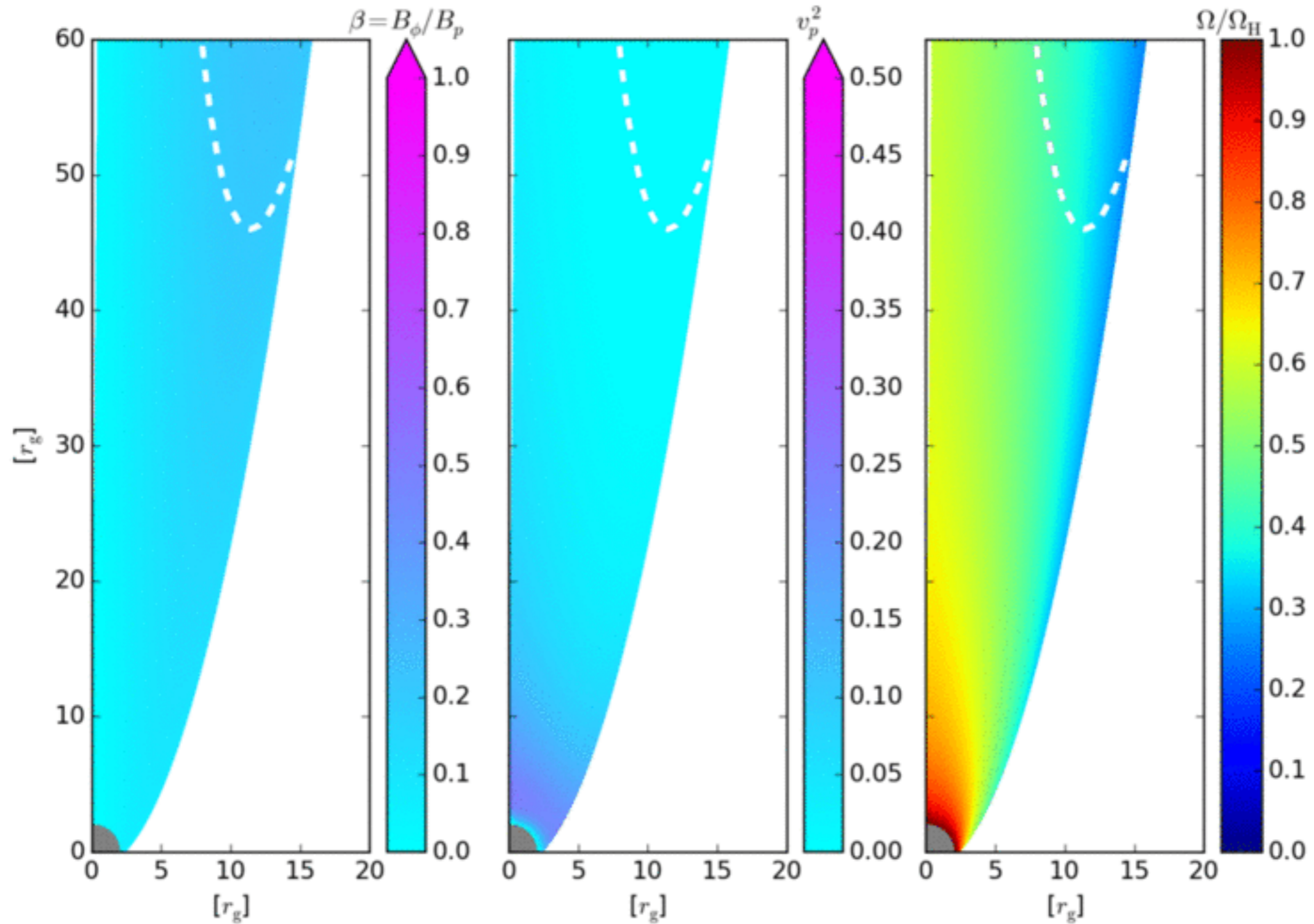


varying BH spin



varying BH spin

$a = 0.20$ $\Psi = r(1 - \cos\theta)$ $\Omega_F(\Psi) = \text{non-constant}$



applying to VLBI observations

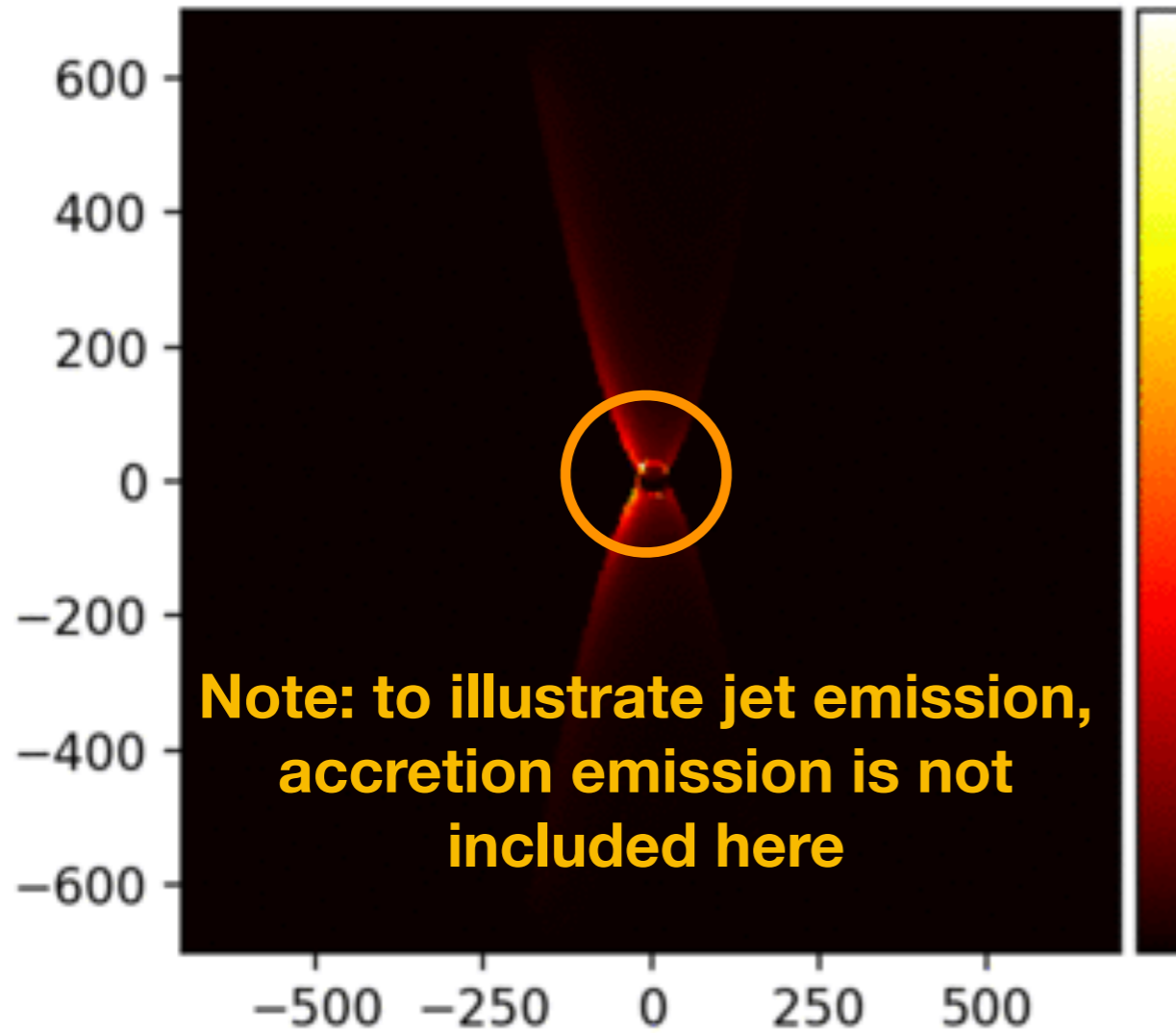
- **dynamics vs. mass-loading**; the later is treated as free parameter, as the injection of non-thermal electrons are unknown (see also **Ogihara's talk**)
- Could the stagnation surface be a possible injection site? (Pu+2018, Kawashima+2020)

applying to VLBI observations

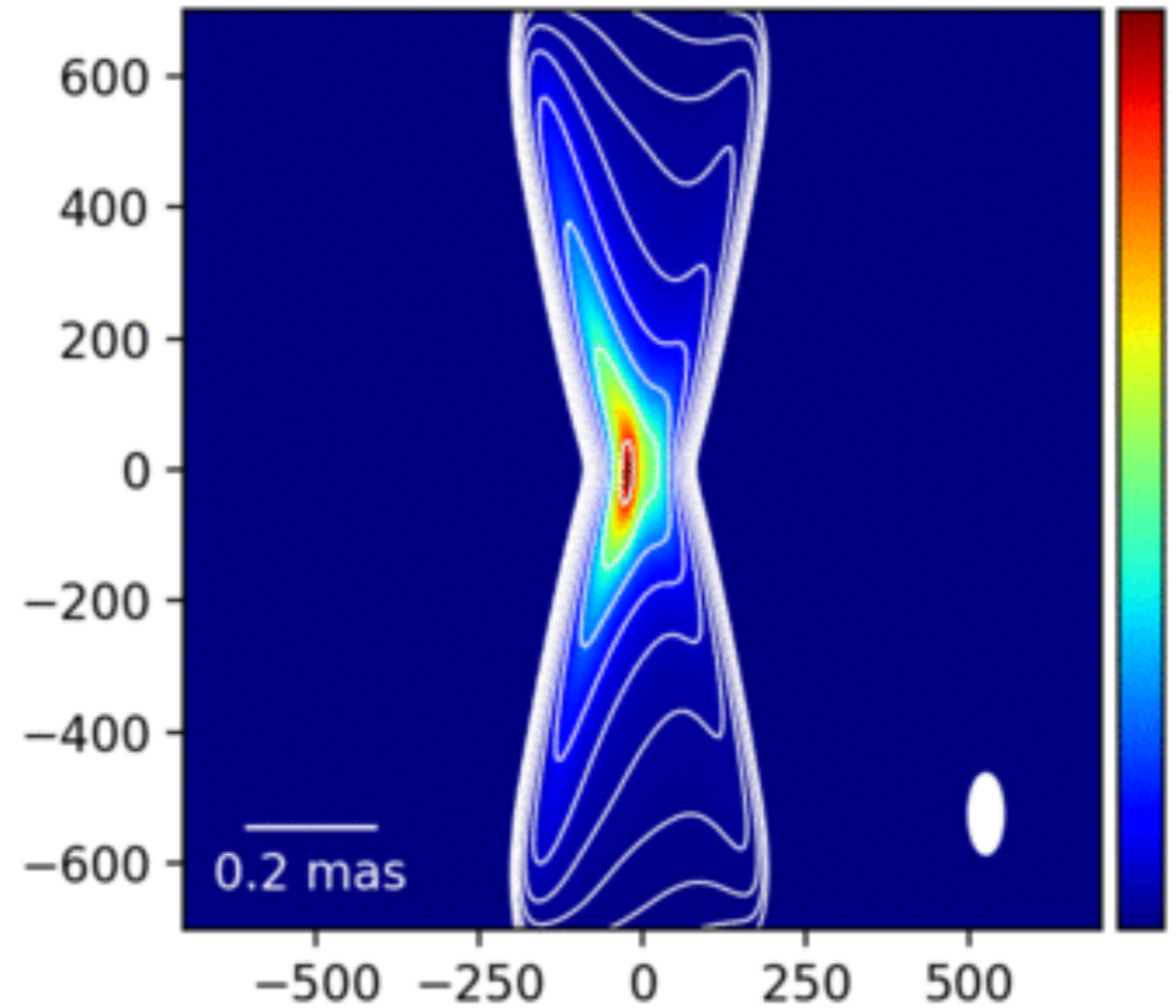
- fiducial parameters for the following movies:
 - adopt similar mass and distance as M87 for LLAGN parameters, $i=60$ degree, beam size: 50×120 uas (GMVA observation for M87, Kim+18)
 - non-thermal electron with a distribution follows
 1. electron number follows power-law decay with distance
 2. more mass loading near the jet edges
 3. emission contributed only from black hole threading field lines

varying black hole spin

86 GHz (jet emission only)

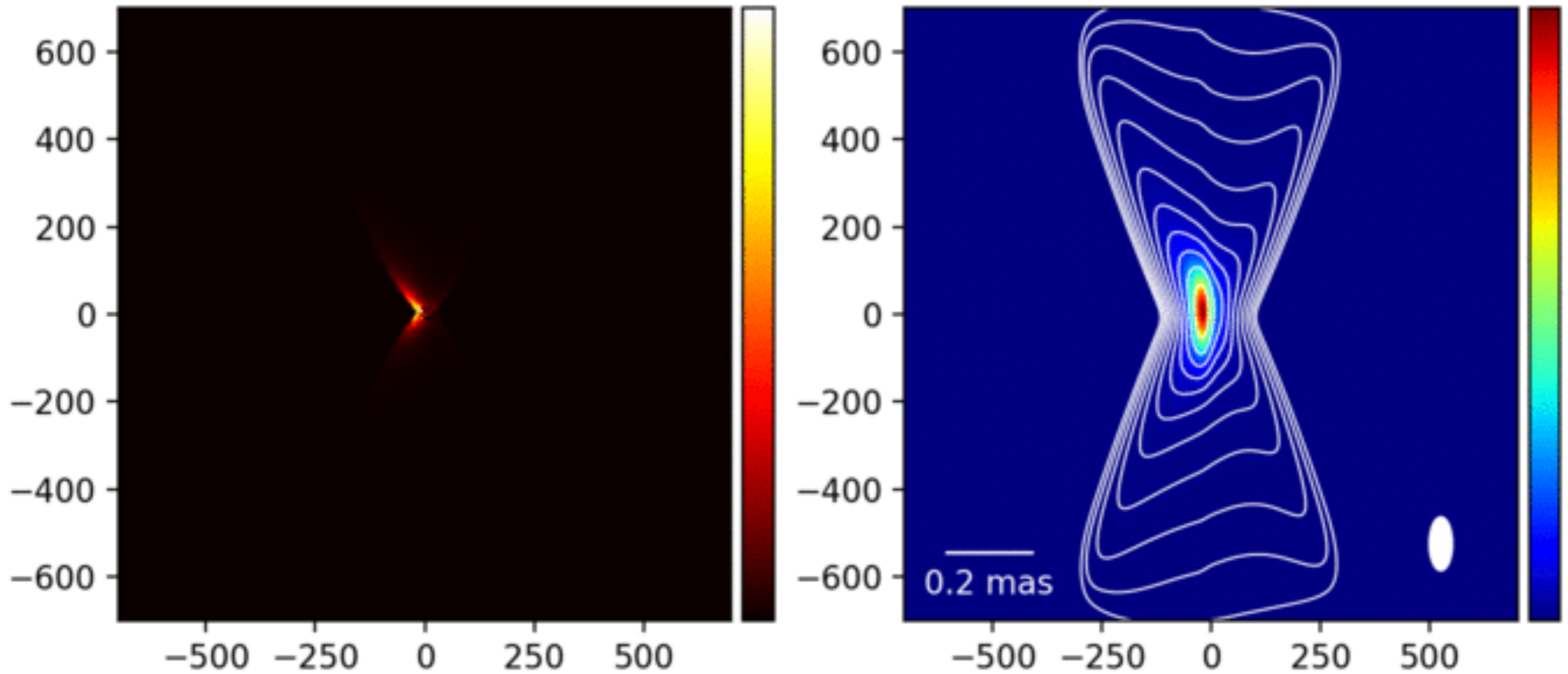


$a = 0.30$

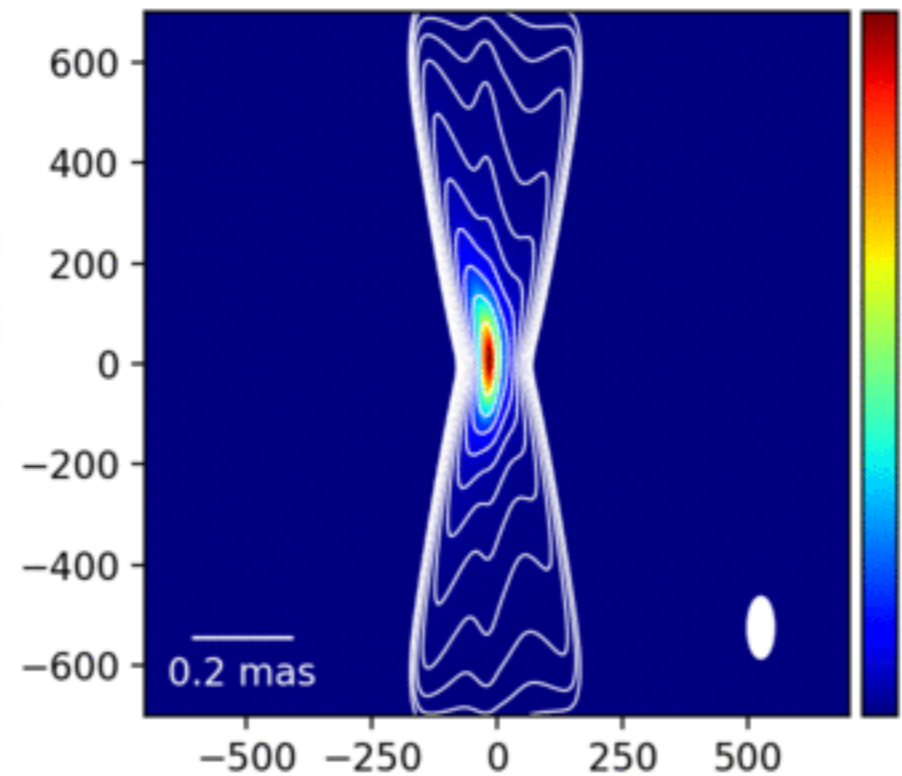
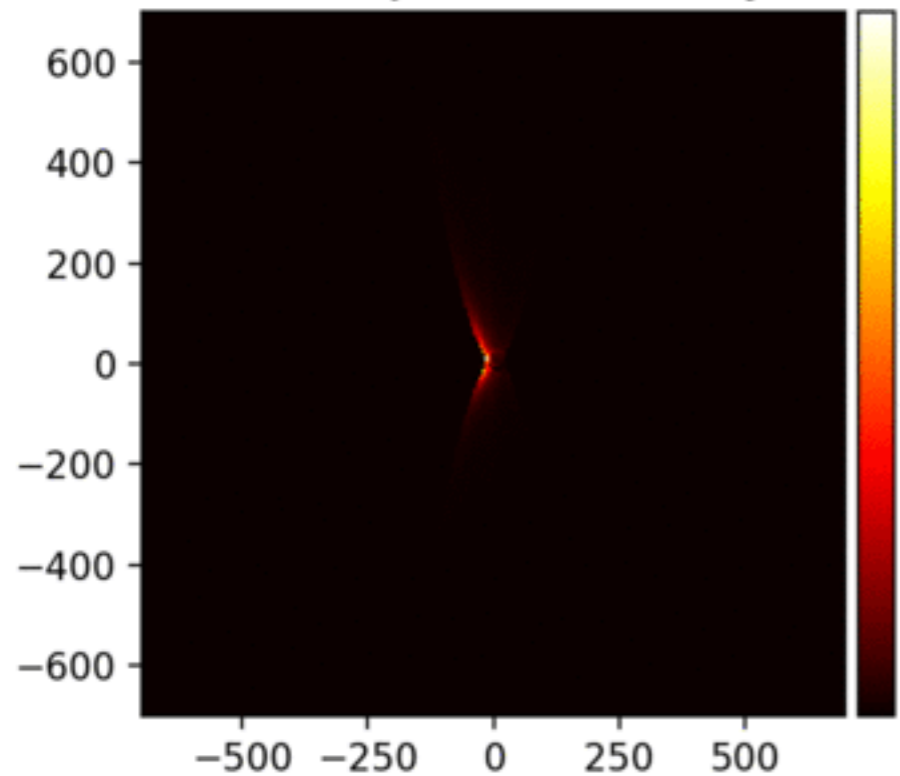


varying jet opening angle

86 GHz (jet emission only)



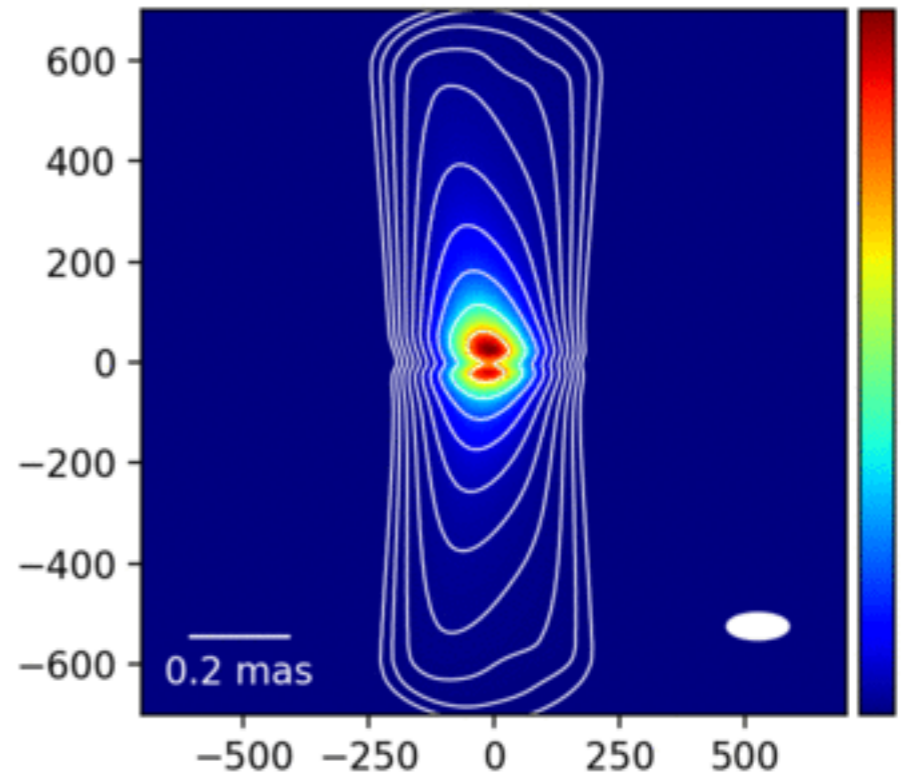
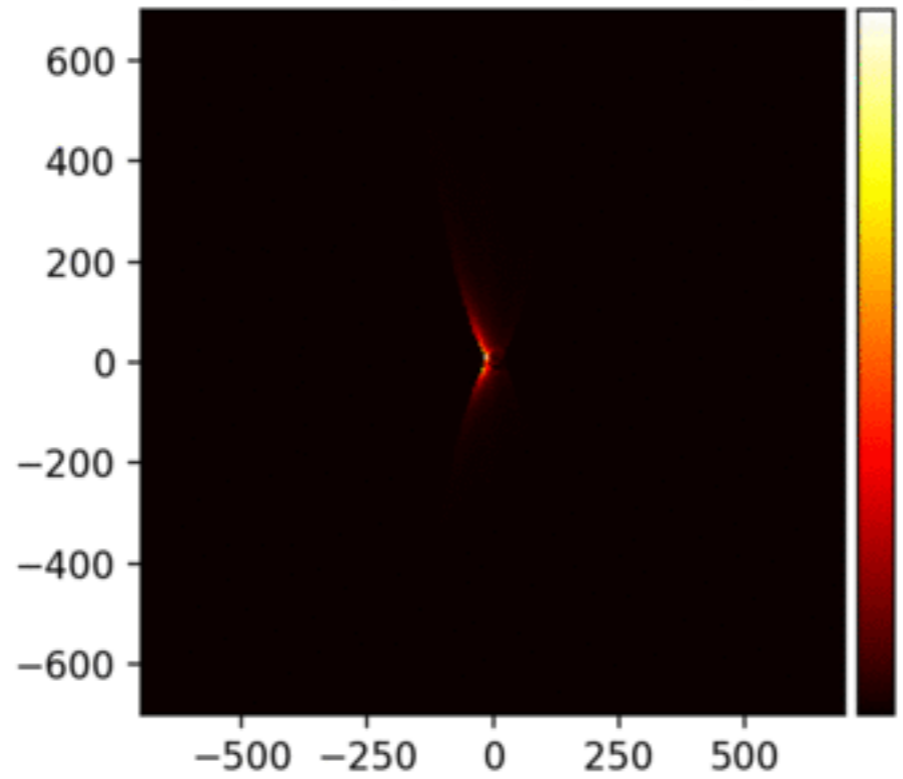
86 GHz (jet emission only)



varying mass loading

86 GHz (jet emission only)

beam direction matters!



summary

- key direction in VLBI: connecting **horizon-scale to large scale** accretion/jet physics
- **challenge**: origin of non-thermal electrons, subsequent cooling, parameter space survey, shock, entrained and other physics beyond horizon scale
- Phenomenological models (with possible implement of new physics) for BH accretion and jet are **ready for help!**

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