Doctoral Thesis

Observation of TeV Gamma-ray from the Active Radio Galaxy Centaurus A with CANGAROO-III Imaging Atmospheric Cherenkov Telescope

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Abstract

We have observed the giant radio galaxy Centaurus A (Cen A) in the TeV energy region and report the result. Cen A is categorized as Seyfert II, Fanaroff Riley I and shows violent activity in various wavelengths. It is believed to have an Active Galactic Nuclei in its center. A jet structure ejecting from the nucleus is clearly observed at radio wavelengths. The viewing angle with respect to its jet axis is estimated to be 60°. This differs from typical TeV AGNs such as Mkn421 and Mkn501, the jets of which were observed at very low angles. Observations of Cen A in TeV gamma-rays are important because AGN theories also predict gamma-ray production of angles as large as 60°. The CANGAROO-III stereoscopic system has been in operation since 2004 and is an array of four Imaging Atmospheric Cherenkov Telescopes (IACT) with about a 100 m spacing. They detect nano-second flashes of Cherenkov light emitted from secondary electrons (and positrons) produced in the electro-magnetic shower initiated by a gamma-ray at the top of the atmosphere. The stereoscopic reconstruction of electro-magnetic showers enables us to precisely measure energies and directions of incident gamma-rays event by event. Furthermore, we adopt a new observation mode, previously used by other groups, in which the ON and OFF source data can be simultaneously taken in the same view. The two important features were not possible in the monoscopic observation. In this thesis we report the first results of stereo observations of CANGAROO-III. The observations were carried out in March and April 2004. In total 20-hour data were obtained from Cen A. No statistically significant gamma-ray signal has been found above 530 GeV, and we obtain an integral flux upper limit of 3.2 ×10^{-12} cm^{-2} sec^{-1} on gamma-ray flux from Cen A above 530 GeV. This upper limit is less than 7 % of the Crab gamma-ray flux. Although some groups reported detections of Cen A in the past, we give upper limits more than one-order of magnitude lower for this object. This result is the world best limit in the TeV energy region.
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Chapter 1

Introduction

High energy particles arriving at the top of the atmosphere are called "cosmic rays". They were discovered by Victor Hess in 1912 [56]. In his balloon flight, Hess brought sealed ionization chambers to the upper atmosphere in order to measure the rate of induced ionization as a function of height and found unknown radiations from the space. Several observations revealed that space is filled with intense cosmic rays. Cosmic rays are observed in the energy range from 1 GeV to $\sim 10^{20}$ eV and they come uniformly from all directions. Integrating the cosmic ray spectrum, the total energy density of cosmic rays is about 2 eV/cm$^3$, which is one order larger than that of star light or the Cosmic Microwave Background (CMB). They are one of the most energetic phenomena in our Galaxy. Cosmic rays consist of 98% proton (hydrogen) and nuclei, and the other 2% of electron. Their rate at the top of the atmosphere is about 1000 particles per square meter per second. Although cosmic rays have been observed by many detectors, it is difficult to show where the acceleration sites are and which acceleration mechanism is dominant, because cosmic rays with electric charges are deflected in direction by the interstellar magnetic field during the propagation from the sources to the Earth.

The candidates of the origin of cosmic rays are thought to be Supernova remnants (SNRs), pulsars in our Galaxy, and active galactic nuclei (AGN) from outside. The best way to search for the acceleration site of cosmic rays is to detect neutral particles, such as gamma-rays and neutrinos, generated in interactions of high-energy cosmic rays with ambient matter, the cosmic microwave background radiation and/or the interstellar magnetic field. Neutrinos, however, are difficult to detect because of their weak interac-
tions. Therefore at present gamma-rays are the best probe to find the acceleration sites of cosmic rays [92] [55].

If cosmic rays of energy less than 100 TeV produce gamma-rays via known fundamental interactions, it should result in gamma-rays in the 1-10 TeV region, which cannot be produced in any thermal equilibrium. Observations of TeV gamma-rays from AGNs are one of the key items to be explored for understanding the origin of cosmic rays. Astrophysics of relativistic jets in AGN essentially rely on gamma-ray observations in the MeV, GeV, and TeV energy regions. The relativistic energy is the key. The radio and X-ray data are also important but their fluxes are significantly dependent on the ambient magnetic field strength which was not yet well determined accurately, due to synchrotron processes.

Many AGNs were reported as TeV gamma-ray sources. First of all, the BL Lac population of blazars of the four objects, Mkn421, Mkn501, 1ES1959+650, and 1ES1426+428 are established as TeV gamma-ray emitters by independent observations of several groups. Since the discovery of Mkn421 as a TeV source by the Whipple group [100], these objects have been extensively studied through multi-wavelength observations. Mkn421 is a time-variable source with typical average TeV flux between 30% to 50% of that of the Crab Nebula. The time variation is as rapid as 0.5h, called “flares”[45, 3]. Two more BL Lac objects were also detected. They are 1ES2344+51 and PKS2155-304, as reported by the Whipple [23] and Durham [25] groups, respectively, and H.E.S.S. also recently detected gamma-rays from PKS2155-304 [6]. Although the TeV blazars were not initially predicted as TeV emitters, now it has become clear and an accepted fact that such intense TeV fluxes from these distant objects are the products of the relativistic bulk motions of jets close to the line of sight of the observers, resulting in Lorenz boosts of the flux by several orders of magnitude. Isotropically radiating sources of similar intrinsic energy are difficult to image. On the other hand, the technical requirements for gamma-ray detections from sources within 10 Mpc become quite modest. One may expect detectable gamma-ray fluxes from some nearby galaxies. The radio galaxies such as Centaurus A (Cen A) and M87, as well as the star-burst galaxies M82 and NGC253 should be detectable TeV source candidates, basically because of their high luminosities shown by the lower energy measurements. Although the first positive measurements of TeV gamma-rays from Cen A
was reported in the 1970s [53], many later attempts have failed. Reliable measurements in the TeV region were awaited. Recently the technology of the Imaging Atmospheric Cherenkov Telescopes (IACTs) was established as follows. The Whipple group started practical steps in the direction of improving the sensitivity of Cherenkov telescopes by implementing the imaging technique. The idea was that the analysis of the angular distribution of Cherenkov radiation of the air showers should allow significant reduction of cosmic ray background. Hillas [57] clearly demonstrated that indeed the analysis of the second moments of the Cherenkov images of air showers should be able to discriminate between the gamma-ray and proton induced showers, and thus to improve significantly the signal-to-noise ratio. The exploitation of this technique by the Whipple telescope resulted in the first high-confidence (9 \( \sigma \)) detection of TeV gamma-rays from the Crab Nebula [127]. The Whipple group also detected gamma-rays from AGNs, Mkn421 [100] and Mkn501 [101].

The construction and operation of multi-10m-scale reflectors has now been realized by VERITIES, HEGRA, CANGAROO-III, and H.E.S.S. The parameters of the four new generation telescopes are summarized in Table 1.1. The CANGAROO-III is one of two

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<td>Namibia</td>
<td>Kitt Peak</td>
<td>La Palma</td>
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<td>23°S, 15°E</td>
<td>32°N, 111°W</td>
<td>28°N, 17°W</td>
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<td>Mirror dishes</td>
<td>57.3 m²</td>
<td>108 m²</td>
<td>100 m²</td>
<td>234 m²</td>
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<tr>
<td>Diameter</td>
<td>10 m</td>
<td>12 m</td>
<td>12 m</td>
<td>17 m</td>
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<td>F</td>
<td>0.77</td>
<td>0.8</td>
<td>1.2</td>
<td>1.03</td>
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<td>Mirrors</td>
<td>114 × 80 cm</td>
<td>380 × 60 cm</td>
<td>350 × 60 cm</td>
<td>956 × 50 cm square</td>
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<td>Cameras</td>
<td>427 pixels</td>
<td>960 pixels</td>
<td>499 pixels</td>
<td>inner 397, outer 180</td>
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<td>Pixel size</td>
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<td>0.16°</td>
<td>0.15°</td>
<td>inner 0.1°, outer 0.2°</td>
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<td>4°</td>
<td>5°</td>
<td>3.5°</td>
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**Table 1.1**: Summary of IACT in the world.

which cover the southern hemisphere where Cen A can be observed near zenith. Here
we report the establishment of more than one order better sensitivity for gamma-rays compared to the previous instruments. With those, we can verify the cosmic ray property in the nearby galaxy Cen A.

The purposes of this thesis are to establish stereo-analysis for imaging Cherenkov telescope and to present observation results of the active galactic nuclei Cen A with the CANGAROO-III Imaging Atmospheric Cherenkov Telescope system which is the one of major next generation telescopes.

This thesis is organized as follows. In Chapter 2, active galactic nuclei and relativistic processes are reviewed. In Chapter 3, a theoretical review of the acceleration of cosmic rays is introduced. In Chapter 4, the production mechanisms of gamma rays is reviewed. In Chapter 5, the technique of TeV gamma-ray detection is explained. In Chapter 6, the details of the CANGAROO-III telescope are described. My contribution which is development of the camera and HV system are described in this chapter. Chapter 7 summarizes observations of Cen A and the Crab. The calibration procedure can be found in Chapter 8. I wrote the calibration program for the group. The Monte-Carlo simulation is described in Chapter 9. The analysis method of CANGAROO-III is described in Chapters 10-12, and in Chapters 13 and 14, the discussions and the conclusions are described, respectively.
Chapter 2

Review of Active Galactic Nuclei and Centaurus A

2.1 Active Galactic Nuclei

Active galactic nuclei (AGN) represent a large population of compact extragalactic objects characterized by extremely luminous electromagnetic radiation produced in very compact volumes. An image of an AGN is shown in Fig 2.1. The formation of an AGN likely proceeds as follows.

Long ago when a galaxy was cosmologically young, star collisions and mergers occurred, resulting a single massive black hole (MBH) with perhaps $10^6$-$10^{10}$ M☉ in the center. Gas from the galaxy’s interstellar medium, from a cannibalized galaxy, or from a star that strays too close, falls onto the MBH. As in X-ray binary star systems, an accretion disk forms, emitting huge amounts of energy across the electromagnetic spectrum (infrared to gamma-rays). The MBH plus accretion disk produces the various phenomena seen in AGN.

Although this source population consists of several classes of galaxies with substantially different characteristics, the current classification schemes are characterized by random pointing directions of the jet axes rather than by intrinsically different physical properties [123], The currently most popular picture of physical structure of AGN is illustrated in Fig 2.3 (unified AGN model).

In 10 % of the AGN, the MBH + accretion disk produce narrow beams of energetic
Figure 2.1: A stylized image of an AGN [133]. A MBH is located in the center, and an accretion disk surrounds it. Jets which are observed in the center emerge at nearly the speed of light. Sometimes the jet speed apparently exceeds the speed of light, an effect known as "superluminal motion". The physical phenomenon peculiar to a jet is observed.

particles and magnetic fields, and eject them outward in opposite directions away from the disk. These make the radio jets, which emerge at nearly the speed of light. Sometimes the jet speed apparently exceeds the speed of light, an effect known as "superluminal motion". Jet phenomena are described in the following section.

2.2 Superluminal Motion of Jet

“Superluminal motion” describes proper motion of a source structure (traditionally mapped at radio wavelength) that, when converted to an apparent speed $v_{\text{app}}$, gives $v_{\text{app}} > c$, as shown in Fig 2.2. The phenomenon occurs for emitting regions moving at very high speeds (close to c) at small angle to the line of sight [103]. Relativistically moving sources “run after” the photons they emit, strongly reducing the time interval separating any two events in the observer’s frame and giving the impression of faster than light motion.

In the relativistic beaming model, the observed transverse velocity of an emitting knot, $v_{\text{app}} = \beta_{\text{app}} c$ is related to its true velocity $v = \beta c$, and the angle to the line of sight, $\theta$,
by
\[ \beta_{\text{app}} = \frac{\beta \sin \theta}{1 - \beta \cos \theta}, \] 
resulting in superluminal motion as \( \beta_{\text{app}} > 1 \). When the viewing angle is such that \( \cos \theta = v/c \), the apparent speed maximized and this value is \( \gamma v \).

\[ \text{Figure 2.2: Schematic view of superluminal motion. Knots are emitted at angle } \theta \text{ to the line of sight [102].} \]

### 2.3 Unified AGN Model

Many AGN are observed at various wavelengths and have various features. AGN are classified into roughly four categories and we explain each category in the following.

**Seyfert galaxy:** The galaxies in this category were first listed by Seyfert in 1943. Many strong emission lines are observed in their spectra. They are further classified into two types (Seyfert I and Seyfert II) owing to their emission line profile. Most of Seyfert galaxies are classified morphologically as spirals.

**Blazar:** Compared with Seyfert galaxies, no remarkable emission lines are observed in their spectrum, and their continuum emission is highly polarized. Jets are thought to be ejected from blazars to the direction of the earth. BL Lac objects and OVV (Optically Violent Variable) Quasars belong to this category.

**Radio galaxy:** Most of elliptical galaxies are not observable at radio wavelengths, but a few of them are strong sources in radio. They are called radio galaxies. There are two kinds of radio galaxy; type I called Broad Line Radio Galaxy (BLRG), and type II called Narrow Line Radio Galaxy (NLRG). Furthermore, NLRG are classified into
Fanaroff-Riley (FR) type I in which the optical intensity is high and FR type II in which the optical intensity is low [40].

**Quasar:** The spectral profile of quasars is similar to Seyfert type I galaxies, but quasars have very high redshifts, up to $z \sim 4$. Some of them are also bright at radio wavelengths (radio loud quasar).

Thus, there are a number of AGN classes. However, although such a diversity reflects the individuality of AGN, it should be unified. A good example is the difference between Seyfert type I and Seyfert type II. Type I shows both broad line, and narrow line emission, and type II shows only narrow line emission. However, as a result of polarization measurements, it became clear that broad line emission also exists in type II. Since the light is polarized, it is thought that this line is dispersed by some medium.

So, in the Seyfert type II, although there is BLRG-like emission, the photons have been observed. For Seyfert type I and Seyfert type II, the observed spectrum differences can be explained by different viewing angles with respect to the center of the AGN. To conclude, they are completely the same AGNs.

The summary of these AGN classification is shown in Table 2.1 and Fig 2.3.

<table>
<thead>
<tr>
<th></th>
<th>Angle to the center: small</th>
<th>Angle to the center: large</th>
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</tr>
<tr>
<td>radio intensity:</td>
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<tr>
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<tr>
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<td>radio intensity:</td>
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<td></td>
</tr>
<tr>
<td>high</td>
<td>RLQ</td>
<td>FR type 2</td>
</tr>
</tbody>
</table>

*Table 2.1: Summary of the AGN classification*
2.4. SPECTRAL SHAPE OF THE BLAZAR

One of the features of the spectrum of blazar objects is shown in Fig 2.4 [42]. The observed 126 blazars were divided into five groups in total radiation intensity, and the average spectrum was plotted for every group. Generally, spectral shapes of Blazars have a form with double peaks. It is one of the reason that emission mechanism of gamma-rays is considered to be the synchrotron self-Compton (SSC) model (later described in the section 4.6). Assuming the SSC model, the maximum acceleration energy of electron ($\gamma_{\text{max}}$) is given as,

$$\gamma_{\text{max}} \propto (U_B + U_{\text{sync}})^{-\frac{1}{2}}$$
where, $U_B$ is energy density of magnetic field, and $U_{sync}$ is energy density of synchrotron photon. Hence, the galaxy accelerates an electron to high energy. The spectrum of a typical AGN is, $\nu_{ssc}/\nu_{sync} \sim 10^{8\pm2}$, and $L_{ssc}/L_{sync} \sim 1$, where $\nu_{ssc}$ is the frequency of Compton peak, $\nu_{sync}$ is the frequency of synchrotron peak, $L_{ssc}$ is the luminosity of Compton peak, and $L_{sync}$ is the luminosity of synchrotron peak.

Figure 2.4: Schematic spectral energy distributions (SED) of blazars from radio through gamma rays [42]. The observed blazars were divided into five groups by total radiation intensity, and the average spectrum was plotted for every group.

The observed radio emission is considered to be due to synchrotron emission while high X-ray fluxes would be expected from inverse Compton scattering with the synchrotron photons (so called synchrotron self-Compton process: SSC).

The X-ray spectrum becomes harder while the gamma-ray spectrum softens with increasing luminosity, indicating that the second (Compton) peak of SEDs also moves to lower frequencies from $\sim 10^{24} - 10^{25}$ Hz for less luminous sources to $\sim 10^{21} - 10^{22}$ Hz for the most luminous ones.
2.5 Giant Radio Galaxy Centaurus A

Centaurus A galaxy is a giant elliptical (S0) galaxy as shown in Fig 2.5. The prominent twisted disk of gas and dust contains many H II regions and is lying approximately along the galaxy minor axis, obscuring the nucleus at optical wavelengths. The dust lane structures detected by optical images are thought to be remnants of a recent ($10^7$ - $10^8$ years ago) merger of a giant elliptical galaxy with a smaller spiral galaxy [88].

![Optical image of Centaurus A](image)

**Figure** 2.5: Optical image of Centaurus A obtained with the Wide-Field Imager (WFI) camera at the ESO/MPG 2.2-m telescope on La Silla. It is based on a total of nine 3-min exposures made on March 25, 1999, through different broad-band optical filters (B(lue) - total exposure time 9 min - central wavelength 456 nm - here rendered as blue; V(isual) - 540 nm - 9 min - green; I(nfrared) - 784 nm - 9 min - red) [134].

The giant double jet lobe of Cen A which extends $10^\circ$ was observed by radio [90, 21]. High resolution radio observations provide sub-arc-second resolution in the inner radio structures.

Cen A is classified as a Fanaroff - Riley (FR) type I [40] radio galaxy and Seyfert 2 object in the optical [32]. Furthermore, Cen A is classified as a "misaligned" BL Lac type
AGN at higher energies [93]. It is one of the best examples of a radio-loud AGN viewed from the side ($\sim 60^\circ$) of the jet axis [50, 33, 70]. The distance of Cen A is reported by many people around 2~8 Mpc. In this paper, we adopt a distance to Cen A of 3.5 Mpc [62] which is standard value.

The optical observations show filaments related to the inner jet seen in the radio and X-ray regimes, and X-ray observations have detected flux variations on timescales of days, like typical high energy BL Lac. Cen A is also a very important source of gamma-rays which has been observed repeatedly by all Compton Gamma Ray Observatory (CGRO) instruments [76, 97, 110, 117]. COMPTEL sky map of the energy band 1 - 30 MeV is shown in Fig 2.7.

<table>
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<th>Units</th>
<th>Reference</th>
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<td></td>
<td>$201.3650633$</td>
<td>[136]</td>
</tr>
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<td>$\delta$(J2000)</td>
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<td>[136]</td>
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<td>kpc</td>
</tr>
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<td>kpc</td>
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<tr>
<td>Dust lane radius</td>
<td>7</td>
<td>kpc</td>
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</table>

Table 2.2: Centaurus A basic data [64]
2.6 Spectral Energy Distribution for Centaurus A

The Spectral Energy Distribution (SED) for Cen A is shown in Fig 2.8. The figure is based on NASA Extragalactic Database [136]. Cen A was observed all over the wavelength because of nearby giant radio galaxy.

1. Radio observations

Radio observations were carried by NRAO [46], PARKS [131, 52, 132, 17], MOST [69, 87, 82], and CCA [107]. The radio observations reveal the existence of high energy electrons. Also two kinds of jets are observed. One extends to the outer regions of the galaxy and another exists in the inner region as shown in Fig 2.6.

2. Infrared and optical observations

It is difficult to observe the nucleus of Cen A in the IR and optical. Because the dust lane obstructs light of these wavelengths. The dust lane of Cen A is shown in Fig 2.5

3. X-ray observations
Figure 2.7: COMPTEL gamma-ray map [135]. A list of the 63 gamma-ray sources detected by the COMPTEL. This figure shows the gamma-ray map in the energy band 1 - 30 MeV. Cen A is located on (RA, Dec.)=(13 h 25 m 27.6 s, -43°01'08.805")

X-ray observations were also performed by ROSAT [19], EINSTEIN [39], and Chandra [79]. Chandra observations find X-ray emission in the outer half of the southwest radio lobe.

4. MeV gamma-ray observations

MeV gamma-rays were detected by CGRO, OSSE [112], EGRET, COMPTEL, and BATSE [110, 111]. COMPTEL observed a flare of Cen A in 1991, as shown in Fig 2.7. Another satellite, SIGMA, also found a flare in 1991.

5. TeV gamma-ray observations

A positive detection of TeV gamma-rays was claimed in 1975 by Narrabri Observatory [53]. The Buckland Park air shower array also reported a detection of Cen A [28]. This observation had a confidence level of 99.4% and the average flux over the period 1984-1989 was \((7.4 \pm 2.6) \times 10^{-12} \text{cm}^{-2} \text{sec}^{-1}\) above 150 TeV.

JANZOS have searched for gamma-rays from Cen A during the period 1987 - 1992 [9]. No significant excess for steady gamma-ray emission was found, and an upper limit on integral flux above 110 TeV was obtained at \(3.2 \times 10^{-13} \text{photons cm}^{-2} \text{sec}^{-1}\) at 95 % confidence level conflicting with the above measurement. However, an en-
hancement of events was found in a short period of 1990. The flux observed during this period was $(5.5 \pm 1.5) \times 10^{-12}$ photons cm$^{-2}$sec$^{-1}$ at energies $\geq$ 110TeV, assuming an index of -2.0 for the differential energy spectrum. This flux is consistent with the Buckland Park result[28].

However, recent observations did not detect any excess. CANGAROO-I gave an upper limit of $1.28 \times 10^{-11}$ photons cm$^{-2}$ sec$^{-1}$ (3-$\sigma$ upper limit at 1.5 TeV).

The sensitivity of CANGAROO-III is $0.25 \times 10^{-11}$ cm$^{-2}$ sec$^{-1}$ at 500 GeV. It is 10 to 1000 times higher sensitivity than the previous measurements. Although the H.E.S.S. telescope is also one of the new generation telescopes which has bigger reflectors, they have not observed Cen A yet. The CANGAROO-III observations are the first with a new generation.

Summary of Cen A SED is shown in Fig 2.8
Figure 2.8: Spectral Energy Distribution for Cen A. The figure is based on NASA Extragalactic Database [136]. The red line is the CANGAROO-III sensitivity. The radio observations which are plotted around $10^{-7} \sim 10^{-4}$ (eV) are NRAO [46], PARKS [131, 52, 132, 17], MOST [69, 87, 82], and CCA [107]. The infrared and optical observations which are plotted around $10^{-2} \sim 10^{1}$ (eV) are CTIO [51, 12, 60, 43], AAO [8], IRAS [49, 78, 63, 104, 94], 2MASS [66, 122], Johnson [1], and RC3.9 [12]. The X-ray observations which are plotted around $10^{3} \sim 10^{4}$ (eV) are ROSAT [19] and EINSTEIN [39]. The MeV gamma observations which are plotted around $10^{4} \sim 10^{8}$ are OSSE [112], EGRET, COMPTEL and BATSE [110, 111]. The TeV gamma observations which are plotted around $10^{11} \sim 10^{13}$ (eV) are Narrabri Observatory [53], JANZOS [9], Buckland Park air shower array [28], and CANGAROO-I [105].
Chapter 3

Acceleration Mechanisms of Cosmic Rays

Charged particles are accelerated by electric fields induced by the motion of magnetic fields $B$. The energy-gain rate of these relativistic particles with the electric charge $Ze$ can be written as,

$$\frac{dE}{dt} = \xi Z e c^2 B,$$

where $\xi < 1$ and it depends on the acceleration mechanism, which is described in this section.

### 3.1 Second-Order Fermi Acceleration

The idea of statistical accelerations was first introduced by Fermi [41]. Molecular clouds extend to the order of 10 pc with a higher density than the interstellar matter. From the Doppler effect of the absorption lines, their dispersion velocity $v$ was obtained to be 30 km/s. The conductivity in the clouds is high because their densities are extremely low and also they are highly ionized. Magnetic irregularities are generated by Alfvén waves due to such moving plasma in the interstellar magnetic field and generally occupy the interstellar space. Motions of charged particles in these magnetic irregularities in the clouds can be considered as if they were elastic collisions against a very large mass, i.e. the particles gain energies from the clouds. Assuming the particles collide randomly, the average gain in energy per collision is an order of $(v/c)^2$ [41]. The energies of the particles increase
The average particle energy after \( n \) collisions with the clouds is given as
\[
E_n = E_0(1 + \xi)^n \simeq E_0 \exp(\xi n),
\] (3.2)
where \( E_0 \) is the initial energy of the particle, and \( \xi \) is the average energy-gain per one collision. The probability of a particle making \( n \) collisions before escaping from the acceleration region is given as
\[
P_n = (1 - P_{\text{esc}})^n,
\] (3.3)
where \( P_{\text{esc}} \) is the probability that particles escape from the acceleration region per collision, which was calculated from the mean free path of the collision with the interstellar matter in Fermi’s case. Using Equations (3.2) and (3.3), the number of accelerated particles with energies more than \( E \) can be calculated as
\[
N(E) \propto \sum_{m=n}^{\infty} (1 - P_{\text{esc}})^m = \frac{(1 - P_{\text{esc}})^n}{P_{\text{esc}}} \propto \frac{1}{P_{\text{esc}}} \left( \frac{E}{E_0} \right)^{-\delta},
\] (3.4)
where \( \delta \) is given as
\[
\delta = \frac{\ln[1/(1 - P_{\text{esc}})]}{\ln(1 + \xi)} \approx \frac{P_{\text{esc}}}{\xi}.
\] (3.5)
This theory naturally leads a power-law energy spectrum of accelerated particles. In this case, however, \( \xi \) is quite small and, considering Equations (3.4) and (3.5), \( \delta \) is likely to be smaller than the power-law index of the cosmic-ray spectrum 2.7. It is difficult to explain Galactic cosmic rays only by this mechanism because of the low acceleration efficiency.

### 3.2 First-Order Fermi Acceleration (Diffusive Shock Acceleration (DSA))

Instead of Fermi’s idea above, a more efficient acceleration mechanism by collisions was introduced, i.e., accelerations in shock fronts. Figure 3.1 shows a schematic view of particle acceleration around a shock front in the laboratory frame. The interstellar matter in the upstream region flows into the downstream region, through the shock front with velocity \( v_1 \) in the rest frame of the shock front, which is greater than the sound speed in the upstream region, i.e. faster than the speed for transmitting information, and becomes slower and denser in the downstream region. Suppose that there are cosmic rays with
an initial energy of \(E_1\), assuming they are relativistic for simplicity. In the rest frame of downstream region, the energy of the accelerated cosmic ray is given as

\[
E'_1 = \gamma_v E_1 (1 + \beta_v \theta_1),
\]

where the prime (') denotes a quantity in the rest frame of the downstream region, \(\gamma_v\) is the Lorentz factor with velocity \(v\), \(\beta\) is \(v/c\), and \(\theta_1\) is the incident angle of the particle.

After multiple elastic scattering with magnetic irregularities, the particle again crosses the shock and enters into the upstream region with some probability. The energy \(E_2\) is that after the interaction with the downstream medium. The energy gain is given as

\[
\frac{\Delta E}{E_1} = \gamma_v^2 (1 + \beta_v \cos \theta_1 - \beta_v \cos \theta'_2 - \beta_v^2 \cos \theta_1 \cos \theta'_2) - 1,
\]

where \(\Delta E\) is \(E_2 - E_1\). This value should be averaged over the particle’s incident angles with respect to the shock front. If the isotropic intensity of the number of particles were given by \(I\), the average of \(\cos \theta_1\) is given as

\[
\langle \cos \theta_1 \rangle = \frac{2\pi \int_0^1 \cos \theta \cdot I \cos \theta d(\cos \theta)}{2\pi \int_0^1 I \cos \theta d(\cos \theta)} = \frac{2}{3}.
\]
Similarly, \( \langle \cos \theta' \rangle = -2/3 \). From \( \beta_v \ll 1 \) of the shock waves, the energy gain is approximated as

\[
\frac{\Delta E}{E_1} = \frac{4}{3} \frac{v_1 - v_2}{c}.
\] (3.9)

The probability \( P_{\text{esc}} \) that the scattered particles escape from the acceleration region for each round trip was calculated by Bell [13]. In the rest frame of the shock front, the flux of non-thermal particles penetrating into the shock front is given as

\[
\int_0^1 d\cos \theta \int_0^{2\pi} d\phi \frac{c\rho_{\text{CR}}}{4\pi} \cos \theta = \frac{c\rho_{\text{CR}}}{4}.
\] (3.10)

The flux of non-thermal particles which escape from the downstream regions is \( \rho_{\text{CR}} v_2 \). The \( P_{\text{esc}} \) is given as

\[
P_{\text{esc}} = \frac{\rho_{\text{CR}} v_2}{c\rho_{\text{CR}}/4} = 4 \frac{v_2}{c}.
\] (3.11)

Using Equation (3.9) and (3.11), the power-law index \( \delta \) of the integral flux in Equation (3.5) is given as

\[
\delta = \frac{3}{v_1/v_2 - 1}.
\] (3.12)

The compression ratio \( v_1/v_2 \) can be estimated by the thermo-dynamics of thermal particles. In the rest frame of the shock front, conservation of mass, momentum, and energy are described as

\[
\frac{\partial \rho}{\partial t} + \frac{\partial (\rho v)}{\partial x} = 0,
\] (3.13)

\[
\frac{\partial \rho v}{\partial t} + \frac{\partial (\rho v^2 + P)}{\partial x} = 0,
\] (3.14)

\[
\frac{\partial}{\partial t} \left\{ \rho \left( \frac{1}{2} v^2 + E \right) \right\} + \frac{\partial}{\partial x} \left\{ \rho \left( \frac{1}{2} v^2 + E \right) v + P v \right\} = 0,
\] (3.15)

where \( \rho, v, P, \) and \( E \) are the density, velocity, pressure, and internal energy per unit mass, which is the sum of the kinetic energies of thermal particles, respectively. Assuming a steady state \( (\partial/\partial t = 0) \) and applying these equations to the shock front shown in Figure 3.1, the relations between the physical parameters in the upstream and in the downstream (Rankine-Hugoniot relations) are given as

\[
\rho_1 v_1 = \rho_2 v_2
\] (3.16)

\[
\rho_1 v_1^2 + P_1 = \rho_2 v_2^2 + P_2
\] (3.17)

\[
v_1 \left\{ \rho_1 \left( \frac{1}{2} v_1^2 + E_1 \right) + P_1 \right\} = v_2 \left\{ \rho_2 \left( \frac{1}{2} v_2^2 + E_2 \right) + P_2 \right\},
\] (3.18)
where subscripts 1 and 2 denote upstream and downstream, respectively. By Equation (3.16), Equation (3.18) reduces to
\[
\frac{1}{2}v_1^2 + E_1 + \frac{P_1}{\rho_1} = \frac{1}{2}v_2^2 + E_2 + \frac{P_2}{\rho_2}.
\] (3.19)
Assuming the plasma behaves as an ideal gas, and using Mayer’s relation, \( E \) can be written as
\[
E = C_V T = \frac{C_V P}{n R \rho} = \frac{C_V}{C_P - C_V} \frac{P}{\rho} = \frac{1}{\gamma - 1} \frac{P}{\rho},
\] (3.20)
where \( C_V \), \( C_P \), and \( \gamma \) are the molar heat at constant volume and pressure, and the specific heat, respectively. Using Equation (3.16) and the Mach number \( M \equiv v/a = v/\sqrt{\gamma P/\rho} \) in the adiabatic gas, where \( a \) is the sound speed, Equations (3.17) and (3.18) become
\[
\left( 1 - \frac{1}{r} \right) \gamma M_1^2 = s - 1,
\] (3.21)
\[
\left( 1 - \frac{1}{r^2} \right) M_1^2 = \frac{2}{\gamma - 1} \left( \frac{s}{r} - 1 \right),
\] (3.22)
where \( r \equiv \rho_2/\rho_1 = v_1/v_2 \) (compression ratio), and \( s \equiv P_2/P_1 \). \( r \) and \( s \) are given by
\[
r = \frac{(\gamma + 1)M_1^2}{(\gamma - 1)M_1^2 + 2},
\] (3.23)
\[
s = \frac{2\gamma M_1^2 - (\gamma - 1)}{\gamma + 1}.
\] (3.24)
In the case of \( M_1 \gg 1 \) (strong shock) approximation, \( r \) becomes
\[
r = \frac{\gamma + 1}{\gamma - 1}.
\] (3.25)
Adopting \( \gamma \) of 5/3 for a mono atomic gas, the compression ratio becomes 4. Using this ratio and Equation (3.12), the index of the integral spectrum of the accelerated particles is unity, i.e. the index of the differential spectrum is 2. This result is consistent with low-energy observations at various wavelengths.
Chapter 4

Gamma-ray Emission Mechanisms

4.1 \( \pi^0 \) decay

\( \pi^0 \)s are produced in collisions of accelerated cosmic rays and target nuclei which are mainly protons (interstellar matter). The \( \pi^0 \)s immediately decay into two gamma-rays with a mean lifetime of \( \approx 10^{-16} \gamma_\pi \) seconds, where \( \gamma_\pi \) is the Lorentz factor of the secondary \( \pi^0 \)s. These gamma-rays have a similar energy spectrum to that of the parent high-energy particles because of the scaling hypothesis. Details of calculations of differential cross section are given in Appendix B.

A rough estimation of gamma-rays from Cen A is shown below. First, let us consider the emission from a SNR as an example. Here we assume that 10% of shock-wave energy of \( 10^{51} \) erg becomes cosmic ray energy. In eV unit,

\[
E_{CR} = \frac{10^{50}}{1.6 \times 10^{-12}} = 6.25 \times 10^{61} \text{eV}
\]

and cross section is described as,

\[
\sigma = 30 \text{mb} = 30 \times 10^{-3} \times 10^{-24} \text{cm}^2 = 3 \times 10^{-26} \text{cm}^2.
\]

The flux of cosmic rays from a certain object is

\[
\text{flux} = E_{CR} \times c \sigma n d / 4 \pi d^2,
\]

where, \( c \) is light speed, \( n \sim 1/\text{cm}^3 \) is number of density of protons, giving \( c \sigma n = 9 \times 10^{-16} \).

Here we assume the distance between earth and the SNR is \( d = 3 \text{kpc} = 9.3 \times 10^{21} \text{cm}, \)

\[
4 \pi d^2 = 1.1 \times 10^{55} \text{cm}^2.
\]
Then,

\[
flux = \frac{6.25 \times 10^{49} \text{[TeV]} \times 9 \times 10^{-16} \text{[sec\textsuperscript{-1}]}}{1.1 \times 10^{45} \text{[cm\textsuperscript{2}]}} = 5 \times 10^{-11} \text{[TeV cm\textsuperscript{-2} sec\textsuperscript{-1}]}
\]

\[
\sim 2 \text{Crab}
\]

Therefore,

\[
Integral flux = 5 \times 10^{11} \left( \frac{E}{10^{50} \text{[erg]}} \right) \left( \frac{n}{1 \text{[proton cm\textsuperscript{-3}]]} \right) \left( \frac{d}{3 \text{[kpc]}} \right)^{-2} \left[\text{TeV cm\textsuperscript{-2} sec\textsuperscript{-1}}\right]
\]

Cosmic rays from Cen A is estimated below. The distance of Cen A to earth is \(d = 3.5\text{Mpc}\) and we assume that Cen A is similar to our galaxy having \(E_{CR} \sim 10^{54}\) erg. Here a SN rate of 1/300 year, and a cosmic ray lifetime of \(\tau \sim 300 \times 10^4\) year were assumed. The number of SNR contributing is 1000 and \(\sim 1000 \text{SN} \sim 10^{54}\) erg is obtained. The flux is estimated as

\[
Integral flux \sim 5 \times 10^{11} \left( \frac{10^{54}}{10^{50}} \right) \left( \frac{n}{1 \text{[proton cm\textsuperscript{-3}]]} \right) \left( \frac{3500}{3 \text{[kpc]}} \right)^{-2}.
\]  

That is,

\[
Integral flux \sim 0.7 \times 10^{-11} \text{[cm\textsuperscript{-2} sec\textsuperscript{-1}]}.
\]  

This estimate for Cen A is 27% of the Crab flux. CANGAROO-III sensitivity is less than 1/10 Crab. We should detect the gamma-rays from Cen A.

### 4.2 Synchrotron Radiation

Photons are emitted from charged high-energy particles accelerated in the magnetic field as described in the previous section. This radiation is known as synchrotron radiation. The charged particle trajectory in the magnetic field is a helix. Seen from a distance, and from a certain angle such as perpendicular direction with respect to the magnetic field, it can be seen as a harmonic oscillator. Such motion of charges cause electro-magnetic waves, i.e., real photons, which can be transported to infinite distance. The mean energy of these photons can be described by the weighted mean of \(\sum n \nu_{cycotron} f(n)\), \(f(n)\) can be determined from the Maxwell equation.
The following is a simple estimation of the total emitted power of an electron [102]. The Lorentz force is produced by only $v_\perp$ which is the velocity of the electron perpendicular to the direction of the magnetic field. We, therefore, should see only $v_\perp$. The magnetic field $B$ in the laboratory frame can be regarded as the electric field $E'$ in the rest frame of the electron. This can be derived using the Lorentz transformation as

$$E' = \gamma \beta_\perp B,$$

where $\gamma$ and $\beta_\perp$ are the Lorentz factor of the electron ($1/\sqrt{1-(v/c)^2}$) and $v_\perp/c$ ($c$ is light speed), respectively. The total emitted power in the rest frame is given as

$$P' = \frac{2e^2}{3c^3} \left( \frac{e \gamma \beta_\perp B}{m} \right)^2 = 2\sigma_T c \gamma^2 \beta_\perp^2 \frac{B^2}{8\pi},$$

where $e$ and $m$ are the charge of the electron and the rest mass of the electron, respectively.

For an isotropic distribution of velocities it is necessary to average this formula over all angles for a given speed $\beta$. Let $\alpha$ be the pitch angle, which is the angle between the magnetic field and the velocity. Then we obtain

$$\langle \beta_\perp^2 \rangle = \int \beta_\perp^2 \sin \alpha^2 d\Omega = \frac{2\beta^2}{3},$$

where $\Omega$ is the solid angle. The result is

$$P' = \frac{4}{3} \sigma_T c \gamma^2 \beta^2 U_B,$$

where $\sigma_T$ and $U_B$ are the cross section of Thomson scattering and the energy density of the magnetic field, respectively. The total emitted power $P'$ is Lorentz invariant. Therefore $P$ is given as

$$P = \frac{4}{3} \sigma_T c \gamma^2 \beta^2 U_B.$$

Synchrotron radiation is important only for electrons since $P$ is proportional to $1/m^2$ for high-energy particles from Equation (4.7). The frequency spectrum can extend to many times the gyration frequency. Figure 4.1 shows the spectral distribution of the power of the total (over all directions) radiation from charged particles moving in a magnetic field as a function of $\nu/\nu_C$, where $\nu$ is the frequency of the emitted photons and $\nu_C = 3eB\gamma^2/4\pi mc$. The spectrum has a roughly monochromatic peak when the energy distribution of electrons is monochromatic.
Figure 4.1: Spectral distribution of the power of the total (over all directions) radiation from synchrotron radiation. $\nu$ is the frequency of the emitted photons and $\nu_C = 3eB\gamma^2/4\pi mc$.

4.3 Inverse Compton Scattering

When relativistic electrons move in a background photon field, the Compton scattered photons gain energy from the electrons. This process is called inverse Compton (IC) scattering. The energy of the ambient photon in the rest frame of the electron is given as

$$h\nu^* = \gamma h\nu (1 + \beta \cos \theta), \quad (4.8)$$

where $h$, $\nu^*$, $\gamma$, $\nu$, $\beta$, and $\theta$ are Planck constant, the frequency of the photon in the laboratory frame, the Lorentz factor of the electron, the frequency of the electron in the rest frame of the electron, the velocity of the electron, and the incident angle of the electron in the laboratory frame, respectively. Assuming the distribution of the ambient photons is isotropic, the mean photon energy becomes $\gamma h\nu$. In case of $\gamma h\nu \ll mc^2$, where $m$ is the rest mass of the electron, the scattering in the rest frame of the electron is approximately Thomson scattering, i.e. elastic. Hence the energy of the scattered photon is given as

$$h\nu' = \gamma h\nu^*(1 + \cos \varphi) \approx \gamma^2 h\nu, \quad (4.9)$$

where $\nu'$ and $\varphi$ are the frequency of the scattered photon in the laboratory frame and the scattering angle of the photon in the rest frame, respectively. The energy of the scattered
photon averaging over solid angles is given by

\[ h'\nu = \frac{4}{3} \gamma^2 h\nu. \] (4.10)

Using Equation (4.10), the energy loss rate of the electron is given by

\[ P_{\text{compt}} = \frac{dE}{dt} = \frac{4}{3} \sigma_T \gamma^2 U_{ph}, \] (4.11)

where \( \sigma_T \), and \( U_{ph} \) are the cross section of Thomson scattering and the energy density of the background photons respectively.

### 4.4 Bremsstrahlung

When charged particles pass near the Coulomb field of a nucleus, photons are emitted. This is called bremsstrahlung. The cross section and the emitted power of bremsstrahlung are proportional to \( 1/m^2 \), where \( m \) is the rest mass of the charged particle. Therefore bremsstrahlung is important only for electrons and is negligible for nuclei.

### 4.5 Relativistic Beaming Effect

For the discussion of AGN jets, we should understand how a relativistic jet is observed at the earth. Here, assuming the velocity of the jet is \( V = \beta c \), then the Lorentz factor is \( \gamma = 1/\sqrt{(1 - \beta^2)} \), where the angle of the line of sight is \( \theta \) with respect to the jet axis.

The beaming factor of a relativistically moving source is defined as \([102]\),

\[ \delta = \frac{1}{\gamma(1 - \beta \cos \theta)}, \] (4.12)

where \( \beta \) is its bulk velocity in units of the speed of light, \( \gamma = (1 - \beta^2)^{-1/2} \) is the corresponding Lorentz factor, and \( \theta \) is the angle between the beam and line of sight. The beaming factor has a strong dependence on the viewing angle (as shown in Fig. 4.2 and Eq 4.12).

We define the beaming factor \( \delta \) as,

\[ \nu_{\text{obs}} = \nu_s \delta, \] (4.13)

\[ \Delta t_{\text{obs}} = \frac{\Delta t_s}{\delta}, \] (4.14)
where $\nu_{\text{obs}}$ is the frequency in the observer frame and $\nu_s$ is the frequency in the rest frame of the particle. The beaming factor is a decreasing function of $\theta$, e.g., if $\cos \theta$ equal to $\beta$, $\delta = \gamma$ is obtained.

The relationship of the direction of the photon $\theta$ in the fixed frame of the jet and the direction to the observer is

$$
\cos \theta_s = \frac{\cos \theta - \beta}{1 - \beta \cos \theta}.
$$

(4.15)

Because of the above formula, the light emission angle is $\Delta \Omega_s = \delta^2 \Delta \cos \theta$, therefore we can see in the observer frame,

$$
\Omega = \gamma^2 \Delta \Omega.
$$

(4.16)

The relationship of the luminosity in the jet frame and the observed flux is

$$
\frac{L_s}{\nu_s} \Delta t_s \frac{\Delta \Omega_s}{4\pi} = \frac{F}{\nu_{\text{obs}}} d^2 \Delta \Omega \Delta t_{\text{obs}}
$$

(4.17)

and,

$$
L_s = 4\pi d^2 F \gamma^{-4}.
$$

(4.18)

For $\gamma$-ray-loud blazars, relativistic beaming is necessary not only to enable the $\gamma$-ray to escape from the source, but also to amplify the flux and therefore make the source more easily detectable [126, 48]. The observed ($F_{\text{obs}}$) and intrinsic ($F_0$) fluxes from a continuous relativistic jet are related by

$$
F_{\text{obs}} = \delta^{2+\alpha} F_0 = \delta^p F_0,
$$

(4.19)

where $\alpha$ is the spectral index. It is notable that beaming does not affect the relation between the two components discussed above [26].
Based on the TeV variability, Mkn 421 (z=0.031) and Mkn 501 (z=0.034) were estimated to have $\theta \lesssim 6^\circ$ [96]. If they were observed at $\theta = 60^\circ$, according to Equation (4.19) and assuming typical values for $\gamma$ and $p$, i.e., $\gamma \sim 5$ and $p \sim 3$, the observed flux would decrease by a factor of $1.3 \times 10^{-4}$, which is not detectable even during the largest outburst. However, if they were placed at the distance of Cen A, z=0.0008, the observed flux would increase by a factor of about $(0.034/0.0008)^2$. Mkn 421 and Mkn 501 at the Cen-A position with a viewing angle of 60°, or moved to the position of M87 (z=0.0043) with $\theta = 35^\circ$, can be detectable at TeV energies even in stable periods.

### 4.6 Synchrotron self-Compton Model

The gamma-ray-loud BL Lac objects have some common properties which imply physical similarities among different objects. It is believed that gamma-ray emission from TeV BL Lac objects is dominantly produced by the synchrotron self-Compton (SSC) process.

The gamma-ray-loud blazars have the following properties which should also be shared by HBL-like FRI radio galaxies, according to the unified AGN scheme. In the context of the SSC model, the relation between the peak frequencies of the synchrotron component $\nu_s$ and the Compton component $\nu_c$ is $\nu_c/\nu_s \propto \gamma^2_{\text{peak}}$, where $\gamma_{\text{peak}}$ is the characteristic electron energy [124]. For TeV BL Lac objects, the averaged up-shifting factor of the Compton peak relative to the synchrotron peak is $\sim 10^{8\pm1}$, i.e.,

$$\frac{\nu_c}{\nu_s} \approx 10^{8\pm1}. \quad (4.20)$$

For example, Mkn 421 has a synchrotron emission peak at or slightly above $10^{17}$Hz, and a Compton peak just below 1 TeV [124], yielding $\nu_c/\nu_s \lesssim 10^9$. The synchrotron peak of Mkn 501 is located in the range $2 - 100$keV [22, 99, 124, 24], and the Compton component peaks at $\sim 1$ TeV [98, 106], with $\nu_c/\nu_s \sim 10^7 - 10^9$. PKS 2155 − 304 has a peak in its synchrotron component in the soft X-ray range [34, 125, 27] and a peak in its Compton component between $10$GeV and 1 TeV (Fig. 5 of Chadwick et al. 1999 [25]), probably at 100GeV [72, 27] with $\nu_c/\nu_s \sim 10^{8-9}$.

The candidates of seed photon of IC are roughly divided into three.

1) Cosmic microwave background (CMB) photon. : $U_{\text{CMB}} \sim 3 \times 10^{-12} (\text{erg cm}^{-3})$. 


2) External photon: \( U_{\text{ext}} < 3 \times 10^{-5} \text{(erg cm}^{-3}) \)

3) Synchrotron photon: \( U_{\text{syn}} \sim 3L_{\text{syn}}/4\pi R^2 c \delta^4 \sim 3 \times 10^{-4} \text{(erg cm}^{-3}) \)

Where, \( R \) is the radius of radiation sphere in the jet \((R \sim c t_{\varphi} \delta \sim 10^{16} \text{ cm and } t_{\varphi} \text{ is the time scale of time variation (1 day)})\). Thus synchrotron photon are dominant. Some of the synchrotron photons will be scattered via inverse Compton scatterings to higher energies by relativistic electrons, which is known as the synchrotron self-Compton (SSC) process. Assuming that the luminosity of the synchrotron emission \((L_{\text{syn}})\) is in agreement with the IC emission \((L_{\text{SSC}})\), using equation (4.7) and (4.11), we have:

\[
\frac{L_{\text{SSC}}}{L_{\text{syn}}} = \frac{U_{\text{syn}}}{U_B},
\]

that is, the radiation losses due to synchrotron emission and to inverse Compton effect are proportional to the magnetic field energy density and photon energy density.

Even if we can not detect gamma-rays from Cen A in our observation, we can give an lower limit on the magnetic field and angle of the jet,

\[
Lower \ limit > L_{\text{SSC}}(60^\circ) = \frac{U_{\text{syn}}}{U_B} L_{\text{syn}}.
\]
Chapter 5

Atmospheric Cherenkov Techniques

5.1 Overview

VHE gamma-rays are detected using the atmospheric Cherenkov telescopes. Charged particles in an extensive air shower (EAS) initiated by a VHE gamma-ray lose their energy ionizing atmospheric molecules and most of them are attenuated in the atmosphere before arriving at the ground. However, the atmosphere is quite transparent for Cherenkov light radiated from charged particles in the EAS. "Cherenkov observations" are possible only on moonless clear nights because of the low intensity of the Cherenkov light, and the use of large reflectors is necessary to collect Cherenkov photons. Cherenkov observations have the advantage of a large gamma-ray detection area, which is more than $10^4$ times larger than the typical area in satellite measurements.

Atmospheric Cherenkov telescopes detect not only gamma-rays but also numerous background events due to cosmic-ray nuclei. Therefore, it is important to eliminate background events for effective detections of gamma-rays. Fortunately, there are several important differences in the development between EASs initiated by gamma-rays ("gamma-ray showers") and EASs initiated by cosmic ray nuclei ("hadronic showers"). These differences in the Cherenkov light profile can be used to enhance gamma-ray signals. There are currently a few kinds of techniques for Cherenkov observations. Of these, the "imaging technique" is the most powerful for rejecting background events. In this technique, Cherenkov images on the focal plane are recorded with an imaging camera consisting of many small photomultiplier tubes (PMTs), and the images well reflect the profile of the
5.2. **EXTENSIVE AIR SHOWERS**

A gamma-ray entering the earth’s atmosphere initiates an electromagnetic cascade. In the electromagnetic cascade, many electrons, positrons and gamma-rays are produced and the average particle energy decreases as it develops in the atmosphere. Finally, ionization losses become dominant for the shower particles and the cascade will stop.

On the other hand, a primary cosmic ray nucleus entering the atmosphere interacts with an atmospheric nucleus and generates a nuclear cascade. The nuclear interaction causes multi-particle production, in which secondary nucleons and mesons are produced. Pions constitute the biggest fraction of cascade particles. Neutral pions decay into gamma-rays via \( \pi^0 \rightarrow 2\gamma \) and electromagnetic cascades are produced by the gamma rays as mentioned above. Since the lifetime of the charged pion is \( 2.603 \times 10^{-8} \gamma_m \) sec, where \( \gamma_m \) is the Lorentz factor of the charged pion, and the mean free path for the charged pion in the atmosphere is \( \sim 80 \) g cm\(^{-2} \), charged pions of energies greater than 10 GeV further interact with atmospheric nuclei, while the others decay as follows:

\[
\begin{align*}
\pi^+ & \rightarrow \mu^+ + \nu_\mu, \quad (5.1) \\
\pi^- & \rightarrow \mu^- + \bar{\nu}_\mu. \quad (5.2)
\end{align*}
\]

High energy muons generated in the early stage of the development of the EAS can reach the ground without decaying (the lifetime of muon is \( 2.197 \times 10^{-6} \gamma_\mu \) sec). The above elements in the development of an EAS are summarized in Fig 5.1.

There are several important differences in the development of EASs between gamma-ray and hadronic showers. First, hadronic showers are more elongated because of the longer interaction length than the radiation length (about three times). Second, nuclear cascades have larger transverse momenta (\( \sim 300 \) MeV) than electromagnetic cascades (\( \sim 0.5 \) MeV) and, as a result, hadronic showers are more extended transversely than gamma-ray showers as shown in Fig 5.2. These two facts also mean that there are larger fluctuations in the development of hadronic showers. Third, comparing at the same

development of the EAS. Imaging telescopes have an angular resolution as good as \( 0^\circ.1 \) and this is an epoch-making improvement in the signal-to-noises ratio compared to the angular resolution of \( \sim 1^\circ \) in non-imaging telescopes such as gamma-ray satellites.
Figure 5.1: Schematic interaction processes in hadronic showers [127].
5.3. CHERENKOV RADIATION

energy of the primary particles, the number of particles in hadronic showers is roughly one third of that in gamma-ray showers. Almost equal numbers of $\pi^+$, $\pi^-$ and $\pi^0$ are produced in hadronic showers due to the isospin symmetry but only 1/3, i.e., $\pi^0$ makes electro-magnetic cascades. The final products of charged pions are muons.

![Shows showers initiated by 1 TeV gamma-ray (left) and 1 TeV proton (right). Both were simulated by Monte Carlo simulations [137].](image)

**Figure 5.2:** Showers initiated by 1 TeV gamma-ray (left) and 1 TeV proton (right). Both were simulated by Monte Carlo simulations [137].

### 5.3 Cherenkov Radiation

In a medium of refractive index $n$, the velocity of light is $c' = c/n$, where $c$ is the velocity of light in vacuum. A charged particle which is moving at a velocity $v > c'$ in the medium radiates Cherenkov radiation. The radiation is only permitted at a particular angle $\theta$ ("Cherenkov angle") with respect to the track of the particle:

$$\cos(\theta) = \frac{1}{\beta n}, \quad (5.3)$$

where $\beta = v/c$. For a wavelength of 400nm, the refractive index of the air at temperature $T$ K can be expressed as:

$$n = 1.0 + 0.000296 \left( \frac{x}{1030 \text{g cm}^{-2}} \right) \left( \frac{T}{273.2K} \right)^{-1}, \quad (5.4)$$
where \( x \) \( \text{g cm}^{-2} \) is the atmospheric depth and \( T \) is written as \( T=204+0.091x \). Since \( n \sim 1.00027 \) at 1 atm, the Cherenkov angle is \( \theta < 1^\circ.3 \) for relativistic particles \( (\beta \sim 1) \). Therefore Cherenkov photons from the EAS are radiated within a few degrees with respect to the shower axis.

The energy output of Cherenkov radiation per unit path is represented by the following equation;

\[
\frac{dE}{dl} = \frac{Z e^2}{c^2} \int_{\beta n > 1} \left( 1 - \frac{1}{\beta^2 n^2} \right) d\omega, \tag{5.5}
\]

where \( Ze \) is the particle charge and \( \omega \) is the frequency of the Cherenkov radiation. From Eq 5.5, the number of photons radiated into wavelength range between \( \lambda_1 \) and \( \lambda_2 \) is

\[
N = 2\pi \alpha Z^2 l \left( \frac{1}{\lambda_1} - \frac{1}{\lambda_2} \right) \left( 1 - \frac{1}{\beta^2 n^2} \right) \tag{5.6}
\]

where \( \alpha = e^2/\hbar c \) is the fine structure constant (Frank-Tamm eq.). The total number of Cherenkov photons radiated from an EAS is proportional to the number of electrons in the EAS, and therefore, from the above mentioned discussions, the number of Cherenkov photons which form a gamma-ray shower is three times more than that from a hadronic shower. The lateral distribution of Cherenkov light initiated by a gamma-ray is almost flat and extended to a radius \( \sim 150 \text{ m} \) at sea level. Therefore the detection area is of the order of \( 10^4 \text{ m}^2 \), which is much larger than the typical area of 1 m\(^2\) in satellite experiments. Moreover, the Cherenkov photon density detected at sea level is a good parameter to estimate the primary gamma-ray energy. Some of the Cherenkov photons are Rayleigh and/or Mie scattered in the atmosphere before arriving at the ground, and the number of Cherenkov photons from EAS initiated by 1 TeV primary gamma-ray is \( 20 \sim 50 \text{ m}^{-2} \) in \( 350 \sim 550 \text{ nm} \) at sea level. Therefore, it is necessary for the efficient detection of VHE gamma-rays to use large reflectors collecting Cherenkov photons from the EAS.

5.4 Shape Analysis

5.4.1 Hillas Parameters

The Cherenkov light which is emitted from a shower is spread according to the Cherenkov angle. Shower images from gamma-rays are sharp and the axes of the images are directed to the primary direction. On the other hand, shower images of protons are somewhat
scattered in the FOV. Cherenkov telescopes catch these images and separate gamma-rays and protons via shape analysis as shown in Fig 5.3.

The analysis method of Cherenkov light from gamma-rays is called “imaging analysis [57]” (Appendix C). Cosmic rays are dominated by hadrons by 99% or more. We must select gamma-ray events which are present at the 1% level. These analysis parameters, called “Hillas parameters”, are shown in Fig 5.4. If single telescope data are used, the number of parameters is 4. “Length” is defined as the long axis of ellipse, and “Width” is defined as the short axis of ellipse. Center of Gravity (CoG) is defined as the centroid of the image. “Distance” is defined as the distance between the source position and the CoG. “Alpha” is a parameter which describes the orientation of the image. If there are many gamma-rays, we can see the excess events of gamma-rays in the “Alpha” plot.

**Figure 5.3:** Cherenkov images of a gamma-ray (left) and a proton (right). Both were simulated by Monte Carlo simulations [137].

**Figure 5.4:** Hillas parameters in the case of the single observations. “Length” is defined by a long axis of ellipse, and “Width” is defined by a short axis of ellipse.
5.4.2 Stereoscopic Observation

We can determine geometrical information of showers using stereoscopic observations. As shown in Fig 5.5, in the stereo analysis the arrival direction of showers can be calculated on an event-by-event basis. The distance between the intersection point and the source position is defined as $\theta$. Considering the phase space of the $\theta$ distribution, the distribution of $\theta^2$ is expected to be flat. The calculation of the $\theta^2$ is described in Section 10.3.2.

Furthermore we can calculate the altitude of showers using stereoscopic observations. It gives a better energy resolution than in the case of single telescope analysis.

![Figure 5.5: Image parameters in stereoscopic observations. The distance between the intersection point and the source position is defined as $\theta$.](image)

5.5 Stereo Mode Design for CANGAROO-III

5.5.1 Study of the Stereo Design

The stereo design study by Monte-Carlo simulations (MC) for CANGAROO-III has been summarized in Ref [35]. For gamma-ray cascades, we chose $E^{-2.5}$ spectrum (Crab-like spectrum [?]). The minimum and maximum of the generated energies were 50 and 5000 GeV, respectively, for cascades from the zenith, with the maximum increased for
the low elevation simulations. The design of stereo observations is explained along with the previous study in this section.

**Pixel Size**

The pixel spacing of the CANGAROO-II camera is 0.112° [115]. This is based on the use of 1/2" PMTs. This setup has a FOV of 2.7° × 2.7° square. We are forced to conduct so-called “Long ON/OFF” mode for observations [75]. As described in Appendix A, we need a larger FOV of approximately 4° in order to estimate background levels during ON source observations. As the major part of shower images of gamma-rays are typically contained within a 1° circle, we selected a larger FOV (4°) to contain two of 1 degree circles. The PMT size was selected to be 3/4". Because of the weight limitation due to the telescope structure, we cannot increase the camera weight very much [73]. We compared the shower image parameters for several cases;

**setup 1:** Five hundred and seventy-six 1/2" PMTs of the same type as the CANGAROO-II camera [73]. With a pixel spacing of 0.112°, the total field of view (FOV) was 2.7° × 2.7° square.

**setup 2:** Five hundred and seventy-six 3/4" PMTs. A pixel spacing of 0.168° yields a FOV of 4° × 4° square.

**setup 3:** Two hundred and fifty-six 3/4" PMTs, with a FOV of 2.7° × 2.7° square.

**setup 4:** Five hundred and seventy-six 3/4" PMTs with a smaller PMT separation than in setup 2: a pixel spacing of 0.147° and a FOV of 3.5° × 3.5°.

The deterioration of the α resolution is very small, as shown in Fig 5.6c. The small width is better than large. The width deteriorates slightly due to the pixel-size effect, but not that greatly (Fig 5.6b). The length, however, changed significantly (Fig 5.6a). In order to check the FOV effect, we tried setup 3, i.e., the same PMT size as setup 2 but the same FOV as setup 1. From Figure 5.6a, it is apparent that the deterioration in length is due to the change in the FOV. We conclude that the smaller FOV deforms the length distribution and that setup 2 is preferred.

A 3/4" PMT has a diameter of 18.6 ± 0.7 mm[59]. In setup 2, the spacing of the PMTs was 24 mm. A clearance of 5 mm was kept. We tried setup 4 with a spacing of
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Figure 5.6: Comparison of parameters (a) length, (b) width, and (c) $\alpha$ for three camera designs. The curves are for gamma-rays MC whose index is -2.5 (Crab-like spectrum) and cascade from the zenith. The solid lines are for setup 1, the dashed lines are setup 2, and the dotted lines are setup 3.

21 mm (clearance of 2 mm). Comparisons in Fig 5.7 show that the difference is sufficiently small. We again concluded that the setup 2 is a reasonable choice. In the real design of the CANGAROO-III camera, we have selected a “hexagonal” arrangement of PMTs with alternate rows offset by half a pixel.

Study of the Effective Area vs. Telescope Spacing

This study is based on the effect of the telescope spacing [138, 61]. First, the calculations of the “effective area” versus the incident gamma-ray energy have been done as shown in Fig 5.8. Gamma-rays of various energies from the zenith were generated.

Here, the effective area is the product of the “real” effective area ($m^2$) and a Crab-like spectrum $(E/\text{GeV})^{-2.5}$ in order to show the effective threshold. These curves were obtained after smoothing. They peak at around 200 GeV and decrease as a function of the telescope spacing. The light pool on the surface has a radius of approximately 100 m. The coincidence rate of the two telescopes, therefore, decreases as the spacing increases.
Figure 5.7: Comparisons between setup 4 and setups 1 and 2 for (a) length, and (b) width: The solid line is setup 1, the dotted setup 2, and the dashed setup 4. The curves are for gamma-rays MC whose index is -2.5 (Crab-like spectrum) and cascade from the zenith.

Figure 5.8: “Effective area” versus the incident gamma-ray energy. The vertical axis is the effective area (m$^2$) multiplied by the $(E/\text{GeV})^{-2.5}$-energy spectrum. The telescope spacing was varied from 60 to 140 m; the solid line is for 60 m, the dashed 80 m, the dot-dashed 100 m, the solid with triangles 120 m, and the dashed with squares 140 m [35].
5.5.2 Study of Angular Resolution vs. Telescope Spacing

In the stereoscopic mode, the images from the two telescopes should point to the same direction, i.e., to the source direction. The angular resolution of this method should improve with a larger telescope spacing due to the larger opening angles of images. This is shown in Figure 5.9.

![Figure 5.9: Angular resolution versus the incident gamma-ray energy. The resolution is obtained on an event-by-event basis. The solid line is for 60 m, the dashed 80 m, the dot-dashed 100 m, the solid with triangles 120 m, and the dashed with squares 140 m.](image)

In the figure, the curves “wiggle” below 100 GeV due to the lack of Monte-Carlo statistics, but the general trend of worsening angular resolution with decreasing energy is clear. This has an opposite energy dependence compared with that of the effective area. Typically, at around the threshold (\(\sim 200 \text{ GeV}\)), the angular resolution is 0.2° per shower.

5.5.3 Figure of Merit for the Stereo Mode

For a point-source observation, we can define the following figure of merit (FOM) using the above two parameters:

\[
FOM = \frac{\text{“effective area”}}{\sqrt{\text{angular resolution}}} \left[ \text{m}^2 \text{GeV}^{-2.5} \text{(degree)}^{0.5} \right].
\]
This value is proportional to the statistical significance of the observations. The energy dependence of the FOM for various telescope spacings are plotted in Figure 5.10.

**Figure 5.10:** Figure of merit (described in the text) versus the incident gamma-rays energy. The solid line is 60 m, the dashed 80 m, the dot-dashed 100 m, the solid line with triangles 120 m, and the dashed with squares 140 m.

The FOM is maximized at the 80 m spacing. The spacing dependence, however, is small within this range. Also, the energy threshold for the stereoscopic mode was estimated to be 200 GeV by MC. In conclusion, telescope spacings between 60 and 140 m are all acceptable. We selected a 95–100 m spacing for the CANGAROO-III experiment.
Chapter 6

CANGAROO-III Telescope System (Imaging Atmospheric Cherenkov Telescopes)

The CANGAROO (Collaboration of Australia and Nippon for a GAmma Ray Observatory in the Outback) is an international collaboration for very high energy gamma-ray observations using IACTs since 1992. The observation site is located near Woomera.

1The collaborators of CANGAROO group:

1Institute for Cosmic Ray Research, University of Tokyo, Kashiwa, Chiba 277-8582, Japan
2Department of Physics, Graduate School of Science, Kyoto University, Sakyo-ku, Kyoto 606-8502, Japan
3Research School of Astronomy and Astrophysics, Australian National University, ACT 2611, Australia
4Department of Physics and Mathematical Physics, University of Adelaide, SA 5005, Australia
5Department of Physics, Yamagata University, Yamagata, Yamagata 990-8560, Japan
6Institute of Space and Astronautical Science, Sagamihara, Kanagawa 229-8510, Japan
7Faculty of Management Information, Yamanashi Gakuin University, Kofu, Yamanashi 400-8575, Japan
South Australia (136°46'E, 31°06'S, 160m a.s.l.) as shown in Fig 6.1.

Figure 6.1: The CANGAROO-III observation site is located near Woomera, South Australia (136°46'E, 31°06'S, 160m a.s.l.).

The CANGAROO-II project started in 1999 using a segmented reflector of 7m diameter consisting of 60 spherical mirrors of 80 cm diameter. This telescope was equipped with a fine-resolution imaging camera consisting of 512 photomultiplier tubes of 1/2” diameter. This telescope could observe gamma-rays with energies greater than 500 GeV.

The CANGAROO-III project started in 2000 and constructed four telescopes. The first telescope was an upgraded version of the CANGAROO-II telescope. In December 2002, stereoscopic mode with two telescopes started and stereoscopic mode with three

The CANGAROO-III telescopes are shown in Fig 6.2. The telescopes are located at the corners of a diamond shape. The distance between the telescopes is about 100 m in order to observe the maximum number of Cherenkov photons. The details of this telescope are described in the following sections.

Figure 6.2: CANGAROO-III four-telescope array. The telescopes are located at the corners of a diamond shape. The distance of telescopes is about 100 m.

6.1 Reflector

The main frame of the CANGAROO-III 10 m reflector is a paraboloid with a focal length of 8 m, whose design is inherited from an existing radio. The reflector surface was covered with 114 spherical mirror segments of 80 cm diameter (Fig 6.3) to have a collection area of $5.7 \times 10^5$ cm$^2$. Each FRP composite segment weighs only 5.6 kg for an 80 cm diameter and the total weight of the reflector is reduced to 6.6 tonne. The orientation of each
6.2 IMAGING CAMERA

The spot-size of the second telescope after the alignment is $0^\circ.21$ (FWHM) as shown in Fig 6.4. This value is comparable to that of the first telescope. Further fine tuning narrowed the spot-size down to $0^\circ.18$. We measured the spot-size by observing 25 stars which have elevation angles from $15^\circ$ to $85^\circ$ in order to check the effect of gravitational deformation of the telescope on the image quality. The correlation between elevation angle and spot-size in horizontal/vertical directions were -0.1 and 0.1 degrees respectively (Fig 6.4) and it was confirmed that gravitational effects are negligibly small compared to the spot size.

### Table 6.1: Summary of the CANGAROO-III 10-m reflector.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frame</td>
<td>parabolic</td>
</tr>
<tr>
<td>Diameter</td>
<td>10.4m</td>
</tr>
<tr>
<td>Focal length</td>
<td>8m</td>
</tr>
<tr>
<td>$f$</td>
<td>0.77</td>
</tr>
<tr>
<td>Number of segmented mirrors</td>
<td>114</td>
</tr>
<tr>
<td>Mirror diameter</td>
<td>80cm</td>
</tr>
<tr>
<td>Total collecting area</td>
<td>57.3 m$^2$</td>
</tr>
<tr>
<td>Mirror segment shape</td>
<td>spherical</td>
</tr>
<tr>
<td>Mirror curvature</td>
<td>16.4m</td>
</tr>
<tr>
<td>Mirror material</td>
<td>GFRP</td>
</tr>
</tbody>
</table>

The spot-size of the second telescope after the alignment is $0^\circ.21$ (FWHM) as shown in Fig 6.4. This value is comparable to that of the first telescope. Further fine tuning narrowed the spot-size down to $0^\circ.18$. We measured the spot-size by observing 25 stars which have elevation angles from $15^\circ$ to $85^\circ$ in order to check the effect of gravitational deformation of the telescope on the image quality. The correlation between elevation angle and spot-size in horizontal/vertical directions were -0.1 and 0.1 degrees respectively (Fig 6.4) and it was confirmed that gravitational effects are negligibly small compared to the spot size.

### 6.2 Imaging Camera

The camera is contained in a cylindrical vessel of 800 mm in diameter and 1000 mm in length, which provides shielding from both rain and light [71]. The vessel is made of an
aluminum alloy (A5052) in order to reduce the weight and provide sufficient rigidity.

Inside the camera vessel, 427 PMT modules, regulator circuit panels, an LED (light emitting diode) light diffuser for gain calibration, and several other instruments such as a thermometer and etc., are contained. The camera frame consists of two aluminum templates (5 mm in thickness), in which holes (21 mm in diameter) are drilled at locations corresponding to each of the 427 pixels. Every PMT module, consisting of a PMT and a preamplifier (20.5 mm in diameter), is held by its template. Light guides are attached to the front panel. The photo-cathode plane of the PMT module is held close to the back plane of the light guide. The front panel, on which the light guides are attached, is fixed at the focal plane. All segmented mirrors can be seen from every pixel position.

The pixels are arranged in a hexagonal shape in order to maximize the collection efficiency of the Cherenkov light. The pixel size was determined to be 0.17 degrees from a simulation study [35], taking into account the spot size of the 10 m composite mirror [95].

6.2.1 PMT Module

The PMT module is cylindrical with a diameter of 20.5 mm and a length of 173.5 mm. Three types of cables (the signal, the D.C. power and the high voltage supply) are passed through from the back end of the module. The module consists of a 19 mm (3/4 inch) PMT (Hamamatsu R3479UV), a bleeder circuit and a pre-amplifier which are attached
6.2. IMAGING CAMERA

Figure 6.4: Spot-size of the second telescope is shown in the left figure. Elevation dependence of the spot-size is the right figure.

The quantum efficiencies were measured for 10 out of 450 PMTs as a function of the wavelength, as shown in Fig. 6.6 (right). The efficiency curves were very similar for all 10 PMTs. Since it is difficult and expensive to measure the quantum efficiency for all PMT modules, the quantum efficiencies of the other PMTs were estimated for all modules from the $Skb$ parameter, which gives the sensitivity of the cathode for blue light. $Skb$ was measured with light from a Tungsten lamp filtered with an optical filter to a narrow wavelength band around 400 nm. Its unit is $[A/\text{lm}]$. It is easier to use $Skb$ than to measure the quantum efficiency. We confirmed that there is a good correlation between the quantum efficiency at 400 nm and $Skb$ for the 10 PMTs for which full measurements were made, and then estimated the quantum efficiency based on the measured $Skb$ and the relation derived from the 10 PMT sample. The average of the quantum efficiency was
estimated to be $25.0 \pm 1.4\%$ (a variation of 7.3\%).

### 6.2.2 Performance of each PMT module

Before each camera was installed on its telescope, the characteristics of all PMT modules were calibrated individually.

The typical distribution of a single photo-electron peak is shown in Fig. 6.7 (left). The peak due to a single photon signal can be clearly separated from the background.

The linearity of all the PMT modules is shown in Fig. 6.7. The gain including pre-amplification was set to $1.2 \times 10^7$. Data points were fitted using the following empirical formula:

$$F(x) = \begin{cases} 
 x & (x \leq a) \\
 \frac{(x-a+c)^b-c^b}{b} e^{(1.0-b)} + a & (x > a),
\end{cases}$$

where $a$ approximately corresponds to the turning point of the line. The resulting average and standard deviation of $a$ were $202.1 \pm 12.7 \ (1\sigma) \ p.e.$, as shown in Fig. 6.7 (right, upper). The deviation from a linear line at 250 p.e. of the input light was estimated to be $-5.1 \pm 2.0 \ (1\sigma) \ %$ from Fig. 6.7 (right, lower). The mean curve was input to the MC simulation code. PMT modules with a deviation from a linear line worse than $-20\%$ were rejected.
Figure 6.6: Photograph of the PMT module (left) and the quantum efficiency of the photo-cathode (right). Each module consist of a 3/4-inch PMT(R3479) and Preamp(MAX4107). The quantum efficiencies have been measured for 10 PMTs as a function of the wavelength measured for 10 PMTs.

6.2.3 Light-guide

The camera consists of PMTs with a significant amount of dead space between them, amounting to \( \sim 65\% \) of the total surface area. Light-guides reflect photons which would otherwise be incident upon the dead space to the photo-cathode area of the PMTs, thus increasing the light-collection efficiency. A photograph is shown in Fig 6.8.

Another advantage is that light-guides reduce the background of photons coming from outside of the mirrors, i.e., at a shallow angles with respect to the light-guide plane.

Various types of light guides made by combining the Winston cone, paraboloids and flat planes were examined by simulations. The Winston cone as shown in Fig 6.9 is a non-imaging optical shape used to concentrate all photons whose incident angle is less than a certain angle \([129, 130]\). The maximum incident angle to the light guide is determined by the incident light from the outer edge of a 10 m mirror dish, which ranges from 32.2° to 35.2°, depending on the position on the camera surface. Although the entrance shape of the original Winston cone is round, as the PMT’s are arranged on a hexagonal grid with a spacing of 24 mm, a hexagonal entrance aperture was chosen so as to obtain less dead space and a better light-collection efficiency. The ratio of the entrance area to the exit area of the light-guides is 2.68 for the case of a gap of 0 mm between the light-guides and
Figure 6.7: PMT performance. The left is a single photon spectrum. The histogram is the ADC data and the curve is the fitting line. The right shows linearity (upper) and saturation (lower).

2.46 for a 1 mm gap.

The reflectance of the samples was measured by a spectrophotometer. One of the results is shown in Fig. 6.9 as a function of wavelength. The incident angles for this measurement were 5° and 60°. The reflectance value ranges from 75% to almost 90% above a wavelength of 310 nm for both incident angles. These data were input to the MC simulation code.

6.2.4 Uniformity of Gain

Fig. 6.10 shows the uniformity of the gain for all PMT modules. We should note that the measured gain by this calibration should be slightly different from that measured by a single p.e. calibration because the dependence of the quantum efficiency for the respective PMTs cannot be estimated by a single p.e. calibration. Therefore, the measured ADC value should be divided by the $Skb$ value, which can be considered to correspond to the quantum efficiency. The average ADC/$Skb$ over all PMT modules was $44.4 \pm 4.8$ [counts·lm/A], with a deviation of 11%. 
6.2. IMAGING CAMERA

Figure 6.8: Photograph of the Light-guide

Figure 6.9: Performance of the Light-guide. The left figure shows a comparison of light collection efficiency between a simulation and a measurement of the light-guide. The right figure shows the measurement reflectance of the new light-guide.

6.2.5 High-voltage Supply

A multi-channel and individually controllable high-voltage (HV) supply system is required in order to obtain a uniform pixel gain. The high-voltage supply system (CAEN SY527) controls up to 10 modules of CAEN A932AP, each of which contains 24 channels of the voltage supply. The primary voltage of the board can be changed over the range of 0–2550 V.

This high-voltage system can be controlled via the VME module, CAEN V288. A computer-displayed control program was constructed using a Tcl/Tk graphical interface, making it possible to monitor the status and to control the supplied high voltage. Also, the monitor program can calculate the positions of a bright star images on the camera.
Figure 6.10: Gains for all PMT modules in the camera. The histogram is the ADC data and the curve is the fitting line.

plane using the SAO (Smithsonian Astronomical Observatory) catalog and accordingly control the gain of the corresponding pixels on a timescale which is much faster than the passage of the bright stars around the field of view (≈30 sec/pixel).

6.3 Electronics and Data Acquisition System

An overview of the DAQ system of the second, third, and forth telescope is shown in Fig 6.12.

The Discriminator and Summing Module (DSM) of CANGAROO-III adopted a VME-bus (CERN V430 type). In the DSMs the signal is inverted according to the polarity of the VME9U-bus (CERN V430 type) 32ch with 15bit. There are 32 ADC chips (BB ADS7805 or ADS7815) to convert signals in parallel. The outputs of the updating discriminator in the DSM is fed to VMEbus 128ch TDC (CAEN V673) which has 0.78 nsec time resolution, and both the leading and trailing edges are recorded.

In the DSM, the signal from each PMT is amplified with a fast shaping amplifier, and the summed signal of 16 channels is output (hereafter ASUM).

The amplified signal is fed to two discriminators; one measures the hit timings by TDC, while the other measures the counts over the threshold during about 700 μsec with a 12 bit scaler. Both act to reduce the night-sky background in the offline analysis: the
former based on the fact that the telescope is parabolic and the time propagation of a shower can be reconstructed with high accuracy, and the latter is used to reject PMTs affected by starlight. The thresholds of both discriminators are adjustable via the VME-bus. The window of the latter discriminator is set to be about 20 ns, the same as the time dispersion of showers, and the output of 16 channels is summed (hereafter $LSUM$).

The DAQ trigger is generated as shown in Fig. 6.11. $LSUM$ signals from the DSMs connected to the 427 PMTs are summed, and discriminated to determine the number of PMTs hit at the same time. The threshold of the discriminator is set to be 4 or 5 PMTs. On the other hand the $ASUM$ signal is discriminated to select concentrated hit patterns, then the summed signal is discriminated. PMT triggers are generated from the coincidence of the outputs from the two discriminators, then DAQ triggers are generated.

![Figure 6.11: DAQ trigger of the CANGAROO-III telescope.](image)

If the threshold in the DSM is lowered, the trigger rate due to the fluctuating night-sky background increases. However it is possible to reduce the night-sky background because the hit pattern due to the night-sky background is random, while that of showers is concentrated. Thus the pattern trigger module gets hit signals of all 427 PMTs and recognizes the 2-dimensional pixel pattern, then generates a trigger if there is a pattern of $N(\geq 3)$ adjacent pixels. PLDs (Altera EPF10k130) are used for the pattern selection, and take less than 100 ns from input to output according to the design simulator. The power of this module is supplied via the VME-bus.

Both weather and cloud monitors are connected to a PC (PC1 in Fig. 6.12) with RS232C lines, and read out once per minute. The Linux operating system is adopted, because the context switching time of Linux is as fast as $2\mu s$. The PC1 also collects scaler counters in the DSMs via a PCI-VME bridge every 10 seconds, and the real time pointing direction of the telescope via a 100-Base network from a PC (PC2 in Fig. 6.12)
which controls the telescope. The sizes of local stored data for each telescope are T1: 9G, T2: 27G, T3: 25G, and T4: 33G / month.

Figure 6.12: Overview of the data acquisition system of the second telescope.

6.4 Local and Global Trigger

The four telescopes of CANGAROO-III were placed at the corners of a diamond with sides of about 100 m. In stereoscopic observations with more than two telescopes, a global DAQ trigger is generated as shown in Fig 6.13. The local trigger is converted to an optical signal and transmitted to the central electronics hut through optical fibers about 100 m in length. The local trigger of each telescope is delayed with a VME-bus
controlled delay generator, whose delay is set by real-time calculation of the difference of path lengths of Cherenkov light, using the pointing directions of the four telescopes. The four delayed triggers are fed to a coincidence unit, which generates a global trigger that stops a TDC to measure the time difference between the local triggers of the four telescopes.

The global trigger is distributed to the four telescopes, and it interrupts the VME-CPUs (Pentium III 700MHz) via a VME interrupt register, then starts collecting data from the ADCs, TDCs, scalers, and GPS via the VME-bus. The system is designed to accept triggers up to 100 Hz. However, if a global trigger is not generated within 2\(\mu\)s after a local trigger is generated, these readout modules are reset in each telescope. The VME-CPUs transmit all collected data to a PC (PC3 in Fig 6.12) in the central electronics hut, and the data is stored on hard disks connected with PC3.

PC3 is also a disk-less server for the VME-bus CPUs of the four telescopes. Another PC (PC4 in Fig 6.12) in the central electronics hut reconstructs the event from the data for quick analysis, and plays a role as a NTP server.

![Diagram of the trigger system](image)

**Figure 6.13:** Global trigger of the four telescopes. The area enclosed with dashed lines shows a local trigger of each telescope.
Chapter 7

Observation

7.1 Observations

In this section, we explain how data were taken.

Crab

We took data on the Crab in 2003 from 15 to 28 Dec. The pointing position for the Crab is \((\alpha, \delta) = (83.625^\circ, 22.0167^\circ)\) which is the position of the Crab pulsar. The observation mode was chosen to be the Wobble mode (Appendix A). These data were taken at very large zenith angles such as \(53^\circ \sim 65^\circ\). The average zenith angle was estimated to be 55 deg. The observation period is summarized in Table 7.1.

<table>
<thead>
<tr>
<th>Observation Date</th>
<th>Observation time (T2-T3)</th>
<th>average zenith angle</th>
</tr>
</thead>
<tbody>
<tr>
<td>15-Dec – 28-Dec 2003</td>
<td>1215 min.</td>
<td>55 deg.</td>
</tr>
</tbody>
</table>

Table 7.1: Summary of the Crab observation periods.

Centaurus A

We took data on Cen A in 2004 Mar and Apr. The pointing position for Cen A is \((\alpha, \delta) = (201^\circ.365, 43^\circ.019)\). This position is the center of galaxy. The observation mode was again chosen to be the Wobble mode. The average zenith angle is 17 deg. The observation periods are summarized in Table 7.2.
7.1. **OBSERVATIONS**

### Table 7.2: Summary of the Cen A observation periods.

<table>
<thead>
<tr>
<th>Observation Date</th>
<th>Observation time (T2-T3)</th>
<th>Observation time (T2-T4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>15-Mar. – 28-Mar. 2004</td>
<td>643 min</td>
<td>452 min</td>
</tr>
<tr>
<td>15-Apr. – 28-Apr. 2004</td>
<td>554 min</td>
<td>468 min</td>
</tr>
<tr>
<td>Total</td>
<td>1197 min</td>
<td>920 min</td>
</tr>
</tbody>
</table>
Chapter 8

Calibration of CANGAROO-III Data

8.1 Pedestal Data

Individual ADC channels have finite charge pedestals. First, we need to know this value. Two kinds of the pedestal runs are taken during observation time. One is a “dark pedestal run” in which data are taken with the lid of the camera closed. The other data are taken with the lid of the camera opened, and used to monitor the effect of the night-sky background (Section 9.1.1) as shown in left figure of Fig 8.1. The pedestals obtained by the two methods were confirmed to be the same.

We checked the time dependence of the pedestal fluctuation. The pedestal data were taken before starting observation runs. The time dependence of the pedestal of a certain pixel is shown in the middle figure of Fig 8.1. These data were taken during the Crab observation term. The pedestal means are shown in the right figure. The pedestal fluctuation was 2 % in RMS when the camera lid was opened. The dispersion of the pedestal fluctuation was 6.5 ADC counts. This value corresponds to 0.07 p.e. The hardware threshold of each pixel is 7.5 p.e. (Section 9.1.2). So, this pedestal fluctuation was 1 % level for the hardware threshold. This value was small enough to analyze our data.
8.2. GAIN CORRECTION

Figure 8.1: The typical pedestal distribution of a certain pixel is shown in the left figure. These data were taken with the camera lid opened. The time dependence of the pedestal is shown in the middle figure. The X-axis is the elapsed time from the observation start. We could find no significant fluctuation according to the time. The pedestal means are shown in the right figure. The pedestal fluctuation was 2% compared with the RMS of the pedestal distribution. Also the pedestal fluctuation was about 1% compared to the hardware threshold.

8.2 Gain Correction

The gain of each channel is measured by the "KONAN LED" system [71]. This system is schematically shown in Fig 8.2. The "KONAN LED" system is able to illuminate the whole camera by a blue LED with uniform brightness.

The average number of photoelectrons (µ_{p.e.}) can be obtained from the ADC distribution, assuming that it obeys a Poisson distribution and the ADC counts are proportional to the number of generated photoelectrons. The Poisson distribution satisfies the following formula:

\begin{equation}
\mu_{p.e.} = \left( \frac{\mu_{ADC}}{\sigma_{ADC}} \right)^2, \quad (8.1)
\end{equation}

\begin{equation}
ADC \text{ constant} = \frac{\mu_{ADC}}{\mu_{p.e.}}, \quad (8.2)
\end{equation}

where \( \mu_{ADC} \) and \( \sigma_{ADC} \) are the average and standard deviation of the ADC distribution, respectively.

Mean pedestal values are subtracted from ADC values of each channel. The pedestal-subtracted ADC values divided by the ADC constant (ADC-to-p.e. conversion factor)
**Figure** 8.2: Schematic view of the "KONAN LED" system (left). Calibration runs taken using this system are used for flat-fielding of the camera. Light uniformity of this system is shown in the right.

gives the number of p.e., that is,

$$Number\ of\ p.e. = \frac{ADC - \text{pedestal}}{ADC\ constant}. $$

### 8.3 Time-Walk Correction

The time-walk correction of each channel is done by using the "CENTER LED" system. A blue LED of this system is set at the center of each reflector. The "CENTER LED" system is shown in Fig 8.3.

The shift of discrimination timing ($T_{\text{start}}$) can analytically be calculated when the shape of the signal pulse is approximated by a Gaussian distribution $\exp(-at^2)$ and the timing correction is applied according to the following formula:

$$T_{\text{start}} = \sqrt{P_1 \cdot \log(ADC) + P_0}. $$

In practice, a profile histogram of $TDC^2$ vs $\log ADC$ is filled for each pixel, and fitted by the following linear function:

$$f(x) = P_0 + P_1 x,$$

where $P_0$ and $P_1$ are calibration parameters for the time-walk correction and obtained individually for all PMTs. An example of the fit is shown in Fig 8.4. Finally, TDC start
**Figure 8.3**: "CENTER LED" system. The effect of time-walk is corrected by using this system.

**Figure 8.4**: Scatter plot of $TDC^2$ vs $\log(ADC)$ using LED calibration data, fitted by a linear function (the green line).
information is expressed as follows:

\[ T_{\text{start}} = 0.78 \cdot \left( \sqrt{P_1 \cdot \log(\text{ADC}) + P_0} - T_{\text{DC}} \right), \]

where 0.78 is the TDC time resolution in nanosecond. \( T_{\text{start}} \) corresponds to the relative hit timing of a channel in nanosecond.

### 8.4 Removing Bad Channels

The definitions of bad channel are as follows;

- The ADC constant of the channel is a factor of 5 smaller or larger than the mean of ADC constants in the camera.

- The RMS of the \( T_{\text{DC}}^2 \) distribution of the channel is larger than 20000 (about 3 times of the RMS mean in the camera).

Those channels were not used in the rest of the analysis.

**Time dependence of bad channels**

We checked the time dependence of the bad channels. That for the Crab observation is shown in Fig 8.5. That for Cen A observation is also shown in Fig 8.6. Before starting the observation of April, some bad PMTs were exchanged to the new one. We inputted the maximum number of bad channels both to the experimental and to the MC analysis.
Figure 8.5: Time dependence of number of bad pixels for the Crab observation. That of bad channels of T2 is shown in the left figure and T3 is shown in right figure.

Figure 8.6: Time dependence of number of bad pixels for the Cen A observation. That of T2 is shown in the left figure, T3 is shown in the middle, and T4 is shown in the left.
Chapter 9

Monte-Carlo Simulation

9.1 Monte-Carlo Setups

Simulations of electromagnetic and hadronic showers in the atmosphere were carried out using a Monte-Carlo simulation code based on GEANT3.21[47]. In this code, the atmosphere was divided into 80 layers of equal thickness (~12.9g/cm²) [35]. Each layer corresponds to less than a half radiation length. The dependence of the results on the number of layers was checked by increasing the number of layers, and was confirmed to be less than a 10% effect. The lower energy threshold for particle transport was set at 20 MeV, which is less than the Cherenkov threshold of electrons at ground level. Most Cherenkov photons are emitted higher in the atmosphere, at lower pressure and a higher Cherenkov threshold. The geomagnetic field at the Woomera site was included in the simulations (a vertical component of 0.520 G and a horizontal component of 0.253 G directed 6.8° east of south).

In order to save CPU time, Cherenkov photons were tracked in the simulations only when they were initially directed to the mirror area. The average measured reflectivity of 80% at 400 nm and its wavelength dependence [73] and the measured PMT quantum efficiency were multiplied using the Frank-Tamm equation to derive the total amount of light and its wavelength dependence. A Rayleigh-scattering length of $2970(\lambda/400\text{nm})(\text{g/cm}^2)$ [11] was used in transport to the ground. No Mie scattering was included in this study. The contribution of Mie scattering is thought to be at most the 10–20% level; we therefore consider this study to have uncertainties of at least this level. When Rayleigh scattering
occurred, we treated it as absorption. Finally, the electronics simulation was carried out. In the following section, we discuss the estimation of several electronics parameters.

### 9.1.1 Estimation of Night Sky Background

Atmospheric Cherenkov telescopes have the problem of the Night Sky Background (NSB). NSB is estimated by Jelly\[67\] and the following formula is called “Jelly’s formula”. NSB is generally written by this formula.

\[
\frac{dN}{dt} = \frac{d^3W}{dSdtd\Omega} \cdot S_m \cdot \Omega_\theta \cdot \epsilon_q \cdot \epsilon_r \cdot \epsilon_l = 7.2 \times 10^{-2} \text{p.e.} \cdot \text{ns}^{-1} \cdot \text{PMT}^{-1} \tag{9.1}
\]

\[
\frac{d^3W}{dSdtd\Omega} = 2.55 \times 10^{-4} \text{erg} \cdot \text{cm}^{-2} \cdot \text{s}^{-2} \cdot \text{sr}^{-1} \tag{9.2}
\]

- \( W \): photon energy
- \( t \): NSB / unit time
- \( \Omega \): Solid angle
- \( S \): Area
- \( S_m \): Collecting area of reflector, \(5.74 \times 10^5 \text{ cm}^2\)
- \( S_\theta \): Field of view of pixel, \(6.28 \times 10^{-6} \text{ sr}\)
- \( \epsilon_q \): Averaged quantum efficiency of spherical PMT, 0.2
- \( \epsilon_r \): Averaged reflectivity of spherical mirror, 0.8
- \( \epsilon_l \): Efficiency of photo-sensitive area including the light-guide, 0.78

According to the calculation, NSB for CANGAROO-III observation is \(7.2 \times 10^{-2} \text{p.e. ns}^{-1} \text{ PMT}^{-1}\).

In order to check this, a method to estimate NSB is needed. If NSB obeys the Poisson distribution, NSB can be measured from ADC fluctuations. In the Poisson distribution, the probability of \( n \) is given by the following formula,

\[
P(n) = e^{-\mu} \frac{\mu^n}{n!} , \tag{9.3}
\]

where \( \mu \) is the average number of occurrences. The most important thing is;

\[
\mu = \sigma^2 \tag{9.4}
\]
Figure 9.1: ADC distribution in random trigger events. The X-axis is p.e. and the Y-axis is number of events. NSB is estimated from the σ of this distribution to be 9 p.e. Here a Poisson distribution is assumed.

Where σ is the standard deviation. The standard deviation of the distribution is equal to mean NSB rate.

In CANGAROO-III, random trigger data are taken and monitored every observation day. A typical distribution of ADC is shown in Fig 9.1. The RMS of this histogram is 3 p.e. and NSB rate is estimated to be 9 p.e./100 nsec (100 nsec is ADC gate range.). This value corresponds to $9 \times 10^7$ p.e. ns$^{-1}$ PMT$^{-1}$. Hence the result of Jelly's formula is in good agreement with the monitor data. In the MC, we use the $8 \times 10^7$ p.e. ns$^{-1}$ PMT$^{-1}$ which is between Jelly's value and observed value.

### 9.1.2 Trigger Threshold

The hardware parameters are input after the discussions above. Finally, we tuned the parameters of the CANGAROO-III trigger logic. As we mentioned previously in Section 6.3, the parameters of the trigger logic are the Hit threshold and $LSUM$ threshold. The Hit threshold is the number of hit pixels which detect Cherenkov light. In the present CANGAROO-III, five pixels are the default value. The $LSUM$ threshold is the threshold for the pulse height of sum of hit pixels.
9.1. MONTE-CARLO SETUPS

Figure 9.2: Oscilloscope image of single photo-electrons. This figure was obtained using an LED light source at the single photo-electron level as input to a PMT module. The average pulse height for a single photo-electron is 4 mV.

As shown in Fig 9.2, the single photo-electron pulse height is estimated to be about 4 mV. This figure was obtained when LED light equivalent to 1-p.e. was input to the PMT module. The $LSUM$ threshold was determined to be 240 mV in hardware. This threshold was selected to obtain a comfortable trigger rate. The upper limit of this rate is restricted by capability of the DAQ system. This voltage was reduced to 1/8 by the electronic circuit. Then, $240 \times 1/8 = 30$ mV is set at the discriminator. The signal from the PMT module was fed through this discriminator. If 4 mV per 1 p.e. is adopted, the $LSUM$ threshold is estimated to be 7.5 p.e. This estimate, however, has an uncertainty in the single-photon pulse-height as shown in Fig 9.2. The fluctuation of the pulse height of a PMT module is large. Because of above reason, we need to study the $LSUM$ threshold in the following MC study.

The Hit threshold ($N_{hit}$) is decided to be 50 mV in hardware. The hit threshold is set a logical signal (10 mV per 1 hit) and the logical signal is added according to the number of hits. So 50 mV corresponds to 5 hits. This is digital logic, therefore, we used this value in the MC simulation code.

The figure which determined the $LSUM$ threshold is shown in Fig 9.3. These distributions are filled from right to left with the highest ADC, the second highest ADC, the third highest ADC ..., etc. If the Hit threshold is assumed to be 5 hits, the fifth ADC
Figure 9.3: “LSUM threshold” estimation. From right to left, histograms for the highest ADC hit values, 2nd highest, 3rd highest, ..., etc were plotted. The red line is the peak of histograms of the fifth highest ADC values.

distribution from right is important. This distribution shows that the \textit{LSUM} threshold of hardware should be 7.5 p.e. as shown by the fifth histogram. It turns out that this offline estimation of the \textit{LSUM} threshold is consistent with estimate from the hardware setting.

9.2 On Reflectivity of Mirrors at the Observation Periods

The mirror reflectivity is a function of time. Our telescopes are exposed to the unfavorable circumstances of a desert for many years. Sometimes we washed them using a water gun and sometimes by ”natural” rain. We need to monitor the reflectivity and incorporate it into the Monte-Carlo simulation. For the T1 telescope, we brought some mirrors back to Japan and measured the reflectivity. It is shown in Fig. 9.4. The measurements were carried out in total five times. The black points were obtained at the initial time of production. They were measured by Mitsubishi in 1999. The red ones were measured on
9.2. **ON REFLECTIVITY OF MIRRORS AT THE OBSERVATION PERIODS**

![Graph showing reflectivity of mirrors against wavelength](image)

**Figure 9.4:** Reflectivity of mirrors of T1. The measurements were carried out in total five times. The black points were obtained at the initial time of the production. They were measured by Mitsubishi in 1999. The red ones were measured on 2003/Dec/17 at National Astronomical Observatory Japan (NAOJ). It was taken after exposure of more than three years at Woomera. The green ones were obtained after washing the above mirror with water. The blue ones were measured on 2004/Sep/16 at NAOJ. This mirror was exposed for four years at Woomera. The yellow ones were obtained after washing. Note that the measurement by Mitsubishi was carried out using a "cheap" monochromator, therefore, the reflectivity in the ultra-violet region is uncertain. We had better concentrate on the visible region such as $\lambda > 400$ nm.

2003/Dec/17 at National Astronomical Observatory Japan (NAOJ). It was taken after exposure of more than three years at Woomera. The green ones were obtained after washing the above mirror with water. The blue ones were measured on 2004/Sep/16 at NAOJ. This mirror was exposed for four years at Woomera. The yellow ones were obtained after washing. Note that the measurement by Mitsubishi was carried out using a "cheap" monochromator, therefore, the reflectivity in the ultra-violet region is uncertain. We had better concentrate on the visible region such as $\lambda > 400$ nm.

The next Fig 9.5 plots ratios for the two periods relative to the initial time. Looking at them, one can see that the reflectivity deteriorated and also that it recovered after washing. During observation periods, we washed them once per season. The data for the blue points were obtained after the four-years’ exposure at Woomera. The amazing thing is that the reflectivity only deteriorated to no less than 70%. In this thesis, we used the data from T2, T3, and T4. They were two years, one year, and one month old, respectively. We, therefore, do estimate the reflectivity is reduced to no less than 70%. On
the other hand, the total light yield of Cherenkov images would deteriorate by the light guide reflectivity and PMT’s quantum efficiencies. Including all those effects, we need to monitor it and should reflect these values to the Monte-Carlo simulation. Considering the optics, these effect can be renormalized into the ”reflectivity” of the mirrors. From now on, the word ”reflectivity” includes all these effects.

In order to determine the ”reflectivity” at the period of observation, we need a clearly defined stable observable. We use the shower rate because the cosmic-ray rate is considered to be stable. The cosmic-ray’s spectrum is proportional to $E^{-2.7}$ in the differential definition of $dF/dE$. We measure the cosmic-rays with a certain threshold determined by the ”reflectivity”. The shower rate, therefore, varies ”reflectivity”$^{1.7}$. See the all time shower-rate plot in Fig. 9.6 below. The time origin is 2003/Jan/01. The black points are T2, the red T3, and the green T4, respectively. Although the shower-rate analysis is described in the following section, we here introduce it in advance. The analysis criteria are as follows:

- ADC cut > 5 p.e.,
- TDC interval within ±50 nsec,
- a clustering of T5a was adopted,
9.2. ON REFLECTIVITY OF MIRRORS AT THE OBSERVATION PERIODS

Figure 9.6: All time shower rate plot. The time origin is 2003/Jan/01.

- select only clear days, meaning if there is a cloud recognized by the shower-rate vs time plot, we rejected,
- the plotted points were average of one month,
- select only runs with more than 1 h,
- select only near-zenith runs, i.e., we rejected Crab observation and etc.,
- and shower rate was defined at zenith angle after extrapolation by a linear function between rate and cos(θ).

At first, we notice the stability of T2 "reflectivity" in the period of 2003 Jan to Nov and the scatter of these measurement is about 10%, which will convert to the "reflectivity" by

$$\left(\Delta \text{shower rate}\right)^{1/1.7},$$

as \(\sim 5\%\). During the period of the Crab observations, it deteriorated in fact. There may be PMT deterioration, because the Crab observations included a bright star of mag 3 well inside the FOV. Another suggestion can be seen in 2004 Jun, right after SN1006 observation, which also included a bright star of mag 2.6 inside the FOV. Concentrating on the periods of Crab and Cen A observations, the estimated "reflectivity" would be the same within the 5% error on them. The "reflectivity" in the Crab period can be estimated from the interpolation from 2003-Nov and 2004-Jan data. That in the Cen A
period can be estimated by the average of 2004-May and 2004-Apr. To conclude, we used the "reflectivity" of T2 as 70% compared to the initial value and T3 as 75%. The initial "reflectivity" was estimated from the average of T4 from 2004-Mar to 2004-Jun. In the following Chapter, we will justify these settings with the Crab observations. The Crab is the standard candle in TeV gamma-rays. If, with these inputs to the Monte-Carlo code, we could obtain the correct Crab flux, we can say our estimation is correct within the error level. Here from the above discussion, we estimated the uncertainty to be at the 10% level.

9.3 On Spot Size of Mirrors at the Observation Periods

One more important input for the Monte-Carlo simulation is a spot size for the mirror system. The spot size is a function of that of each mirror and mirror alignment. The intrinsic spot size of each mirror should be around 0.07–0.1 degree because at production we only selected those mirrors which satisfied the above criterion. The mirror alignment is somewhat more uncertain than 0.07 degree. We adjusted each mirror by changing the alignment while viewing a bright star. The adjustment was carried out for mirror by mirror. This procedure was done integrally, i.e., after one mirror was adjusted, then the next mirror was. The light yield on a white screen was monitored by a CCD camera. Each additional mirror adjustment was carried out after subtracting the previous images from the new images. Therefore, the error should be added one by one. It should result in the final error more than the first mirror spot size. In addition, considering the Woomera climate, sometimes wind exceeds 10m/sec. We observed that some of mirrors were misaligned from initial setups. We, therefore, need to monitor these alignment via observational data. The best astronomical object to monitor these is clearly the Crab. The Crab is known to be a point source, the size of which is considered to be well below our angular resolution by the lower energy measurement. We used the Crab measurement to determine the spot size of each telescope. Although the Crab analysis is described in the following section, we here introduce the spot-size measurement by the Crab in advance. As can be seen in the following section, the Crab is positively detected as a gamma-ray source. We, therefore,
can derive the *Width* distribution of Crab gamma-rays. It is shown in Fig. 9.7. The

**Figure 9.7:** *Width* distribution for Crab signals. The histogram is the -*Width* distribution for Crab excess for T2 (left) and T3(right), respectively. The analysis of which will be given in the following section. Blank histograms were obtained for the Crab data after the background subtraction. The hatched area are the Monte-Carlo estimations with spot-size=0 degree, i.e., an idealistic situation.

histograms are the -*Width* distributions for the Crab excess. The blank histograms were obtained for the Crab data after background subtraction, the analysis of which will be described in the following Chapter. The hatched areas shown are the estimation with spot-size=0 degree, i.e., an idealistic situation. The curve can be considered to be an intrinsic resolution for the IACT measurement. The disagreement between the curve and the measurement is large. A spot size of 0.1 degree is not enough in Fig. 9.7. The peak position of *Width* distribution is not consistent with 0.1 degree. Our conclusion is that there is a significant misalignment in this observation period. From this figure we could estimate the spot size of each telescope system. The results are

0.15 degree (T2) and 0.12 degree (T3).

The T4 spot size was measured in Feb. of 2004, right before Cen A observations, therefore, we adopted the optical measurement value of 0.081 degree. These values are input into the Monte-Carlo simulation and later are proved to be correct by the Crab measurement.
Here, we understand that a future upgrade should be concentrated on these. We need to minimize the spot size. Later we will see the Crab spectrum and find that the signal-to-noise ratio (S/N) of the Crab spectrum is bad in our measurement compared to that by H.E.S.S. This is all due to this effect. The other components are working very well, even compared to H.E.S.S. We had spent many years in developing a light plastic mirror. We, however, should consider changing it to a metal mirror such as developed by the MAGIC collaboration in future.
Chapter 10

Analysis of CANGAROO-III Data

10.1 Data Cleaning

The CANGAROO-III data were taken at ground level and, therefore, suffered from circumstances. There are backgrounds from star light and artificial light. Such light (called Night Sky Background (NSB)) should be removed from the data because atmospheric Cherenkov light is faint. In order to separate Cherenkov light from the NSB, data cleaning is necessary. The data cleaning is based on the three methods, ADC cut, Clustering cut, and TDC cut. The timing distribution of most of the gamma-ray showers should be concentrated within 10 nsec according to the MC predictions, although the NSB should distribute randomly. We can, therefore, use the TDC distribution to check the data quality at each cut level.

10.1.1 ADC Cut

First, a cut on ADC data was carried out. The raw ADC data distribution is shown in Fig 10.1. The peak of this distribution corresponds to the hardware threshold. The signals below the hardware threshold are due to Poisson fluctuation of the NSB, which has a mean of 9 p.e., and electronics noise. Here we had better select a cut as low as possible in order to maximize the acceptance. The relation of the ADC cut and the TDC distribution is shown in Fig 10.2. Note that increasing the ADC cut (0, 3, 5, 7 p.e.), the TDC distribution become thinner and the level of random background is reduced. This indicates that the data become cleaner by this operation. Here, an ADC cut of 5 p.e. was
Figure 10.1: Raw ADC distribution. The X-axis is p.e. and the Y-axis is number of events. The red line indicates a cut at 5 p.e. The peak at 0 p.e. is due to bad channels which were identified by the calibration.

Figure 10.2: TDC distribution. The X-axis is nanosecond and the Y-axis is number of events. The black line is obtained with no ADC cut. The red line is obtained with a 3-p.e. ADC cut. The blue line is obtained with a 5-p.e. cut. The green line is obtained with a 7-p.e. cut.
Figure 10.3: ADC cut location versus TDC hit rate (the left panel). The black line is the NSB rate estimated from the TDC sideband (\(60 \text{ ns} < \text{TDC} - \text{TDC}_{\text{mean}}, -60 \text{ ns} > \text{TDC} - \text{TDC}_{\text{mean}}\)), which is considered to be proportional to the background. The red line indicates the signal rate estimated from the on-time (\(|\text{TDC} - \text{TDC}_{\text{mean}}| < \pm 25 \text{ ns}\)) pixels from the TDC distribution which is considered to be proportional to the acceptance. The horizontal axis is the cut location. The flat region of the NSB rate is the TDC threshold. This is a differential plot. The TDC-hit rates between (ADC-cut) and (ADC-cut+1) are shown. The right panel shows the figure of merit which is on-time-hit rate divided by the square-root of off-time-hit rate. We chose the best point, i.e., 5-p.e. cut.

We check this ADC cut position in the following. The left panel of Figs 10.3 are plots for NSB hit rate estimated from the TDC sideband (the black line),

\[
60 \text{ ns} < \text{TDC} - \text{TDC}_{\text{mean}}, -60 \text{ ns} > \text{TDC} - \text{TDC}_{\text{mean}},
\]

and signal rate estimated from the on-time (the red line),

\[
|\text{TDC} - \text{TDC}_{\text{mean}}| < \pm 25 \text{ ns}
\]

region in the TDC distribution (Fig 10.2) at each ADC cut. This is a differential plot. The TDC-hit rates between (ADC-cut) and (ADC-cut+1) are shown. The black line is the NSB rate obtained from the TDC sideband, which is considered to be proportional to the background. The red line is the on-time hit rate in the TDC distribution which
is considered to be proportional to the acceptance. The region of flat background (the black line) appears around 4-5 p.e. cut, where we estimate the hardware threshold effect emerges. We need to set a threshold higher than this in order to derive correct acceptance by the Monte-Carlo simulation. On the other hand, the red line shows that the higher ADC cut would lose acceptance. The right panel of Fig 10.3 show the figure of merit which is on-time-hit rate divided by the square-root of off-time-hit rate. It peaks around at 5 p.e. and we select 5 p.e. This cut is rather severe, so we need to check the justification of the ADC cut. We will discuss this in the later section 12.2.

10.1.2 Clustering

Next, the following clustering analysis was applied to the data. Here, the “cluster size” is defined as the number of adjacent pixels including possible signals. Also the threshold for the cluster size is represented as “TNa”, where N is the minimum cluster size. In the same way as the ADC cut, increasing the N of the TNa cut (N=0, 3, 6, 9), the TDC distribution became thinner and the flat background became less (Fig 10.4). We need

**Figure 10.4:** TDC distribution. The X-axis is nano second and the Y-axis is number of events. The black line is obtained with no clustering cut. The red line is obtained with a T3a cut, the blue line is obtained with a T6a cut, and the green line is obtained with a T9a cut.

same way as the ADC cut, increasing the N of the TNa cut (N=0, 3, 6, 9), the TDC distribution became thinner and the flat background became less (Fig 10.4). We need
Figure 10.5: Clustering cut location versus TDC hit rate. The black line is the NSB rate estimated from the TDC sideband, which is considered to be proportional to the background. The red line was the signal rate from the on-time number of pixels in the TDC distribution which is considered to be proportional to the acceptance. The horizontal axis is the cut location. This plot is a differential one. The TDC-hit rates between \([T(N)a\text{-cut}]\) and \([T(N+1)a+1]\) are shown.

To check the N value as we did for the ADC cut. Here, a cluster size of 6 was chosen. Figure 10.5 is a plot for the NSB hit rate estimated from the TDC sideband,

\[
60 \text{ ns} < \text{TDC} - \text{TDC}_{\text{mean}}, -60 \text{ ns} > \text{TDC} - \text{TDC}_{\text{mean}},
\]

and signal rate estimated from the on-time,

\[
|\text{TDC} - \text{TDC}_{\text{mean}}| < \pm 25 \text{ ns}
\]

region in the TDC distribution (Fig 10.4) at each clustering cut. The black line is the NSB rate obtained from the TDC sideband, which is considered to be proportional to the background. The red line is the on-time hit rate in the TDC distribution which is considered to be proportional to the acceptance. The horizontal axis is the cut location. The clustering size of 5, 6, and 7 is reasonable. The cut value exceeding 7 reduces the excess event and the image size is small when cut value is less than 5. We need to
consider the image size for calculating the axis of the image. A small image is difficult to reconstruct the axis of the image. So the default cut location of T6a is reasonable. We also need to check the justification of clustering size N. We will discuss this in the later section 12.2.

10.1.3 TDC Cut

Finally, a TDC cut was applied. Before the TDC cut, we have to consider the problem of the difference of the TDC distribution between each telescope. Because TDCs are measuring relative arrival times of Cherenkov photons, the time offset of the telescopes should be different from each other, owing to the difference of cable length, etc. Shown in Fig 10.6 is difference of the TDC distribution between the two telescopes. The central timing of each distribution is determined by the trigger timing of each telescope. We should correct the data in order to apply the same cut for each telescope. The mean TDC is calculated for each event and subtracted from every TDC value of the same event. After this correction, a common TDC cut of ±25 nsec was applied to each pixel.

Figure 10.6: TDC distribution. The X-axis unit is nano second and the Y-axis unit is number of events. The raw TDC distributions are shown in upper panel, mean TDC distributions are shown in middle panel, and corrected TDC distributions are shown in lower panel. The hatched histograms are the T2 and blank are the T3.
10.1.4 Scaler Cut

In CANGAROO-III, pixel hit rates (called Scaler data) were recorded in order to monitor the influence of star light and electronic noise. The hit counts within a 700 µsec window were recorded for all pixels every second. The distributions of the Scaler data for the Crab are shown in Fig 10.7. The aim of the Scaler cut is to remove hot pixels, especially due to star light. The black histogram is the raw Scaler distribution. First, the “hottest 40 pixels” were removed. These “hottest 40 pixels” were selected by number of scaler hits in all data. The red histogram is obtained after applying this cut. Second, in order to remove the influence of star light, the pixels with 250 counts or more are removed on an event-by-event basis. The blue histogram is obtained after applying all these cuts. Furthermore, we checked whether the influence of star light had been removed correctly.

![Scaler distributions graph](image)

**Figure 10.7:** Scaler distributions. The X-axis unit is Scaler count and the Y-axis unit is number of events. The black histogram is a raw Scaler distribution. The red one is after a cut of the hottest 40 pixels. The blue distribution is after a cut at 250 TDC counts.

Removed pixels were plotted in histograms shown in Fig 10.8. The upper left panel represents the tracks of bright stars on the focal plane and the upper right represents the star positions after rotation correction and the average telescope pointing direction was located to the center. The lower left panel shows the removed pixels and the lower
right panel shows these after the rotation correction. This figure shows that the influence of a magnitude 3 star and 4.2 star were clearly removed. As mentioned in the previous Section 6.2.5, the HV system disabled pixels around the magnitude 3 star. However, some influence has remained due to the distribution tails of the spot for a bright star. As a result, we removed ten percent of pixels including bad channels. This effect was taken into account by the MC simulation.

Figure 10.8: Scaler and star plots. X-axis unit is deg and Y-axis unit is deg. The details are described in the text.
10.2 Data Reduction

10.2.1 Cloud and Elevation Cuts

As discussed in the previous section, it is quite likely that only proton and gamma-ray showers have been chosen at this time. This shower rate, therefore, should be stable (Fig 10.9). This means that it can be used to monitor conditions of the atmosphere.

From this point of view, we can remove bad condition data, i.e., cloud data. Also a cut was applied to the elevation angle. If a target source goes down to lower elevation, the path of Cherenkov light in the atmosphere becomes large ($1/\cos(\text{zenith angle})$) and then the acceptance is reduced for low energies. There is no reason to save these events. We verified this effect using MC simulation and verified the acceptance significantly reduced for low elevation observations. We selected the cut at 30° in the case of the Crab analysis.

Typical event rates and shower rates are shown in Fig 10.9. Both sides of the upper panels are event rates, the middle panels are shower rates, and the lower panels show the
elevation angles. Data with a good weather condition are shown in the left figure. Those with a cloudy weather condition are shown in the right figure. Reading the "experimental-log book", we could find the description that cloud passed in the line of sight to the target and the timing was confirmed to be correct. Therefore this cut is very effective to remove "cloud", and is called the 'Cloud cut'.

10.3 Image Analysis

Now, we can concentrate on the separation of proton and gamma-ray data. We discuss the effective method to separate gamma-rays from protons. Also this thesis describes a first result by a new instrument CANGAROO-III. In order to justify the following result, we need to show the result of the "standard TeV gamma-ray candle", i.e., the Crab Nebula. We will first show the result of the Crab, show its consistency with the previous results and then go into Cen A results.

10.3.1 Distribution of Hillas Parameters

As discussed in Chapter 5.4, the principle of the imaging analysis technique is mainly based on the three parameters, i.e., Length, Width, and Distance. In order to separate protons from gamma-rays, cuts on these three parameters were always used. In order to show differences between gamma-rays and protons, we compared these parameters with the gamma-rays by MC and OFF data obtained by the Wobble mode which are considered to be dominated by protons. Shown in Fig 10.10 are the histograms of these parameters. The differences are clear. The gamma-ray images are relatively sharp compared to those of protons. The conventional way is to set multiple cuts on these parameters in comparing with the MC simulation. In this thesis we introduce the method to combine these multi-parameters into one parameter called "Likelihood", the details of which will be described in the next section. This method was invented originally by Super Kamiokande and by high energy physics experiments and also introduced to CANGAROO-II in the RX J1713 (supernova remnant) analysis [37]. It became one of standard methods and follows mathematics. Before introducing it, we explain the intrinsic problem in the low elevation observation such as the Crab, i.e. elevation angle $\sim 30$ degrees.
10.3. IMAGE ANALYSIS

![Hillas parameters graph]

Figure 10.10: Hillas parameters. *Length* is shown in the left, *Width* is shown in the middle, and *Distance* is shown in the right. The blank histograms are the OFF data of the Crab and the hatched histograms are the data of the gamma-ray MC simulation (Crab-like spectrum).

10.3.2 Event Reconstruction

The analysis method of the stereo data was described in Section 5.4.2. In this section, we explain the stereo analysis method in detail. As shown in Fig 10.11, the arrival direction of shower can be calculated in event-by-event basis. The intersection point (IP) is determined from the intersection of two major axes of the images and has only one solution. The major axes images are written as:

\[
\begin{align*}
y - y_{2\text{cog}} &= u_2 \cdot (x - x_{2\text{cog}}), \\
y - y_{3\text{cog}} &= u_3 \cdot (x - x_{3\text{cog}}),
\end{align*}
\]

where \((x,y)\) is defined on focal plane and unit is degree as shown in Fig 10.11. The \(u_2\) (\(u_3\)) is defined as inclination of major axis of T2 (T3). \(x_{2\text{cog}}\) (\(x_{3\text{cog}}\)) and \(y_{2\text{cog}}\) (\(y_{3\text{cog}}\)) are the Center of Gravity (CoG) of T2 (T3) which are defined as the centroid of the image. The distance between the intersection point and the source position is defined as \(\theta\). The intersection point is calculated by the following formulas,

\[
\begin{align*}
\theta_x &= -\frac{((y_{2\text{cog}} - y_{3\text{cog}}) - (u_2 \cdot x_{2\text{cog}} - u_3 \cdot x_{3\text{cog}}))}{u_2 - u_3}, \\
\theta_y &= u_2 \cdot \theta_x + y_{2\text{cog}} - u_2 \cdot x_{2\text{cog}}.
\end{align*}
\]
where $\theta_x$ and $\theta_y$ are the components of the intersection point. Considering the phase space of the $\theta$ distribution, the distribution of $\theta^2$ is expected to be flat, where $\theta^2$ is written as:

$$\theta^2 = \theta_x^2 + \theta_y^2.$$  

(10.5)

10.3.3 Determination of an Intersection Point by Two Images

The intersection point (IP) is determined from the intersection of two major axes of the images and has only one solution. When the Crab data are analyzed, the following trouble occurred. Here, Armlength is defined as the distance between the intersection point and the center of gravity as shown in Fig 10.12. When the intersection of two gamma-ray images are correctly determined, the Distance and Armlength should be equal. The relation of the Armlength and Distance according to the MC simulations, however, is shown in Fig 10.13 for the low elevation measurement of the Crab case. The correct relation of these parameters is not realized as seen from this figure. This can be interpreted as the results of the small opening angle and "too"-parallel pair of images dominating the events. In such a case, the error of the intersection point diverges mathematically and the constructed the Armlength becomes meaningless. The elevation angle in the Crab observation is about 30 degrees. As such an angle, the shower maximum would be located at a higher altitude than 10 km considering the air thickness sensed by the
Figure 10.12: Determination of intersection point.

Figure 10.13: The relation of the Armlength and Distance for the Crab. These data are the MC simulation of the Crab. The meaning of this figure is that the Armlength is not correctly calculated in the standard method.
gamma-rays, where the refractive index of air is smaller. Shower maximum positions are more distant compared to the zenith observations. The expected results are smaller images, smaller Distances. Also the telescope "effective" spacing becomes smaller in the view of "shower-maximum-located" observers. We, therefore, constrain the Armlength to the expected value of 0.75 degree from the MC(Fig 10.14). This operation does not bias one to the intersection point determination mathematically. It can be understood by seeing the plots of the OFF $\theta^2$ spectra shown later, which are all flat.

In the analysis we use a "constrained-$\chi^2$ fit" using the definition of,

$$\chi^2(x, y) = \sum_{\text{telescope}} \left( \frac{\text{Width}(x, y)}{\sigma_{\text{wid}}} \right)^2 + \left( \frac{\text{Armlength}(x, y) - 0.75^\circ}{\sigma_{\text{arm}}} \right)^2, \quad (10.6)$$

where $\sigma_{\text{arm}} (= 0.26)$ is dispersion of the Armlength derived as MC study (Figure 10.14, left panel) and $\sigma_{\text{wid}} (= 0.168)$ is dispersion of the Width which corresponds to the pixel size.

We use a fine-mesh method which search the best point $(x,y)$ where the $\chi^2$ gets minimum. The Width is redefined which corresponds to the image dispersion around a newly axis in the search of fine-mesh method. When the point $(x,y)$ is decided, The Width and Armlength are decided uniquely. The minimization was applied by a fine-mesh method on the focal plane with a mesh size less than the angular resolution. We chose the size of minimum-mesh size as $0.025^\circ$, while the expected angular resolution is $0.15^\circ$.

**Figure** 10.14: Distance and Armlength distributions. The Distance distribution of the Crab MC is shown in the left. The mean value of the Distance distribution is $0.75^\circ$. The Armlength distribution before and after the IP fit are shown in the middle and right.
10.3. IMAGE ANALYSIS

After applying this IP fit, the signal concentration in the $\theta^2$ plot in the MC gamma-rays improves by a factor of two as shown in Fig 10.15, while the background distributions remain flat as shown in the later introduced figures. The $\theta^2$ distribution of the proton MC after the IP fit is also shown in Fig 10.16. The $\theta^2$ distribution shows a flat distribution, i.e., no spurious effects.

![Graph showing $\theta^2$ distribution before and after IP fit](image)

**Figure** 10.15: Comparison of $\theta^2$ distribution before the IP fit (the hatched histogram) and after the IP fit (the blank). It turns out that the IP fit is working effectively in the case of the large zenith angle observation.

### 10.3.4 Angular Resolution Versus Energy

The angular resolution becomes better in the stereoscopic observation. The angular resolutions of the Crab for each energy bin are estimated by the MC and are shown in Fig 10.17. The detailed Crab analysis is described in Section 10. The angular resolution in the energy of $> 2.4$ TeV is $0.15^\circ$, $> 3.4$ TeV is $0.14^\circ$, and $> 5.4$ TeV is $0.12^\circ$ for the Crab, respectively. The angular resolution of high energy range is slightly better than low energy range.
**Figure 10.16:** $\theta^2$ distribution of proton MC after IP fit. The MC simulation was carried assuming the same elevation angle of the Crab. The average elevation angle is $35^\circ$. The points with error bars are an ON-source data and the hatched area is an OFF-source data. IP fit does not make a peak at the center. The parameter of *Armlength* does not bias the position of the intersection point. The red line indicates the $\theta^2$ cut at $(0.217)^2$.

**Figure 10.17:** Angular resolutions of the Crab for each energy bin. These $\theta$ distributions were projected onto x-axis in y-slices at y=0. The histograms are the $\theta$ distributions and the solid lines are the Gaussian fit lines. The angular resolutions for each energy bin are shown (left : $E > 2.4$ TeV , middle : $E > 3.4$ TeV, right : $E > 5.4$ TeV). The obtained resolutions are $0.15^\circ$, $0.14^\circ$, and $0.12^\circ$ from left to right, respectively.
10.3.5 Angular Resolution Versus Zenith Angle

The angular resolution depends on the zenith angle. Angular resolutions for each zenith angle are estimated by the MC and are shown in Fig 10.18. The left figure shows the $\theta$ distribution of Cen A. The average zenith angle of Cen A is $17^\circ$ and the angular resolution is $0.15^\circ$. The average zenith angle of the Crab is $55^\circ$. The middle figure shows the $\theta$ distribution of the Crab before the IP fit (Section 10.3.3). The angular resolution before the IP fit of the Crab is $0.25^\circ$. The right figure shows the $\theta$ distribution of the Crab after the IP fit. The angular resolution after the IP fit of the Crab is $0.15^\circ$. The angular resolution of high zenith angle is slightly better than low zenith angle. Stereo-event samples in the FOV are shown in Fig 10.19.

![Angular resolution](image)

**Figure** 10.18: Angular resolutions versus zenith angle of Cen A and the Crab. These $\theta$ distributions were projected onto x-axis in y-slices at y=0. The histograms are the $\theta$ distributions and the solid lines are the Gaussian fit lines. The angular resolution of Cen A is $0.15^\circ$ (left). The angular resolution before the IP fit of the Crab is $0.25^\circ$ (middle). The angular resolution after the IP fit of the Crab is $0.15^\circ$ (right).

10.3.6 Energy Resolution per Event

We only can estimate the energy resolution of our device by the Monte-Carlo simulations. The generated energy versus the accepted total ADC counts ($\text{sumADC}$) is shown in Fig 10.20. The left figure is the 2D plot between these two. Here, the $\text{sumADC}$ is defined as:

$$\text{sumADC} = \frac{T2 \times \text{sumADC} + T3 \times \text{sumADC}}{2}.$$
Figure 10.19: Event display of the typical stereo events. The stereo events of real data are shown. T2 hit pixels are colored red, and T3 hit pixels are blue. The intersection points are plotted as red circles.
10.4 “Li and Ma” significance

In order to derive statistical significance of signal yield, we need to discuss the definition of statistical significance. In this thesis, we will discuss the 2σ-upper limits on Cen A. The definition of the standard deviation following the Poisson distribution is important to be shown in advance. We adopted the method by Li and Ma [84]

We define that the telescope points in the direction of a source position for a certain time $T_{on}$, obtaining $N_{on}$ events, and points in the direction of background for a certain time $T_{off}$, and obtains $N_{off}$ events. The quantity $\alpha$ is the ratio of the on-source time to the off-source time, $\alpha = T_{on}/T_{off}$. Then we can estimate the number of background
photons:

\[ \hat{N}_B = \alpha N_{off} \]  \hspace{1cm} (10.7)

The number of gamma-ray events within \( \theta^2_0 \) defined as following formula,

\[ N_{sig} = N_{on} - \alpha N_{off} \quad (\theta^2 < \theta^2_0), \]  \hspace{1cm} (10.8)

where \( \alpha \) is the inverse of the number of background points (appendix A). We here take six points of background. Li and Ma studied three methods of calculation of significance.

Defining the significance \( S \) as a ratio of the excess events above background to its standard deviation,

\[ S = \frac{N_s}{\sigma(N_s)} = \frac{N_{on} - \alpha N_{off}}{\sqrt{N_{on} + \alpha^2 N_{off}}} \]  \hspace{1cm} (10.9)

The above formula is simply derived from the Poisson law for \( N_{on} \) and \( N_{off} \).

If there were no extra sources, and all the observed counts were due to the background, ON-source events, \( N_{on} \), would follow a Poisson distribution and so would OFF-source. Then the formula is,

\[ S = \frac{N_s}{\sigma(N_s)} = \frac{N_{on} - \alpha N_{off}}{\sqrt{\alpha(N_{on} + N_{off})}} \]  \hspace{1cm} (10.10)

Another way of estimating the significance is obtained by use of the method of hypotheses test in mathematical statistics. Then the formula is,

\[ S = \sqrt{-2 \ln \lambda} \]  \hspace{1cm} (10.11)

= \[ \sqrt{2} \left\{ N_{on} \ln \left[ \frac{1 + \alpha}{\alpha} \left( \frac{N_{on}}{N_{on} + N_{off}} \right) \right] + N_{off} \ln \left[ (1 + \alpha) \left( \frac{N_{off}}{N_{on} + N_{off}} \right) \right] \right\}^{\frac{1}{2}} \]  \hspace{1cm} (10.12)

For the purpose of checking the methods of estimating statistical significance, Monte-Carlo results are shown in Fig 10.21. The lines indicate the standard normal distribution. It can be seen from the simulation results shown in Fig 10.21 that compared with the Gaussian probability, Eq 10.9 systematically underestimates the significances, Eq 10.10 is better than Eq 10.9, but the distributions of the significances calculated by Eq 10.12 from the maximum likelihood ratio method are most consistent with the expected Gaussian probabilities. We, therefore, follow formula 10.12.
10.5 Proton/gamma separation by Likelihood analysis

Now we are ready to show how our telescope detected the Crab signals correctly. As discussed in subsection 10.3.1, the image parameters differs significantly between gamma-rays and protons. They, however, are multi-parameters. An easy way is to set a cut interval in each image parameter and see a signal. If there are only one or two parameters, this procedure is un-biased as shown in Fig 10.22, because it can be visually tested by any other reader of these papers. However, typical experiments have more than three cuts parameters. In this case, to show the justification of location of cut values is quite difficult for typical readers in reading the written paper. Because a typicalexperimentalist wants a signal, sometimes he is biased. In order to avoid this effect, we can define a probability like object "Likelihood ratio (L)" as follows and as also can be found in the analysis of "(Super) Kamiokande (atmospheric-neutrino analysis)". So far, the Monte-Carlo is thought to be "tuned" and OFF data is thought to be dominantly protons (backgrounds). Making

Figure 10.21: Integral frequency distributions of the significance of the MC samples. The comparison of three methods of calculate significances. Left panel is $\alpha=0.1$ and right panel is $\alpha=0.5$. Plus points were obtained for Eq 10.9, Cross points were obtained for Eq 10.10, and filled circle points were obtained for Eq 10.12. The lines indicate the standard normal distribution.
histograms of various image parameters via the gamma-ray’s MC and via OFF data, we can obtain Probability Density Function (PDF) like objects for these image parameters, however, the normalization factors are still unknown. These histograms can be converted to functions such as PDF(Width), PDF(Length), and etc. with unknown normalization factors. When we have an event with some Width and Length, we can define a value proportional to the probability using these PDFs on an event by event basis using these PDFs. The gamma-ray’s PDFs can be obtained from the MC simulation and proton’s can be obtained from the OFF data. Then we can define,

$$L_{\text{likelihood ratio}} = \frac{\text{PDF}(\text{gamma-ray})}{\text{PDF}(\text{gamma-ray}) + \text{PDF}(\text{proton})}$$  \hspace{1cm} (10.13)$$

where each PDF like object is defined as follows,

$$\text{PDF}(\text{gamma-rays}) = \text{PDF}(MC_{\text{Length}}) \times \text{PDF}(MC_{\text{Width}}) \times ...$$ \hspace{1cm} (10.14)

$$\text{PDF}(\text{proton}) = \text{PDF}(OFF_{\text{Length}}) \times \text{PDF}(OFF_{\text{Width}}) \times ...$$ \hspace{1cm} (10.15)

Although this is not a probability itself, due to normalization ambiguities, gamma-ray events should show a peak at one and proton backgrounds should peak at zero in this notation. We are carrying out multiple telescope observation, so called ”stereoscopic observation”. This method is quite adequate to combine each telescope’s data. The number of image parameters is doubled via two telescope observation and tripped with three telescopes’ observations, and in this operation we only should analyze one cut parameter. This is really avoiding ”human bias”.

These image parameters have an energy dependence. In the case of higher energy events, Width should be larger. These energy dependences are shown in Fig 10.23. To
10.5. PROTON/GAMMA SEPARATION BY LIKELIHOOD ANALYSIS

Figure 10.23: Likelihood input histogram; a) Length vs $\log_{10}(ADC)$, b) Width vs $\log_{10}(ADC)$, c) Opening Angle vs $\log_{10}(ADC)$, d) Center of Gravity Distance vs $\log_{10}(ADC)$ The contours were obtained by the OFF data and the color maps by the MC simulation for gamma-rays which index is -2.5 (Crab-like spectra). We used the 2D PDF of MC gamma-ray and OFF-source data.

...take care of these dependences is easy. A minimum change to the PDF from 1D to 2D function is enough. So we used the 2D PDF of MC gamma-ray and OFF-source data.

The resultant "Likelihood-ratio ($L$)" distributions are shown in Fig 10.24. The hatched histogram is obtained by the MC simulation for gamma-rays and the blank one for OFF data. In the MC simulation we used an averaged Crab energy spectrum of $\propto E^{-2.5}$.

10.5.1 Propriety of Likelihood ratio Cut

We show the likelihood distributions obtained from the Crab data and from the “proton” Monte-Carlo in Fig 10.25. The likelihood ratios of gamma-ray-like events are shown in the left panel. The solid line is the MC distribution and filled circles with error bars are obtained from the excess events of the Crab. The Crab data were analyzed with the square cuts and events were selected by the $\theta^2$ cut. The cut value is $(0.217)^2$. The likelihood ratios of proton-like events are shown in the right panel. The solid line is the MC distribution for “proton” input and the filled circles with error bars are OFF-region data. The distribution of the Crab data agrees with that of the gamma-ray simulation.
Figure 10.24: \( L \) distributions. The hatched histogram is the MC gamma-rays for the Crab. The blank histogram is the MC proton. The points with error bars are the OFF-source data.

Figure 10.25: Likelihood ratio distributions. The likelihood ratios of gamma-ray-like events are shown in the left panel. The solid line is the MC distribution and points with error bars are obtained from the excess events of the Crab. The Crab data were analyzed using the square cuts and events were selected by the \( \theta^2 \) cut. The likelihood ratios of proton-like events are shown in the right panel. The solid line is the MC distribution for “proton” input and the points with error bars are OFF-region data.
Some disagreement in the likelihood ratio was observed near zero between the OFF-source data and the proton simulation. However, the default cut location is $< 0.9$ which were selected by the MC study. Therefore the influence can be estimated to be small. We also checked $L$ distributions for all energy bins (Fig 10.26). These figures show the energy dependence of $L$. We chose the stable cut at 0.9 to all energy regions because we want to eliminate humanic bias on selecting cut value. The resultant $\theta^2$ distribution is shown in Fig 10.27. A peak at zero degree$^2$ is observed. The distribution is consistent with the MC simulation of gamma-rays. The excess number of events is $115 \pm 21$ using Li-Ma method, i.e., 5.7 $\sigma$ peak. Next we show the estimation of the integral flux using this excess.

**10.6 Integral flux**

First, we generated gamma-rays with the MC simulation. The assumption that the power-law index of energy spectrum was -2.5, based on the Crab result of HEGRA [30], was made. We considered the orbit of the Crab. The zenith angle distribution is shown in Fig 10.28. We used this distributions as a probability density function. The range of the generated energy for the Crab is between 200 GeV ($E_{min}$) and 30 TeV. The minimum and maximum energies were determined in order to accept entire events generated via the MC simulation considering the present statistics and observation periods. We define $N_{gen}$
Figure 10.27: $\theta^2$ distribution after $L$ cut for the Crab data. The point with error bars were obtained from ON data and the hatched histogram from OFF data ($sumADC > 0$ p.e.). The red line indicates the $\theta^2$ cut at $(0.217)^2$ (Appendix A).

Figure 10.28: Zenith angle distribution of the Crab. We considered the orbit of the Crab for the MC study.
to be the number of generated events. In order to derive the energy dependence of the flux, we set three cuts on the \( \text{sumADC} \), i.e., no-cut, 100., and 200 p.e. The \( \text{sumADC} \) should be proportional to the initial gamma-ray energy, and these cuts correspond to an initial gamma-ray energy of 2.30, 3.43, and 5.38 TeV according to the Monte-Carlo studies. The number of energy bins \( (n) \) was determined to be three considering the excess signal is 5.7 \( \sigma \). The three cut locations were determined by requiring the \( \text{sumADC} \) distribution of the excess events for each selection to have statistically significant signals.

The corresponding energy for each bin is determined as follows. The generated energy of the accepted events in the \( \text{sumADC} \) intervals were plotted as shown in Fig 10.29. We averaged over these hatched region and define them as specific gamma-ray energy

![Figure 10.29: Energy distributions in each energy bin. The black histograms are a generated energy distribution. The red are those for the accepted events in all region (the left : \( \text{sumADC} > 0. \) p.e. the middle : \( \text{sumADC} > 100. \) p.e. the left : \( \text{sumADC} > 200. \) p.e.). The hatched are those in each energy bin. (the left : 100 p.e. > \( \text{sumADC} > 0. \) p.e., the middle :200 p.e. > \( \text{sumADC} > 100. \) p.e. the left : \( \text{sumADC} > 200. \) p.e.).](image)

Next, we calculate the number of accepted events \( (N_{\text{accept, n}}) \) which remained after the whole analysis. An efficiency can be defined for each energy range. The efficiency is defined above the energy threshold. The efficiency is written as,

\[
\text{Efficiency} (E > E_n) = \frac{N_{\text{accept, n}}}{N_{\text{gen}}},
\]

where \( E_n \) is the threshold energy of each energy bin. We can calculate the expected
number of events as follows,

\[ N_{\text{event},n} (E > E_n) = \text{Time} \times \text{Flux}_{\text{assum}} \times \text{Area} \times \text{Efficiency} \]  \hspace{1cm} (10.17)

where, \( N_{\text{event},n} \) is the expected number of events. \( \text{Time} \) is the live time of the stereo observation. The live time of the Crab is 840.3 min and we use the HEGRA Crab flux [5] as \( \text{Flux}_{\text{assum}} \):

\[ (2.79 \pm 0.02 \pm 0.05) \times 10^{-11} (E/1 \text{ TeV})^{-2.59 \pm 0.03 \pm 0.05} \text{ photons cm}^{-2} \text{s}^{-1} \text{TeV}^{-1}. \]  \hspace{1cm} (10.18)

The definition of \( \text{Area} \) is shown in Fig 10.30. \( \text{Area} \) corresponds to the spread of the Cherenkov light in the surface of the earth calculated from the altitude of shower maximum and the zenith angle. So \( \text{Area} \) has dependence of zenith angle as \( \cos^{-2}(\text{zenith angle}) \). In the present MC code used in CANGAROO-III, \( \text{Area} \) is given as \( \pi r^2 \times \cos^{-2}(\text{zenith angle}) = 250^2 \pi \cos^{-2}(\text{zenith angle}) \text{ m}^2 \). In the MC simulation, we randomize the gamma-rays incidence position using the above definition. Finally, we calculate the integral fluxes as

\[ \text{ Flux}_{\text{excess},n} (E > E_n) = \text{ Flux}_{\text{assum}} \times \frac{N_{\text{excess},n}}{N_{\text{event},n}} \]  \hspace{1cm} (10.19)

\[ = \frac{N_{\text{excess},n}}{\text{Area} \times \text{Time} \times \text{Efficiency}} \]  \hspace{1cm} (10.20)

\[ = \frac{N_{\text{excess},n}}{\text{Time} \times \text{Effective area}}, \]  \hspace{1cm} (10.21)
where \( N_{\text{excess},n} \) is the observed excess events. The excess events of all energy bins are shown in Fig 10.31. The Effective area \( (S_{\text{eff}}) \) is defined as

\[
S_{\text{eff}}(E > E_n) = \text{Area} \times \text{Efficiency.}
\]

Figure 10.31: \( \theta^2 \) distribution of the Crab for each energy bin. From left to right, three energy bins from lower energy to higher energy are shown (Left: \( \text{sumADC} > 0 \) p.e., middle: \( \text{sumADC} > 100 \) p.e., and right: 200 p.e.). The red line is the \( \theta^2 \) cut \((\theta^2 < (0.217)^2, \text{Appendix A})\). The features of energy bins are summarized in Table 10.1.

This Effective area is defined using the Efficiency. The Effective area of the Crab vs threshold energy is shown in Fig 10.32.

The resultant integral fluxes are shown in Fig 10.33. The integral flux means that the intensity of energy greater than the specific energy determined from the \( \text{sumADC} \) cuts. These results are consistent with the previous measurements of the Crab Nebula within the statistical flux errors of typically 20%. Thus, the uncertainty of the absolute value for the measured flux is proved to be correct at the 20% level. These studies are summarized in Table 10.1.

We also carried out a differential flux derivation and obtained it around the energy of 5.38 TeV to be

\[
dF/dE = (3.25 \pm 0.71) \times 10^{-13} (E/5.38 \text{ TeV})^{-2.93\pm0.83} \text{ cm}^{-2}\text{s}^{-1}\text{TeV}^{-1},
\]

where the HEGRA flux is \( 3.59 \times 10^{-13} (E/5.38 \text{ TeV})^{-2.59} \text{ cm}^{-2}\text{s}^{-1}\text{TeV}^{-1} \). Our measurement is consistent with it.
Figure 10.32: Effective area of the Crab vs threshold energy. The definition of Effective area is written as Eq 10.22.

Figure 10.33: Integral flux after the $L$ cut for the Crab. The solid line is obtained by HEGRA group [30] and the dashed line by Whipple [58].
10.7 Distribution of Arrival Directions for the Crab

The stereo analysis can show the arrival direction event by event, but the analysis of single telescope can only show the significance map which can show after the whole analysis. One of the merits of the stereo observation is that the analysis can correctly show the arrival point map. Here, we do not use the $\theta^2$ method but the $L$ cut method because it can make us study about the arrival point map. The arrival direction distribution (2D) of the Crab is shown in Fig 10.34 and Fig 10.35 for two cases of the background assumption. We took the backgrounds as “proton like events” such as those showing low ”Likelihood-ratio”. In the above procedure, we should really select proton like events which would show acceptance in the Field of View. The excess from the Crab is seen at the center of the FOV in both cases. The S/N ratio is not good compared to that of H.E.S.S. results due to the bad mirror alignment described in the previous section. It is better to argue using one-dimensional $\theta^2$ distribution rather than this map.

Here we again explain about the assumed signals and backgrounds. The ON distribution was chosen $L > 0.9$ (gamma-like). We tried two ways of estimation of background. The background was chosen by $L < 0.2$ (most hadron like) of likelihood ratio in Fig 10.34.
and the background was chosen by $L < 0.7$ (modestly hadron like) of likelihood ratio in Fig 10.35. In the subtraction, the normalization between the ON and OFF was simply done using the total entries of each plot.
Figure 10.34: The distribution of arrival direction of the Crab. The bin width is $0.1^\circ$. The upper left panel shows the ON distribution of Crab, the upper right panel shows the background which was chosen by $L < 0.2$. This distributions normalized to ON using total entries. The Lower left panel shows the distribution of after background subtraction. The X-axis is Right Ascension-83.625. The Y-axis is Declination-22.0167. The Crab is located at the center of this figure. We can see a peak in the excess.
Figure 10.35: The arrival direction map of Crab. The bin width is 0.1°. The upper left panel shows the ON distribution of the Crab, the upper right panel shows the background which chosen by $L < 0.7$. This distribution was normalized to ON using total entries. The lower left panel: The ON-OFF distribution. The X-axis is Right Ascension-83.625. The Y-axis is obtained with Declination-22.0167. The Crab is located at the center of this figure. A clear excess from the Crab is seen in the center of this figure.
Chapter 11

Analysis of Centaurus A

11.1 Data Cleaning

We applied the same cleaning procedure as used in the Crab analysis (Section 10.1). The ADC cut of 5 p.e. was chosen and the minimum cluster size of 6 was chosen. We also corrected TDC timing distribution event-by-event basis. Finally, a TDC cut was applied. The TDC cut of $\pm 25$ nsec was selected by inspection of Fig 11.1. The number of hot pixels from the scaler hit rate was determined to be 40, which is the same cut as in the Crab analysis. The scaler cut has no effect in the case of Cen A, because there are no bright stars around Cen A.

11.2 Data Reduction

Then the Cloud and elevation cuts were applied. These cut values differ from the case of the Crab, since the zenith angle of the observation was different (Section 10.2). The rates of showers were different day by day and we chose 7 Hz for the Cloud cut because the shower rate on fine day is over the 7 Hz. The elevation cut was chosen over the 60 degree because the shower image became poor. The event rate, the rate shower, and the elevation is shown in Fig 11.2.

According to this, about 15% of data were rejected. The summary of used periods are shown in Table 11.1.
Figure 11.1: TDC distribution of Cen A. The X-axis unit is nanosecond and the Y-axis unit is number of events. The black line is obtained with the no clustering cut, The red line is obtained with the T3a cut, the blue line is obtained with the T6a cut, and the green line is obtained with the T9a cut.

<table>
<thead>
<tr>
<th>Observation Date</th>
<th>$T_{on}$ (min) (T2-T3)</th>
<th>$T_{on}$ (min) (T2-T4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Before reduction</td>
<td>1112</td>
<td>907</td>
</tr>
<tr>
<td>After reduction</td>
<td>970</td>
<td>794</td>
</tr>
</tbody>
</table>

Table 11.1: Summary of the observation time
11.3 Relationship between ArmLength and Distance for Cen A

The zenith angle of Cen A is smaller than that of Crab, i.e., it culminates near the zenith. Therefore, the telescope spacing seen from the shower maximum point is larger than in the case of the Crab. The intersection point determination should be better in this case, i.e., the calculation of ArmLength is expected to be more correct than that of the Crab (Section 10.3.3). The relation of ArmLength and Distance obtained by the MC simulation is shown in Fig 11.3. This MC was carried out with a Crab index (-2.5) at Cen A elevation angle (average elevation angle is 73°). We can see a good correlation between the two parameters. We checked the concentration of $\theta^2$ distribution for the two cases, i.e., with the IP fit and without the IP fit which was used in the Crab analysis. As shown in Fig 11.4, The determination of the intersection points is accurate enough even without the IP fit. We, therefore, have not used the IP fit in the Cen A analysis.
CHAPTER 11. ANALYSIS OF CENTAURUS A

Figure 11.3: Relationship of *Armlength* and *Distance*. The X-axis is *Distance* [degree]. The Y-axis is *Armlength* [degree]. This MC simulation was carried assuming the Crab gamma-ray energy spectrum with index of -2.5 at Cen A elevation angle (the average elevation angle is 73°). The calculation of *Armlength* is better than for the Crab.

Figure 11.4: $\theta^2$ distribution of MC data. The MC simulation was carried out assuming the Crab gamma-ray energy spectrum with index of -2.5 at Cen A elevation angle (average elevation angle is 73°). The x-axis is $\theta^2$ [degree$^2$] and y-axis is number of events. The blank area was obtained without the fit. The hatched area shows the previous calculation of the IP fit.
11.4  Likelihood Analysis

As discussed in the previous Section 10.5, we applied a \textit{Likelihood} cut in the analysis of Cen A. The PDF inputs for the $L$ are shown in Fig 11.5. The same cut (of 0.9) as the Crab case was used. The $L$ distributions are shown in Fig 11.6.

\textbf{Figure} 11.5: \textit{Likelihood} input histograms; a) \textit{Length} vs. $\log_{10}(ADC)$, b) \textit{Width} vs. $\log_{10}(ADC)$, c) Opening angle vs. $\log_{10}(ADC)$, and d) center of gravity distance vs. $\log_{10}(ADC)$, The contour maps were obtained by the MC simulation for gamma-rays and the color maps were obtained by the OFF data, respectively.

The $\theta^2$ distribution is shown in Fig 11.7. The points with statistical error bars show the $\theta^2$ distributions of the ON-source data. The hatched area shows the OFF-source data. Six places with the same radius as the target position in the Wobble mode were used as OFF-source regions. For this reason, the OFF-source data were normalized by the ratio of 1/6. We cannot see any statistically significant excess events in $\theta^2 < 0.047$ degree$^2$. We also tried this with the IP fit, and found that there is no difference in the results.
Figure 11.6: $L$ distributions for OFF (the blank histogram) and for the MC gamma-rays (the hatched histogram).

Figure 11.7: $\theta^2$ distribution of Centaurus A. The point with statistical errors show the $\theta^2$ distributions of the ON-source data. The hatched area shows the Off-source data. The OFF-source data were normalized by the ratio of number of background points, i.e., 6 regions. The red line is the $\theta^2$ cut ($0.217^2$).
11.5 Upper Limit to the Integrated Flux from Cen A

So far, no evidence for gamma-ray emission was detected from Cen A in our observations and we set the 2-σ flux upper limit. The integral flux definition described in Section 10.6. The live time of Cen A is 1764 min and the Effective area for Cen A (i.e., zenith object) are shown in Fig 11.8. The definition of effective area is described in Section 10.6. The average zenith angle of Cen A is 17°. The definition of 2-σ flux upper limit is

\[
2 - \sigma \text{ flux upper limit} < (2 \times \sqrt{\sigma_{\text{stat.}}^2})
\]

The \(\sigma_{\text{stat.}}\) is the statistical error of Cen A. The \(\theta^2\) distributions are shown in Fig 11.9.

The 2-σ flux upper limits are plotted in Fig 11.10. The flux level is approximately 5 % Crab. Here only statistical errors are included. Later, we will discuss the systematic error and show the final result for Cen A.

The upper limits and various values are summarized in Table 11.2.
Figure 11.9: $\theta^2$ distribution of Cen A for each energy bin. From left to right, three energy bins from lower energy to higher energy are shown (left: \textit{sumADC} > 0 p.e., middle: \textit{sumADC} > 100 p.e., right: \textit{sumADC} > 200 p.e.). The red line indicates the $\theta^2$ cut ($\theta^2 < (0.217)^2$, Appendix A). The features of each energy bin are summarized in Table 11.2.

Figure 11.10: 2-$\sigma$ upper limit after the $L$ cut for Cen A. The arrow indicate the 2-$\sigma$ upper limit of Cen A. The solid line is the Crab flux obtained by the HEGRA group [30] and the dashed line is also the Crab flux obtained by the Whipple group [58].
### 11.6. Time Variability

Centaurus A is thought to be an AGN. Although the fluxes might change day-by-day, the time-averaged 2-σ upper limit of Cen A (Fig 11.9) during this period is less than about 1/100 flux of the highest flux observed in 1975 [53]. For typical AGNs such as Mkn421 and Mkn501, the TeV-flux level changes day-by-day by 1 order of magnitude. Even considering these facts, two orders of magnitude difference between our time-averaged upper limits and the previous burst measurements is inconsistent. Therefore, we searched for a lower level burst in our observation periods. The result of the Rossi X-ray Timing Explorer (RXTE) all sky monitor (ASM) in our observation period is shown in Fig 11.11 and these fluxes were quiet. The result of our TeV observation is shown in Fig 11.12. The observation period is from 16 March to 19 April. There are 7 observation days in this period and the average observation time/run is about 3 hours. As shown in Fig 11.12, there is no significant excess in this day-by-day analysis. We consider the fluctuation of count/min. The average rate is -0.001 count/min and average of dispersion rate is 0.038 count/min. The count rate of the run number 3 (Fig 11.12) is 0.119 count/min. The maximum fluctuation of count/min is calculated as:

\[
0.119 / (2 \times 0.38 - 0.001) = 1.6 \times 2 \sigma
\]
CHAPTER 11. ANALYSIS OF CENTAURUS A

Figure 11.11: Day-by-day flux of Cen A. These data were obtained by the RXTE All Sky Monitor (ASM) which is an X-ray satellite. The red points are more than 3-σ detections.

The maximum fluctuation of count/min is less than 1.6 × 2 σ of the average. From the above result, we concluded that there were no burst of fluxes greater than 12 (= 1.6 × 7 %) % Crab in these periods.

Figure 11.12: Day-by-day result of Cen A with CANGAROO-III. The X-axis is day. The Y-axis is the flux in units of count/minute.
11.7 Distribution of Arrival Directions around Cen A

Here we show the two-dimensional distribution of arrival directions around Cen A. The distributions of these for Cen A are shown in Fig 11.13 and Fig 11.14 for the two different background assumptions. The ON distribution was chosen $L > 0.9$ in the both cases. First, the background was chosen by $L < 0.2$ of likelihood ratio in Fig 11.13. Second, the background was chosen by $L < 0.7$ of likelihood ratio in Fig 11.14. These backgrounds were normalized to the total entry of ON. In the both cases, we could not find significant excess events. Although, there is an excess near the edge of the FOV in Fig 11.13, its statistical significance is below $2 \sigma$, and it is not significant at all.

11.8 Single telescope analysis

In this thesis, we only show the results of the stereo-analysis. The mono-analysis, however, was carried out and described in Appendix D. The results are a factor of two worse than that by the stereo-analysis. We, therefore, concentrate on the stereo-analysis in this thesis.
Figure 11.13: Distribution of arrival directions for Cen A. The bin width is $0.1^\circ$. The upper left panel shows the ON distribution of Cen A. The upper right panel shows the background which was chosen by $L < 0.2$. This distribution is normalized to the total entries in the ON plot. The lower left panel shows the ON-OFF distribution. The X-axis is Right Accession-201.365. The Y-axis is Declination-43.019. Cen A is located at the center of this figure. We can not find any significant excess.
Figure 11.14: Distribution of arrival directions for Cen A. The bin width is $0.1^\circ$. The upper left panel shows the ON distribution of Cen A. The upper right panel shows the background which was chosen by $L < 0.7$. This distribution is normalized to the total entries in the ON plot. The lower left panel shows the ON-OFF distribution. The X-axis is Right Ascension-201.365. The Y-axis is Declination-43.019. Cen A is located at the center of this figure. We can not find any significant excess.
Chapter 12

Systematic Error Estimation

In this section, we estimate systematic errors for the Cen A result. The errors are induced by many reasons, most of which are not well understood, but we need to estimate them in order not to underestimate flux upper limits on Cen A. The Crab observation is very important in this point because the Crab is a well-measured object and known to give stable emission in TeV gamma-rays. HEGRA reported its differential flux of \((2.79\pm0.02\pm0.05) \times 10^{-11} \text{ photons cm}^{-2}\text{s}^{-1}\text{TeV}^{-1}\) with a power law index of \((-2.59 \pm 0.03 \pm 0.05)\) at 1 TeV\[5\]. Whipple reported \((3.20 \pm 0.17 \pm 0.6) \times 10^{-11} \text{ photons cm}^{-2}\text{s}^{-1}\text{TeV}^{-1}\), with a power law index of \((-2.49 \pm 0.06 \pm 0.04)\) at 1 TeV\[58\]. They agree within 15%. This 15% should also be considered to be a minimum error for IACTs. Since we can detect the signals even varying several cuts, we define deviations in the Crab signal obtained by changing various cuts as systematic errors.

12.1 Uncertainty in the Energy Determination

The energy determination should be governed by the so called "reflectivity" and atmospheric conditions. The center values for the former and the systematic uncertainties of the latter were determined both by the shower rate in this thesis. From the description in these determinations done so far, we had better to set 10% for each. In total, the energy ambiguity of our measurement is 15%.

For a reference we show the relationship of T2 sumADC and T3 sumADC in Fig. 12.1. The color contour and black error bars are obtained with real data. The red error bars
12.1. Uncertainty in the Energy Determination

are obtained with MC data. We fitted them with straight lines and summarized the fitted parameters in Table 12.1. This systematic error is smaller than other errors, so we did not use this error for the final result.

![Energy resolution between T2 and T3](image)

**Figure 12.1:** Energy resolution between T2 and T3. The X-axis unit is T2 \( \log(\text{sumADC}) \) and the Y-axis unit is T2 \( \log(\text{sumADC}) \). The color contour and black error bars are obtained with real data. The red error bars are obtained with MC data. The red and black lines are fitted lines (Table 12.1).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>p0</th>
<th>p1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Real data</td>
<td>0.174</td>
<td>0.94</td>
</tr>
<tr>
<td>MC data</td>
<td>0.163</td>
<td>0.95</td>
</tr>
<tr>
<td>Systematic error</td>
<td>6 %</td>
<td>-1 %</td>
</tr>
</tbody>
</table>

**Table 12.1:** The fitting function is \( y = p0 + p1x \). \( p0s \) and \( p1s \) are shown in this Table.

A good correlation between MC and real data was obtained. We also show the ratio of T2- and T3-energies in Fig 12.2. The histograms were obtained from the experiment and the points by the Monte-Carlo simulations. They agree with each other.
12.2 Systematic Error of NSB Uncertainty

Although we used Jelley’s value as a standard NSB rate, this value is an all-sky average. It changes depending on the point in the sky. Although we clean up shower events applying the ADC cut and the clustering cut, there is also town (artificial) light which cannot be well estimated. It might be dependent on the elevation angle of the telescopes. Inputting it correctly into the Monte-Carlo simulation is essentially impossible. We, therefore, tried to check this by changing the above two cut parameters.

Systematic error of the clustering cut

The cut value for the clustering method was determined looking at TDC distributions after the clustering cut as shown in Fig. 10.4. To reduce random timing events, but not to reduce the acceptance for signals, we selected the T6a clustering. Here we check it again from Fig 10.5. The default cut location of T6a is reasonable. The clustering levels have been changed from T5a to T7a for calculating the systematic error. The systematic errors were calculated from comparing the flux of T5a and T6a cut, and comparing T7a and T6a cut. Table 12.2 is the summary of the systematic errors estimated by changing the parameters of the clustering cut in each energy bin. The deviations are small, compared with the statistical errors.
12.2. SYSTEMATIC ERROR OF NSB UNCERTAINTY

<table>
<thead>
<tr>
<th>Cut</th>
<th>Energy bin</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>SumADC cut (p.e.)</td>
<td>0. &lt;</td>
<td>100. &lt;</td>
<td>200. &lt;</td>
<td></td>
</tr>
<tr>
<td>$E_{th}$ (TeV)</td>
<td>2.35</td>
<td>3.42</td>
<td>5.37</td>
<td></td>
</tr>
<tr>
<td>T5a Excess events</td>
<td>110 ± 22</td>
<td>91 ± 19</td>
<td>65 ± 14</td>
<td></td>
</tr>
<tr>
<td>Integral flux ($10^{-12}$ cm$^{-2}$ sec$^{-1}$)</td>
<td>3.76 ± 0.74</td>
<td>1.94 ± 0.42</td>
<td>0.99 ± 0.21</td>
<td></td>
</tr>
<tr>
<td>syst. error ± stat. error (%)</td>
<td>-15 ± 20</td>
<td>-8 ± 22</td>
<td>-5 ± 21</td>
<td></td>
</tr>
<tr>
<td>T6a Excess events</td>
<td>115 ± 21</td>
<td>97 ± 19</td>
<td>68 ± 14</td>
<td></td>
</tr>
<tr>
<td>Integral flux ($10^{-12}$ cm$^{-2}$ sec$^{-1}$)</td>
<td>4.44 ± 0.81</td>
<td>2.11 ± 0.42</td>
<td>1.04 ± 0.21</td>
<td></td>
</tr>
<tr>
<td>syst. error ± stat. error (%)</td>
<td>-26 ± 19</td>
<td>-3 ± 21</td>
<td>1 ± 21</td>
<td></td>
</tr>
<tr>
<td>T7a Excess events</td>
<td>105 ± 20</td>
<td>94 ± 19</td>
<td>68 ± 14</td>
<td></td>
</tr>
<tr>
<td>Integral flux ($10^{-12}$ cm$^{-2}$ sec$^{-1}$)</td>
<td>3.28 ± 0.63</td>
<td>2.04 ± 0.42</td>
<td>1.03 ± 0.22</td>
<td></td>
</tr>
<tr>
<td>syst. error ± stat. error (%)</td>
<td>-32 ± 19</td>
<td>-4 ± 21</td>
<td>1 ± 21</td>
<td></td>
</tr>
</tbody>
</table>

Table 12.2: Summary of the systematic errors estimated from various clustering cuts. The default cut location is T6a and the systematic errors were calculated from comparing the flux of T5a and T6a cut, and comparing T7a and T6a cut.

Systematic error of the ADC cut

Just as for the clustering cut, we show the justification of the default cut value in Fig 10.3. The ADC threshold has been changed from 4.5 to 5.5 p.e, and this interval corresponds to ±10% of the default cut value of 5 p.e., i.e., the same level as the energy uncertainty that will be discussed later. The systematic errors were calculated from comparing the flux of 4.5 p.e. and 5.0 p.e. cut, and comparing 5.5 p.e. and 5.0 p.e. cut. Table 12.3 is the summary of the systematic errors estimated by changing the ADC cuts in each energy bin.

Summary of the systematic error for the NSB uncertainty

Combining the above two tables, we determined the systematic error for the NSB uncertainty. Here, we assumed that these errors listed in Tables 12.2 and 12.3 are dependent on each other. So we took the maximum deviations into account as the NSB uncertainty which is shown in Table 12.4.
<table>
<thead>
<tr>
<th>Cut</th>
<th>Energy bin</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$SumADC$ cut (p.e.)</td>
<td>0. $&lt;$</td>
<td>100. $&lt;$</td>
<td>200. $&lt;$</td>
<td></td>
</tr>
<tr>
<td>$E_{th}$ (TeV)</td>
<td>2.35</td>
<td>3.42</td>
<td>5.37</td>
<td></td>
</tr>
<tr>
<td>4.5 p.e. Excess event</td>
<td>95 $\pm$ 21</td>
<td>92 $\pm$ 19</td>
<td>66 $\pm$ 14</td>
<td></td>
</tr>
<tr>
<td>Integral flux (10$^{-12}$ cm$^{-2}$ sec$^{-1}$)</td>
<td>3.56 $\pm$ 0.78</td>
<td>1.96 $\pm$ 0.41</td>
<td>1.01 $\pm$ 0.21</td>
<td></td>
</tr>
<tr>
<td>syst. error $\pm$ stat. error (%)</td>
<td>-20 $\pm$ 22</td>
<td>-7 $\pm$ 21</td>
<td>-3 $\pm$ 21</td>
<td></td>
</tr>
<tr>
<td>5 p.e. Excess events</td>
<td>115 $\pm$ 21</td>
<td>97 $\pm$ 19</td>
<td>68 $\pm$ 14</td>
<td></td>
</tr>
<tr>
<td>Integral flux (10$^{-12}$ cm$^{-2}$ sec$^{-1}$)</td>
<td>4.44 $\pm$ 0.81</td>
<td>2.11 $\pm$ 0.42</td>
<td>1.04 $\pm$ 0.21</td>
<td></td>
</tr>
<tr>
<td>5.5 p.e. Excess event</td>
<td>106 $\pm$ 22</td>
<td>92 $\pm$ 20</td>
<td>65 $\pm$ 15</td>
<td></td>
</tr>
<tr>
<td>Integral flux (10$^{-12}$ cm$^{-2}$ sec$^{-1}$)</td>
<td>3.50 $\pm$ 0.72</td>
<td>1.96 $\pm$ 0.43</td>
<td>1.00 $\pm$ 0.23</td>
<td></td>
</tr>
<tr>
<td>syst. error $\pm$ stat. error (%)</td>
<td>-20 $\pm$ 21</td>
<td>-7 $\pm$ 22</td>
<td>-4 $\pm$ 23</td>
<td></td>
</tr>
</tbody>
</table>

**Table 12.3:** Summary of the systematic errors estimated from various ADC cuts. The default cut location is 5 p.e. and the systematic errors were calculated from comparing the flux of 4.5-p.e. and 5.0-p.e. cut, and comparing 5.5-p.e. and 5.0-p.e. cut.

<table>
<thead>
<tr>
<th>Cut</th>
<th>Energy bin</th>
</tr>
</thead>
<tbody>
<tr>
<td>$SumADC$ cut (p.e.)</td>
<td>0. $&lt;$</td>
</tr>
<tr>
<td>$E_{th}$ (TeV)</td>
<td>2.35</td>
</tr>
<tr>
<td>systematic error (%)</td>
<td>-26</td>
</tr>
</tbody>
</table>

**Table 12.4:** Summary of the systematic error for the NSB uncertainty
12.3 Systematic Error of the Spot Size Uncertainty Etc.

In the analysis of the Crab data, we found that the spot size of the mirrors is worse than the value of the initial design and we replaced the Monte-Carlo input with the tuned value. There might still be some unknown mis-tuned parameters. Here we concentrate on the major cuts using the Hillas parameters. Difference of the result of these major cuts are regarded as the systematic error of the spot size uncertainty etc.

Systematic error of the likelihood cut

The likelihood cut plays an essential role in gamma-ray selections in the present analysis and is based on the Hillas parameters. The default cut parameter of 0.9 is discussed in Section 10.5. From the Figures 10.25, we tried various likelihood cuts in the region of 0.7 ∼ 0.9. The default $L$ cut location is 0.9 and the systematic errors were calculated from comparing the flux of 0.8 and 0.9 cut, and comparing 0.7 and 0.9 cut. The Table 12.5 is the summary of the systematic errors estimated by changing likelihood cut values in each energy bin.

The deviations are smaller than the statistical errors. Here, we show the histories of $\theta^2$ plots obtained via the above procedures in Figs 12.3. As can be seen from the figures, they are very stable.

Systematic error of the difference between square cut and likelihood cut

We had used a "Likelihood cut" in selecting gamma-ray-like events from the huge backgrounds dominated by proton showers. The "Likelihood" is mathematical and there is no reason why we do not use it. However, the conventional way of analyzing IACT’s data is so called "square cuts" which will be introduced as follows [57]. The deviation of the results might also reflect systematic ambiguities. In the conventional way, we set lower and upper limits on each parameter, such as Width, Length, and Distance, by seeing the differences predicted by Monte-Carlo simulations. Here we show the performance of the "square cuts" and define the difference in the results from the "likelihood cut" as a systematic error.
<table>
<thead>
<tr>
<th>Cut</th>
<th>Energy bin</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$SumADCD$ cut (p.e.)</td>
<td>0. &lt;</td>
<td>100. &lt;</td>
<td>200. &lt;</td>
<td></td>
</tr>
<tr>
<td>$E_{th}$ (TeV)</td>
<td>2.35</td>
<td>3.42</td>
<td>5.37</td>
<td></td>
</tr>
<tr>
<td>0.9 Excess event</td>
<td>115 ± 21</td>
<td>97 ± 19</td>
<td>68 ± 14</td>
<td></td>
</tr>
<tr>
<td>Integral flux ($10^{-12}$ cm$^{-2}$ sec$^{-1}$)</td>
<td>4.44 ± 0.81</td>
<td>2.11 ± 0.42</td>
<td>1.04 ± 0.21</td>
<td></td>
</tr>
<tr>
<td>0.8 Excess event</td>
<td>112 ± 25</td>
<td>86 ± 23</td>
<td>65 ± 16</td>
<td></td>
</tr>
<tr>
<td>Integral flux ($10^{-12}$ cm$^{-2}$ sec$^{-1}$)</td>
<td>3.84 ± 0.87</td>
<td>1.63 ± 0.45</td>
<td>0.86 ± 0.23</td>
<td></td>
</tr>
<tr>
<td>syst. error ± stat. error (%)</td>
<td>-14 ± 21</td>
<td>-23 ± 28</td>
<td>-17 ± 27</td>
<td></td>
</tr>
<tr>
<td>0.7 Excess event</td>
<td>127 ± 28</td>
<td>92 ± 25</td>
<td>57 ± 17</td>
<td></td>
</tr>
<tr>
<td>Integral flux ($10^{-12}$ cm$^{-2}$ sec$^{-1}$)</td>
<td>4.35 ± 0.97</td>
<td>1.65 ± 0.45</td>
<td>0.80 ± 0.24</td>
<td></td>
</tr>
<tr>
<td>syst. error ± stat. error (%)</td>
<td>-2 ± 22</td>
<td>-22 ± 27</td>
<td>-23 ± 30</td>
<td></td>
</tr>
</tbody>
</table>

Table 12.5: Summary of the systematic errors estimated from various likelihood cuts. The default $L$ cut location is 0.9 and the systematic errors were calculated from comparing the flux of 0.8 and 0.9 cut, and comparing 0.7 and 0.9 cut.

First, we used a cut of $Distance$. The upper and lower cut values for $Distance$ were determined to be 0.0 and 1.3 degrees respectively, from Fig 10.10. These reject background events, because the $Distance$ distributions of proton backgrounds should be uniform in the phase space, i.e. directionally uniform in $4\pi$ solid angle (should follow $(Distance)^2$ in the phase space.). Before going into the details of $Length$ and $Width$ cuts, we should consider that $Length$ and $Width$ depend on the primary energy. For higher energy events, they should have larger values. The energy dependences of $Length$ and $Width$ are shown in Fig 12.4. Here the Y-axis is $log_{10}(SumADCD)$ and X-axis is the parameter value. The energy (Y-axis) was divided into four regions shown by the red lines. The cut values were decided as $\pm 3\sigma$ from the MC average value for each energy region. Applying them to the Crab data, we can see an event excess. The $\theta^2$ distribution is shown in left figure of Fig 12.5. The integral fluxes are obtained in the way as described before and shown in right figure. The difference between this analysis and the "default likelihood cuts" are shown in Table 12.6.
12.3. SYSTEMATIC ERROR OF THE SPOT SIZE UNCERTAINTY ETC.

Figure 12.3: $\theta^2$ distributions of the Crab for various cuts. Distributions obtained changing clustering cuts are shown in the top (left:T5a, middle:T6a, and right:T7a). The points with error bars are an ON-source data and the hatched area is an OFF-source data. The red line is $\theta^2$ cut positions ($\theta^2 < (0.217)^2$). The ADC cut results in the middle (left:4.5 p.e., middle:5 p.e., and right:5.5 p.e.). The L ratio cut results in the bottom (left:0.7, middle:0.8, and right:0.9). The energy of these figure is $E > 2.35$ TeV.

Summary of the systematic error of the spot size uncertainty etc.

Combining the above two tables (Table 12.7 and Tables 12.5), we determined the systematic error on the spot size etc. and show it in Table 12.7. Here, we assumed that these errors listed in Tables 12.5 and 12.6 are dependent on each other because both are caused by the systematic error of the spot size etc. We took the maximum deviations into account in Table 12.7.
Figure 12.4: $\log_{10}(\text{sumADC})$ vs. Length and $\log_{10}(\text{sumADC})$ vs. Width. Length is shown in the left. Width is shown in the right. The color contour is MC data and the solid line contour is real data of the Crab.

Figure 12.5: The $\theta^2$ distribution of the Crab is shown in the left figure. The points with error bars are an ON-source data and the hatched area is an OFF-source data. The Red line is $\theta^2$ cut positions ($\theta^2 < (0.217)^2$). The integral flux of the Crab is shown in the right figure. The solid line is obtained by HEGRA group [30].
12.4 SYSTEMATIC ERROR OF ATMOSPHERIC CONDITIONS

<table>
<thead>
<tr>
<th>bin #</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>SumADC cut (p.e.)</td>
<td>0. &lt;</td>
<td>100. &lt;</td>
<td>200. &lt;</td>
</tr>
<tr>
<td>(E_{th} ) (TeV)</td>
<td>2.35</td>
<td>3.42</td>
<td>5.37</td>
</tr>
<tr>
<td>Square cut flux((\times10^{-12})cm(^{-2})sec(^{-1}))</td>
<td>4.7 ± 1.2</td>
<td>2.2 ± 0.58</td>
<td>0.77 ± 0.28</td>
</tr>
<tr>
<td>Likelihood cut flux((\times10^{-12})cm(^{-2})sec(^{-1}))</td>
<td>4.4 ± 0.81</td>
<td>2.1 ± 0.42</td>
<td>1.0 ± 0.21</td>
</tr>
<tr>
<td>Systematic error</td>
<td>7 ± 25 %</td>
<td>5 ± 26 %</td>
<td>-23 ± 36 %</td>
</tr>
</tbody>
</table>

Table 12.6: Summary of the systematic errors estimated by changing analysis methods. We compared the results of the square (conventional) cuts with the likelihood cut.

<table>
<thead>
<tr>
<th>Energy bin</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>SumADC cut (p.e.)</td>
<td>0. &lt;</td>
<td>100. &lt;</td>
<td>200. &lt;</td>
</tr>
<tr>
<td>(E_{th} ) (TeV)</td>
<td>2.35</td>
<td>3.42</td>
<td>5.37</td>
</tr>
<tr>
<td>systematic error</td>
<td>-14 ± 21 %</td>
<td>-23 ± 28 %</td>
<td>-23 ± 36 %</td>
</tr>
</tbody>
</table>

Table 12.7: Summary of the systematic error on the spot size etc.

12.4 Systematic Error of Atmospheric Conditions

The day-by-day trigger rate will reflect atmospheric conditions. The cosmic-ray rate itself should be very stable at the % level. Note that those cannot be simulated by the MC.

Shower rate fluctuation

The fluctuation of the cosmic-ray rate is regarded as efficiency fluctuation. As shown in Fig 12.6 (left panel), the shower rates have a dependency on \(\cos(\text{zenith angle})\). In order to correct these effects, we used the shower rate at the minimum zenith angle (\(z = 53^\circ\)) of the target object (here it is the Crab), which is obtained by fitting a straight line to the cosine zenith angle plot as shown in Fig 12.6. Also to keep good statistical quality, we rejected short-time runs such as 30 min. The short time observation is usually bad condition because we gave up observation on the way. These shower rates are shown in the right panel of the figure. The X-axis is \(\cos(\text{zenith angle}) - 0.6\), and \(\cos(53^\circ)\) is 0.6. The Y-axis is the relative shower rate from the average shower rate. Dispersion of these shower rates is 8 %. The systematic uncertainty due to shower rate variations was
estimated to be smaller than 8 %.

**Figure 12.6:** Relation between shower rates and cos(zenith angle) (left panel). The X-axis is Cos(zenith angle) - 0.6, and Cos(53°) is 0.6 and the Y-axis is the shower rate. The variation of shower rates is plotted run by run in the right panel. The X-axis is run number and the Y-axis is the relative shower rate from the average shower rate. Dispersion of these shower rates is 8 %.

**Time variation of the Crab flux**

The Crab flux is known to be very stable by many previous measurements. Therefore, if a time variation was detected by our measurements, it would indicate serious systematic uncertainty. Considering the statistics of the previously derived Crab signal, we divided them into two datasets. The individual $\theta^2$ distributions are shown in Fig 12.7 and the individual fluxes are shown in Fig 12.8. Dataset 1 corresponds to 508 minutes in the first half of the observation period (Dec 18 - 23) and dataset 2 corresponds to 350.8 minutes in the second half of the observation period (Dec 24 - 28). The excess events and fluxes etc. are summarized in Table 12.8. The maximum deviation of time variation of the Crab flux is 11 % shown in Table 12.8 (dataset# 1, bin# 3). They seem to be dominated by statistical fluctuations.
Figure 12.7: $\theta^2$ distribution for the Crab. The data are divided into 2 sets. Dataset 1 is shown on the left and dataset 2 is shown on the right. Dataset 1 corresponds to 508 minutes in the first half of the observation period (Dec 18 - 23) and dataset 2 corresponds to 350.8 minutes in the second half of the observation period (Dec 24 - 28). We did not use observation runs shorter than 30 minutes. This $\theta^2$ distribution is the result of the lowest energy bin. The points with error bars are an ON-source data and the hatched area is an OFF-source data. The red line is the $\theta^2$ cut ($\theta^2 < (0.217)^2$, Appendix A).
### Data set #1

<table>
<thead>
<tr>
<th>bin #</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>SumADC cut (p.e.)</td>
<td>0. &lt;</td>
<td>100. &lt;</td>
<td>200. &lt;</td>
</tr>
<tr>
<td>$E_{th}$ (TeV)</td>
<td>2.30</td>
<td>3.43</td>
<td>5.38</td>
</tr>
<tr>
<td>$N_{on}$ ($\theta^2 &lt; (0.217)^2$)</td>
<td>254</td>
<td>217</td>
<td>121</td>
</tr>
<tr>
<td>$N_{off}$ ($\theta^2 &lt; (0.217)^2$)</td>
<td>189.0</td>
<td>161.7</td>
<td>78.2</td>
</tr>
<tr>
<td>Excess event</td>
<td>65 ± 16</td>
<td>55 ± 15</td>
<td>43 ± 11</td>
</tr>
<tr>
<td>Observation time (min)</td>
<td>508</td>
<td>508</td>
<td>508</td>
</tr>
<tr>
<td>Significance</td>
<td>4.1</td>
<td>3.8</td>
<td>4.1</td>
</tr>
<tr>
<td>Effective area ($E &gt; E_{th}$, $\times 10^{10}$ cm$^2$)</td>
<td>0.051</td>
<td>0.091</td>
<td>0.128</td>
</tr>
<tr>
<td>Flux ($\times 10^{-12}$ cm$^{-2}$ sec$^{-1}$)</td>
<td>$4.2 \pm 1.03$</td>
<td>$2.0 \pm 0.53$</td>
<td>$1.1 \pm 0.28$</td>
</tr>
<tr>
<td>Event rate (/min)</td>
<td>$0.13 \pm 0.03$</td>
<td>$0.11 \pm 0.03$</td>
<td>$0.08 \pm 0.02$</td>
</tr>
<tr>
<td>Average event rate</td>
<td>$0.14 \pm 0.04$</td>
<td>$0.12 \pm 0.04$</td>
<td>$0.09 \pm 0.03$</td>
</tr>
<tr>
<td>Systematic error</td>
<td>$-7 \pm 23%$</td>
<td>$-8 \pm 25%$</td>
<td>$-11 \pm 33%$</td>
</tr>
</tbody>
</table>

### Data set #2

<table>
<thead>
<tr>
<th>bin #</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>SumADC cut (p.e.)</td>
<td>0. &lt;</td>
<td>100. &lt;</td>
<td>200. &lt;</td>
</tr>
<tr>
<td>$E_{th}$ (TeV)</td>
<td>2.30</td>
<td>3.43</td>
<td>5.38</td>
</tr>
<tr>
<td>$N_{on}$ ($\theta^2 &lt; (0.217)^2$)</td>
<td>184</td>
<td>158</td>
<td>81</td>
</tr>
<tr>
<td>$N_{off}$ ($\theta^2 &lt; (0.217)^2$)</td>
<td>130.8</td>
<td>110.8</td>
<td>49.3</td>
</tr>
<tr>
<td>Excess event</td>
<td>53 ± 14</td>
<td>47 ± 13</td>
<td>32 ± 9</td>
</tr>
<tr>
<td>Observation time (min)</td>
<td>350.8</td>
<td>350.8</td>
<td>350.8</td>
</tr>
<tr>
<td>Significance</td>
<td>4.0</td>
<td>3.9</td>
<td>3.8</td>
</tr>
<tr>
<td>Effective area ($E &gt; E_{th}$, $\times 10^{10}$ cm$^2$)</td>
<td>0.052</td>
<td>0.089</td>
<td>0.152</td>
</tr>
<tr>
<td>Flux ($\times 10^{-12}$ cm$^{-2}$ sec$^{-1}$)</td>
<td>$4.9 \pm 1.3$</td>
<td>$2.5 \pm 0.66$</td>
<td>$1.2 \pm 0.33$</td>
</tr>
<tr>
<td>Event rate (/min)</td>
<td>$0.15 \pm 0.04$</td>
<td>$0.13 \pm 0.04$</td>
<td>$0.09 \pm 0.03$</td>
</tr>
<tr>
<td>Average event rate</td>
<td>$0.14 \pm 0.04$</td>
<td>$0.12 \pm 0.04$</td>
<td>$0.09 \pm 0.03$</td>
</tr>
<tr>
<td>Systematic error</td>
<td>$7 \pm 22%$</td>
<td>$8 \pm 25%$</td>
<td>$0 \pm 27%$</td>
</tr>
</tbody>
</table>

**Table 12.8:** Summary of each dataset and systematic errors. The average event rate is the average of dataset# 1 and dataset# 2. The data of $N_{off}$ were normalized by the ratio of the number of background points i.e. 6 regions. The definitions of integral flux are described in Section 10.6.
Figure 12.8: Difference between the integral fluxes for the Crab for the 2 datasets. Dataset 1 is plotted with the black points and dataset 2 is plotted with red. The solid line is the Crab flux obtained by the HEGRA group [30] and the dashed line is obtained by the Whipple group [58].

Summary of the systematic error of atmospheric conditions

The systematic error of shower rate is estimated as 8 % and The maximum deviation of time variation of the Crab flux is 11 % which id shown in Table 12.8 (dataset # 1, bin # 3). So we adopt this systematic errors of time variation of the Crab flux due to atmospheric conditions. We took the maximum deviations into account in Table 12.9.

<table>
<thead>
<tr>
<th>Energy bin</th>
<th>1</th>
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<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>SumADC cut (p.e.)</td>
<td>0. &lt;</td>
<td>100. &lt;</td>
<td>200. &lt;</td>
</tr>
<tr>
<td>$E_{th}$ (TeV)</td>
<td>2.35</td>
<td>3.42</td>
<td>5.37</td>
</tr>
<tr>
<td>systematic error</td>
<td>-7 ± 23 %</td>
<td>-8 ± 25 %</td>
<td>-11 ± 33 %</td>
</tr>
</tbody>
</table>

Table 12.9: Summary of the systematic error of atmospheric conditions


12.5 Summary of the Systematic Error

Here, we compile the systematic errors for the final results on Cen A. Because the errors on the NSB, spot sizes, air conditions, and etc. are considered to be independent of each other, the obtained errors were quadratically added. But we did not add the uncertainty of energy determination because the value of errors is small. Also we now understand that these errors derived so far are statistically governed. To add all of them quadratically might be an overestimation. But for safety, we chose it. It is shown in Table 12.10.

<table>
<thead>
<tr>
<th>Energy bin</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>SumADC cut (p.e.)</td>
<td>0. &lt; 100. &lt; 200. &lt;</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$E_{th}$ (TeV)</td>
<td>2.30</td>
<td>3.43</td>
<td>5.38</td>
</tr>
<tr>
<td>NSB uncertainty (%)</td>
<td>-26</td>
<td>-8</td>
<td>-5</td>
</tr>
<tr>
<td>Spot size uncertainty etc. (%)</td>
<td>-14</td>
<td>-23</td>
<td>-23</td>
</tr>
<tr>
<td>Atmospheric conditions (%)</td>
<td>-7</td>
<td>-8</td>
<td>-11</td>
</tr>
<tr>
<td>Total systematic error (%)</td>
<td>30</td>
<td>26</td>
<td>26</td>
</tr>
</tbody>
</table>

Table 12.10: Summary of the systematic error.

The final results on the upper limits on Cen A are shown in the following Table 12.11. The final result of the Cen A 2-$\sigma$ upper limit is also shown in Fig 12.9. Several definitions of flux upper limit are used in the world. Since the 2-$\sigma$ flux upper limit has been used in CANGAROO group, we used the 2-$\sigma$ flux upper limit. Definition of 2-$\sigma$ flux upper limit is

$$2 - \sigma \text{ flux upper limit} < \left( 2 \times \sqrt{\sigma_{\text{stat.}}^2} \right) \times \left( \frac{\text{systematic error} \%}{100} + 1 \right)$$

The overall upper limits obtained for Cen A are at about 7% Crab gamma-ray flux level on average.

12.6 Square Cut Result of Cen A

Finally we want to show that the excess signal cannot even be found by the "Square cuts". The procedure of the analysis is the same as for the Crab. The parameters for the square
Figure 12.9: Final 2-$\sigma$ flux upper limit of Cen A including systematic errors. These arrows show the 2-$\sigma$ flux upper limit. The solid line is the Crab flux obtained by the HEGRA group [30].
### Table 12.11: Summary of the upper limit for the integral flux $I(\geq E)$ of Cen A. 2-σ upper limits are calculated by using the total systematic error.

<table>
<thead>
<tr>
<th>parameter</th>
<th>1</th>
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</thead>
<tbody>
<tr>
<td>$SumADC$ cut (p.e.)</td>
<td>0. $&lt;$</td>
<td>100. $&lt;$</td>
<td>200. $&lt;$</td>
</tr>
<tr>
<td>$E_{th}$ (TeV)</td>
<td>0.53</td>
<td>0.70</td>
<td>1.12</td>
</tr>
<tr>
<td>2-σ upper limit Flux</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>($\times 10^{-12}$ cm$^{-2}$ sec$^{-1}$)</td>
<td>3.2</td>
<td>1.9</td>
<td>0.9</td>
</tr>
</tbody>
</table>

cuts are selected to be Length and Width and Distance. We exactly followed the same procedure as the Crab analysis. The result of $\theta^2$ distribution is shown in Fig 12.10. We selected 6 background regions for the $\theta^2$ analysis. The $\theta^2$ distribution is shown below. No significant excess was found.
Figure 12.10: $\theta^2$ distribution of Centaurus A by the "Square cuts". The points with statistical errors show the $\theta^2$ distribution of the ON-source data. The hatched area shows the OFF-source data. The OFF-source data were normalized by the ratio of the number of background points i.e. 6 regions.
Chapter 13

Discussions

13.1 Comparison with the Past Data

Cen A has been observed many times. First, the Narrabri Observatory reported a positive detection with a flux of \( I(> 0.3\text{TeV}) \sim (4.4 \pm 1) \times 10^{-11} \text{ photon cm}^{-2} \text{ sec}^{-1} \) [53]. Buckland Park and the JANZOS Observatory also detected gamma-rays during burst states (Buckland Park : \( I(> 100\text{TeV}) \sim (7.4 \pm 2.6) \times 10^{-12} \text{ photon cm}^{-2} \text{ sec}^{-1} \), JANZOS (burst) : \( I(> 150\text{TeV}) \sim (5.5 \pm 1.5) \times 10^{-12} \text{ photon cm}^{-2} \text{ sec}^{-1} \)). Thus, Centaurus A was one of promising extragalactic sources radiating VHE gamma-rays. However, CANGAROO-I [105], JANZOS (in steady states) [9], and Durham [20] observed the Cen A nucleus part and set upper limits on the emission in the VHE range. The above observation results together with this thesis result are shown in Fig 13.1. We give an upper limit of 1/10 of those reported by Narrabri, Durham, CANGAROO-I, and JANZOS.

13.2 Time Variability of Cen A

Perhaps the most striking property of TeV \( \gamma \)-ray emission by AGN is its time variability. The simultaneous multi-waveband observations for Mkn 421 and Mkn 501 show that the amplitude variation at TeV energies is the largest [98, 22, 113]. For Mkn 421 and Mkn 501, TeV fluxes change together with X-rays without a significant time lag (<24hr). This may be understood if the emission mechanisms of these two energies are due to the same origin, such as high energy electrons. Therefore, the TeV \( \gamma \)-ray outburst of an object can
Figure 13.1: Cen A final results together with the previous observation results. The black line is the Crab flux obtained by the HEGRA group [30]. The red arrows are our results. The J1 - J3 points are the results reported by JANZOS group [9]. The J3 point is the integral flux during burst state. Narrabri Observatory [53] and Buckland Park air shower array group [28] were detected the gamma-ray from Cen A. The another arrows are the flux upper limit of Durham [20] and CANGAROO-I [105].

be predicted by investigating the X-ray variability of the object.

The observation period of Cen A is 15 – 28 Mar. and 15 – 28 Apr (Section 7.1). In our analysis of Cen A, we find no sign of such outbursts. The 2-σ flux upper limit of each day is summarized in Table 13.1. The average flux upper limit is $1.0 \times cm^{-2} sec^{-1}$ from this table. The maximum TeV-variability of Cen A was less than $2.3 \times 2-\sigma$ average flux upper limit. Our result is consistent with no significant bursts found in this period and also a null result was obtained by the RXTE ASM. The typical flux variations of AGNs, such as Mkn 421 and Mkn501, are of one order of magnitude between steady and burst states. Therefore, ten times as higher flux upper limit as ours can also be considered to be an upper limit in the burst state.
### Chapter 13. Discussions

<table>
<thead>
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<th>Mar</th>
<th>Mar</th>
<th>Mar</th>
<th>Apr</th>
<th>Apr</th>
<th>Apr</th>
</tr>
</thead>
<tbody>
<tr>
<td>day</td>
<td>16</td>
<td>18</td>
<td>25</td>
<td>27</td>
<td>15</td>
<td>16</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>2σ upper limit (E &gt; 530 GeV)</th>
<th>(10^{−12} , cm^{-2} , sec^{-1})</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2.3</td>
</tr>
</tbody>
</table>

**Table 13.1**: Summary of Cen A 2-σ upper limit for each day. The observation period of Cen A is 15 – 28 Mar. and 15 – 28 Apr and Cen A has been observed 7 nights in this period.

### 13.3 Estimation of Flux During Outburst

Considering the time variability of the above upper limits and that of typical AGNs, a ten times larger value than our upper limits in the stable period should be considered to be maximum allowable burst limits. We make this assumption in the following discussion.

The two radiation components of blazars are self-similar in shape. The following is discussed in Bai et al [10]. In the context of the SSC model, the luminosities ($L_{\text{sync}}$, $L_{\text{ssc}}$), peak luminosities ($L_{\text{sync}}(\nu_{\text{sync}})$ and $L_{\text{ssc}}(\nu_{\text{ssc}})$), and peak flux densities ($f_{\text{sync}}(\nu_{\text{sync}})$ and $f_{\text{ssc}}(\nu_{\text{ssc}})$) of the two components are related by [109, 119]

$$\frac{L_{\text{ssc}}}{L_{\text{sync}}} = \frac{\nu_{\text{ssc}}L_{\text{ssc}}(\nu_{\text{ssc}})}{\nu_{\text{sync}}L_{\text{sync}}(\nu_{\text{sync}})} = \frac{\nu_{\text{ssc}}f_{\text{ssc}}(\nu_{\text{ssc}})}{\nu_{\text{sync}}f_{\text{sync}}(\nu_{\text{sync}})}. \quad (13.1)$$

During the high state, the TeV blazars all have a Compton luminosity $L_{\text{ssc}}$ comparable to or slightly less than the synchrotron luminosity $L_{\text{sync}}$ (i.e., this should also be the case in the stable state), i.e., $L_{\text{ssc}}/L_{\text{sync}} \sim 1$. Assuming this is also valid for candidate TeV sources, we obtain

$$\nu_{\text{ssc}}f_{\text{ssc}}(\nu_{\text{ssc}}) \simeq \nu_{\text{sync}}f_{\text{sync}}(\nu_{\text{sync}}). \quad (13.2)$$

According to the SSC model, the Compton component has the same spectral shape as the synchrotron component. Thus, with Eq 13.2 and the spectral index of the synchrotron component, the TeV $\gamma$-ray flux of a candidate can be estimated as

$$F_{\gamma} \simeq \nu_{\text{sync}}f_{\text{sync}}(\nu_{\text{sync}})[(\nu_{\text{ssc}}^{1-\alpha_1} - \nu_{\text{ssc}}^{1-\alpha_1}) + \frac{\nu_{\text{ssc}}^{1-\alpha_2} - \nu_{\text{ssc}}^{1-\alpha_2}}{(1 - \alpha_2)\nu_{\text{ssc}}^{1-\alpha_2}}], \quad (13.3)$$

where $\nu_1 (\nu_1 \leq \nu_{\text{ssc}})$ and $\nu_2 (\nu_2 \geq \nu_{\text{ssc}})$ are the energy thresholds of a TeV $\gamma$-ray detector, and $\alpha_1$ and $\alpha_2$ are spectral induces of the synchrotron component below and above $\nu_{\text{sync}}$, respectively.
Fig 13.2 shows spectral energy distributions of Cen A, M87, and Mkn 501 based on several observations in the literature. The X-ray spectral index of the Cen A nucleus at 2-10 keV was found to be less than 1 [91, 121], and hence \( \log(\nu f_{\nu}) \) is a monotonically rising function of \( \log \nu \) in this energy region. At higher energies between 10 and 100 keV, the spectral index is \( 0.7 - 0.8 \) [18, 91, 76, 128], also less than 1, with a monotonically rising SED of Cen A in this energy range. The SED of Cen A continuously rises below \( \sim 150 \) keV where the spectrum breaks, then the SED goes down continuously to the MeV and even GeV energy ranges as shown in Fig 13.2. As in the case of blazars, this hump represents one of the two radiation components, and the radiation mechanism of this component is the same as that of X-rays.

Steinle et al. [112] and Kinzer et al. [76] pointed out that the spectra of Cen A show interesting similarities to those of jet-aligned blazars [89], and in particular to those of the well-studied quasar 3C 273 [68, 85]; both show spectral breaks in the soft gamma-ray regime, and both have intensity-independent power-law shapes below the break. For 3C 273 the peak in the soft gamma-ray regime is a Compton peak, so Steinle et al. and Kinzer et al. regarded the peak of Cen A at \( \sim 150 \) keV as a Compton peak. However, based on the...
observations by *ROSAT* and *ASCA*, Turner et al.[121] found that the spectrum of the jet of Cen A is actually consistent with the spectrum of the nuclear continuum, and that the spectrum of the jet should be flatter than that expected as a result of an inverse-Compton scattering of radio photons but is consistent with that predicted by a simple synchrotron model. Recent Chandra observations also indicated that X-rays from the jet of Cen A comprise synchrotron emission [15]. Therefore, the peak at $\sim 150$ keV is not the Compton peak but the synchrotron peak, as the case of Mkn 501 (Figure 13.2). The peak frequency of Cen A is variable, which is also similar to the behavior in TeV blazars. Observations by *CGRO* OSSE in 1991–1994 revealed peak frequencies of $\sim 150$ keV [76], while Miyazaki et al.[91] reported a peak frequency of 180 keV, based on Welcome-1 observations.

The peak of the Compton component of Mkn 501 is at $\sim 1$ TeV, so that of Cen A is probably also at $\sim 1$ TeV. During the outburst state, the flux density at 100 keV is $\approx 11 \times 10^{-5}$ photons cm$^{-2}$s$^{-1}$keV$^{-1}$ [18] and the synchrotron peak flux density is

$$f_{150keV} = 2.1 \times 10^{-29} \text{erg cm}^{-2}\text{s}^{-1}\text{Hz}^{-1},$$

assuming a spectral index of 0.7. According to Eq. 13.3 with $\alpha_1=0.7$ and $\alpha_2=1.3$, then the estimated TeV flux in $0.25-30$ TeV for Cen A is

$$F(0.25-30\text{TeV}) = 6.4 \times 10^{-9} \text{erg cm}^{-2}\text{s}^{-1}$$

which is about 10 times brighter than that of Mkn 501 in the 1997 outburst, suggesting that Cen A can be a strong TeV ($1 \text{TeV}=2.4 \times 10^{26} \text{Hz}$) $\gamma$-ray source during outburst. In our analysis, the 2-$\sigma$ upper limit is or $\sim 10^{-12} \text{erg cm}^{-2}\text{s}^{-1}$ at about 1 TeV, which is an order of magnitude higher than the flux estimated above. We did not detect the flux at Eq 13.4 level, but we could if it occurred. Since it was already discussed by Section 13.2, burst was not occurred in Cen A observation period. Again the important fact is that the flux of Mkn501 changes only about 10 times between burst and quiet states. Hence, our result is inconsistent with this SSC model which discussed in Bai et al [10].

### 13.4 Comparison with Typical AGNs

Here we assume that Cen A is a typical AGN. In the case of the typical AGNs like Mkn 421 and 501, gamma-rays from the jet reach the earth after being amplified via relativistic beaming. The inclination angle of the jet of Cen A is estimated to be about 60° by several
13.4. COMPARISON WITH TYPICAL AGNS

From the above estimation, the detection efficiency of gamma-rays from Cen A is reduced by a factor of $10^{-4}$ than the cases of Mkn421, 501 etc. On the other hand, Cen A is much closer than Mkn 421 and 501. Here the distance factor between Cen A and those typical AGNs is geometrically $(4 \pi d^2)^{-1}$. This factor becomes $10^3$. Our sensitivity is now $10^{-1}$ improved compared with the experiments of 10 years ago, so we should have enough sensitivity.

13.4.1 Difficulties of the LBL Assumption

First, we assume that Cen A is a LBL, i.e., the radio and IR are considered to be due to synchrotron radiation and the MeV emissions are due to the Inverse Compton Scatterings, respectively. The maximum energy of the synchrotron radiation is located around the IR region. In the SSC model the IC has the maximum energy located around the MeV region. This is consistent with a typical AGN’s value of $\nu_{ssc}/\nu_{sync} \sim 10^8$, and, $L_{ssc}/L_{sync} \sim 1$ is consistent with the peak height of the SED. This assumption seems to support the SSC model. Here, assuming $\nu_{ssc}/\nu_{sync} \sim 10^8$ the $\gamma$ factor of incident electron is $\sqrt{10^8} = 10^4$. Therefore $E_{\text{max}}^e$ is estimated to be $\gamma m_e = 5$ GeV. The blue line which is plotted around the MeV region is the spectrum which is predicted by theory with $E_{\text{max}}^e = 10$ GeV and a spectral index of electrons of -1, which is rather higher than the prediction of the standard diffusive shock acceleration of -2. The electron power-law index from the radio observation is -2.5, which is different from this assumption. According to this model, our observation becomes totally nonsense because $5 \sim 10$ GeV electrons cannot produce TeV gamma-rays.

Next, the magnetic field ($B$) which causes the synchrotron radiation is estimated. Synchrotron radiation is the electron scattering with ambient background photon field of energy proportional to $B^2$ with high energy electrons. The peak energy of synchrotron radiation, therefore, can be considered to be proportional to

$$U_B \times \gamma_e^2$$

. Typically we have the relationship

$$E_{\text{max}}^\text{sync} \text{(keV)} \sim \left( \frac{E_{\text{max}}^e}{100 \text{ TeV}} \right)^2 \left( \frac{B}{\mu\text{G}} \right),$$

(13.5)

where the unit of E is TeV and B is $\mu$G [35]. If $E_{\text{max}}^e$ is 10 GeV, the magnetic field is $O(10^{-2})$ G which is consistent with the values of typical AGNs. As shown in Fig 13.3,
the blue line at lower energies is the expectation of synchrotron radiation when $B=10^{-2}$ G and $E_{\text{max}}^e = 10$ GeV. In this assumption, the total energy of high energy electrons can be calculated to be more than $10^{57}$ erg.

This value is 10000 times bigger than our Galaxy. Although Cen A is a very energetic galaxy, it seems too high.

Furthermore, the measurement of our Galaxy indicates that proton acceleration is 100 times more effective than electron, considering the electron fluxes on the earth. Assuming the above ratio, the total energy of cosmic rays in Cen A becomes more than $10^{59}$ erg, again considered to be too high. With this situation, assuming that the mean interstellar matter density in Cen A is about 1 proton/cc as in our Galaxy, naturally, we can expect gamma-rays which are produced by $\pi \rightarrow \gamma \gamma$ (red line) and due to bremsstrahlung

\[ \text{Figure 13.3: SED for the LBL model. In units of energy flow, this corresponds to an upper limit of } \sim 1 \text{ eV/cm}^2/\text{sec at energies greater than 1 TeV. The red arrows are the CANGAROO-III flux upper limits. The black arrows are the flux upper limits of other measurements. The black points are the results of other measurements. The blue line is synchrotron radiation in which the peak energy is located at } \sim 0.1 \text{ eV. The IC emission is considered to occur in a higher energy region, i.e., MeV region. The red line is a } \pi^0 - \text{decay model and the green shows bremsstrahlung. The details can be found in the text. The light blue line is the sum of all these emissions.} \]
radiation. 1 proton/cc is a minimum estimation as Cen A is considered to be a higher density galaxy. The light blue line is the total flux according to the above discussions. It obviously conflicts with the 100 MeV (EGRET) upper limit, i.e., this assumption is not self-consistent. In these estimations, we only calculate a flux with electron energies up to 300 GeV. This is the reason why these curves fall off at 10 GeV. Because of the above reasons, the LBL assumption cannot reasonably explain the situation. From now on, we need to consider the HBL assumption.

### 13.4.2 Possibility on HBL Assumption

If we assume Cen A as the HBL, the peak frequency of the synchrotron radiation is located at 100 MeV. So we can assume that the synchrotron radiation contributes below 100 MeV. Here, we regard the data of radio and COMPTEL measurements as due to synchrotron radiation. The low and high fluxes of IR and X-ray are problems. It looks inconsistent to the flux of HBL model. The total volume of the jet (Fig 2.6) is located well behind the dust lane (Fig 2.5). The dust lane was observed in various energy region (IR, visible, and X-ray) should cause strong absorption. The points of high flux can be regarded as the thermal emission of Cen A.

Under the assumptions discussed above, we plot the synchrotron radiation as the blue line in Fig 13.4.

The blue line is plotted after the theoretical calculation assuming that electron power-law index is -2.5. This value is consistent with radio observations. Here $E_{\text{max}}^e$ is assumed to be 100 TeV and $B$ to be 0.01 G. This assumption is rather consistent with the typical AGN model. In this case, the maximum energy of TeV gamma-rays which are emitted by the IC scattering is affected by Klein-Nishina suppression and

$$\frac{\nu_{\text{ssc}}}{\nu_{\text{sync}}} \sim 10^7 < 10^8,$$

(light blue line in Fig 13.4). The IC line is the maximum which does not exceed our upper limit ($\sim 1 \text{ eV cm}^{-2} \text{ sec}^{-1}$ at 1 TeV). This flux was obtained using free normalization factors. Here, we find the upper limit,

$$\frac{L_{\text{ssc}}}{L_{\text{sync}}} < \frac{1}{400} \ll 1.$$  \hspace{1cm} (13.6)
Figure 13.4: SED of HBL model. The blue line is synchrotron radiation and the higher-energy light-blue is IC emission line. The radio observations are plotted around $10^{-7} \sim 10^{-4}$ (eV). The infrared and optical observations are plotted around $10^{-2} \sim 10^{1}$ (eV). The X-ray observations which are plotted around $10^{3} \sim 10^{4}$ (eV) are ROSAT [19] and EINSTEIN [39]. The MeV gamma observations which are plotted around $10^{4} \sim 10^{8}$ are OSSE [112], EGRET, COMPTEL and BATSE [110, 111]. The red arrows are CANGAROO-III flux upper limit.

This is largely inconsistent with “typical” AGN models where,

$$L_{ssc}/L_{sync} \sim 1.$$  \hspace{1cm} (13.7)

The result of this estimate leads to the conclusion that it is not a “typical” AGN, although having jets and flares. This relationship, therefore, should be realized in any angle to the line of sight, such as 60° in the case of Cen A. The beaming factor $\delta$ is described as following:

$$\delta = \frac{1}{\gamma(1 - \beta \cos \theta)}.$$  \hspace{1cm} (13.8)

We chose “typical” AGN parameters, $\gamma=10.$, $\beta=1.$, and angle ($\theta = 60^\circ$ as the special value for Cen A). Then we obtain,

$$\delta \sim 0.2.$$  \hspace{1cm} (13.8)

According to the SSC model,

$$L_{sync} = \text{const} \times U_B$$  \hspace{1cm} (13.9)

$$L_{ssc} = \text{const} \times U_{sync}$$  \hspace{1cm} (13.10)
and the constants are common, so that

\[ \frac{L_{ssc}}{L_{sync}} = \frac{U_{sync}}{U_B}. \]  

(13.11)

where \( U_{sync} \) should be,

\[ L_{sync} = 4\pi R^2 c \delta^4 U_{sync}. \]  

(13.12)

Using this formula, we can derive a physical constant which includes the emission size \( R \) and the magnetic field \( B \). Our angular resolution is 0.2\(^\circ\), and assuming \( d=3.5 \) Mpc, temporally we can calculate a corresponding size of \( R=12 \) Kpc and we obtain

\[ U_{sync} \sim 6 \text{eV/cc}. \]  

(13.13)

Finally we obtain,

\[ U_B > 400U_{sync} = 2400 \text{eV/cc} \]  

(13.14)

under the relationship

\[ U_B = \frac{1}{8\pi} B^2. \]  

(13.15)

We, hereby, can give an lower limit to the magnetic field:

\[ B > 210 \mu G. \]  

(13.16)

Here we presume the size equal to our angular resolution 0.15, but, the emission region of the synchrotron radiation should be smaller considering the radio jet structure. The real lower limit on \( B \) should be bigger than this result. Since \( B > 210 \mu G \) is bigger than in our galaxy, obviously Cen A is special. The stronger magnetic field increase the electron energy loss by synchrotron radiation. Electron cooling, therefore, should be very high. Time scale of particle acceleration in Cen A can be , however, cosmological scale, which may allow cosmic-ray energies up to 100 TeV. Considering that our angular resolution is larger than the size of jets observed in the radio, we can consider the size of the emission region as a free parameter. Then we obtain

\[ B > 210 \mu G \times (R/12 \text{kpc})^{-1}. \]
In the cases of Mkn 421 and 501, the TeV emission regions are considered to be $10^{16-17}$ cm. Even in the rest frame of the relativistic jets, it is $10^{17-18}$ cm which is far less than 1 pc. The above formula gives $B >$ a few Gauss which is high even compared to the geomagnetic field of the earth, and too high compared to typical estimations of several mG. A field of this magnitude within such a large volume is unbelievable. This would give more stringent lower limit than the above result.

Although the jet of Cen A behaves like typical AGN and sometimes flares occurred, it is different from Mkn421 and 501.

We here again assume that Cen A is located at the same distance as Mkn421, Mkn501 with an inclination angle=60°, the synchrotron radiation of Cen A is stronger than Mkn412 and Mkn 501. However, we could not see any similarity with Mkn 421 and 501. Cen A seems to be a special galaxy and AGN. The unified AGN model should be reconsidered.

### 13.5 Cold Dark Matter Scenario

Here, we introduce the possibility to search for Cold Dark Matter (CDM) using these data. Cen A is called a ”giant galaxy”, i.e., some estimated its total mass of $3 \sim 5 \times 10^{11} M_\odot$ within 50kpc from its center\cite{86}. It is heavier than the Galaxy and is nearby galaxy. The visible volume can be contained within our field of view. It is one of the best targets for the CDM search.

CDM was introduced by cosmology and particle physics. An important motivation is Super Symmetric theory and recent WMAP measurement\cite{14}. Super Symmetry (SUSY) is considered to be unavoidable in field theory. Typical theories predict the CDM mass around the weak scale, which matches the energy scale of our sub-TeV to TeV measurements. In addition, accelerator searches can not reach over 1 TeV. Our observation is complementary to these. In addition, the recent WMAP result gave very stringent limits to the theory. They derived the relic CDM density ($\Omega_{CDM}$) to be 0.23. In cosmology, the relic density of the CDM is determined mostly by the co-annihilation cross section of these particles. At the big bang they were thermally produced, therefore, the density was determined by the thermal equilibrium. The densities decrease by the co-annihilation
speed and expansion speed of the universe. The thermal equilibrium was broken when the above two speeds were equalized (freeze: so they were called "Cold"). The above relationship and $\Omega_{CDM}$ determines the co-annihilation cross section multiplied by the relative velocity to be

$$3 \times 10^{-26} \text{cm}^3 \text{s}^{-1}.$$ 

This error is less than 1 %. Although this value is higher than the standard weak theory, the broadness of SUSY parameter space allows it[38].

Here, we assume that the annihilation product is mostly quark anti-quark pair ($q\bar{q}$). The detailed argument can be found in reference[36]. The annihilation rate of the CDM can be written

$$< \sigma v > B_{q\bar{q}}n^2,$$  \hspace{1cm} (13.17)

where $\sigma$ is the annihilation cross section, $v$ the relative velocity, $B_{q\bar{q}}$ the branching ratio to quark anti-quark pair, and $n$ the relic density of these particles. The quark anti-quark pair fragments to final products, i.e., stable particles. Gamma-rays are well measured by accelerator experiments[7]. The values of

$$\frac{1}{\sigma_h} \frac{d\sigma}{d(E/M_\chi)}$$  \hspace{1cm} (13.18)

can be obtained from experiment, where $E$ is the gamma-ray’s energy and $M_\chi$ is the CDM particle mass. We, therefore, obtain the following gamma-ray flux estimation at the solar system:

$$\left[ \frac{dF}{dE} \right] = \frac{< \sigma v B_{q\bar{q}} > \rho_{CDM}^3 [V/(4\pi d^2)]}{M_\chi^3} \cdot \left[ \frac{1}{\sigma_h} \frac{d\sigma}{d(E/M_\chi)} \right],$$  \hspace{1cm} (13.19)

where $\rho_{CDM}$ is the energy density of the CDM, $V$ is the volume of the emission region, $d$ is the distance from the solar system to Cen A. For an assumed $M_\chi$, there are no unknown parameters after obtaining the observational flux of $\frac{dF}{dE}$.

Using these arguments, CANGAROO-II carried out such CDM search on the nearby star-burst galaxy NGC 253[36] and the Galactic Center[120]. For NGC 253, within the radius of 27 kpc, the total mass upper limit of $1.7 \times 10^{14} M_\odot$ was obtained which corresponds to 2400 times the mass of NGC 253. For the Galactic Center, within 47 pc, $8 \times 10^7 M_\odot$ was obtained where the gravitational mass inside that volume was estimated to be the same order from the motion of the stars and clouds.
Cen A is an interesting object for this study, because its distance is 3.5 Mpc similar to NGC 253 (2.5 Mpc), while its mass is heavier than NGC253 by an order of magnitude. The total galactic volume is well contained inside our FOV.

### 13.6 Upper Limit on Density of CDM

Here, we discuss the CDM density upper limit with various mass assumptions for the CDM as a byproduct of this study. Another feature of Cen A is that it is a "giant galaxy", the mass of which is an order of magnitude higher than in our Galaxy. Also it is categorized as a "near-by galaxy". We investigate its dark-matter abundance.

In our Galaxy, especially around the solar system, theorists estimate the CDM density to be 0.3 GeV/cm$^3$. In astronomical units, it corresponds to $0.01 M_\odot/pc^3$. Therefore, if we can derive an upper limit 10 times lower than this value, it can constrain cosmology.

The annihilation rate of the CDM should be proportional to its squared density multiplied by its annihilation cross section as shown in Section 13.5. The gamma-ray multiplicity per annihilation can be described by the fragmentation function (Equation 13.18) measured by accelerator experiments. The accelerator measurement of the fragmentation function $\frac{1}{\sigma_h} \frac{d\sigma}{dE/M_\chi}$, however, is measured below 100 MeV. We, therefore, use additionally the EGRET upper limit at 189 MeV for this study\[136\]. The fragmentation function of LEP data ($e^+e^-$ collider experiment at the center of mass energy of $\sim 90$ GeV) was well fitted with the sum of three exponential functions:

$$\frac{1}{\sigma_h} \cdot \frac{d\sigma}{dx} = e^{5.5605-34.482x} + e^{3.1777-10.551x} + e^{7.2391-123.29x},$$

(13.20)

where $x$ corresponds to $E/M_\chi$ and the error of this function was less than 0.1 \%.

The fit was carried out by $\chi^2$ minimization. Both of the EGRET and CANGAROO-III, errors in $\chi^2$-fitting were assumed to be half of 2-$\sigma$ errors in the case of upper limits. The central values of 0.1$\sigma$ were used for fitting in those cases. The function of $\chi^2$ is defined as follows. The $\chi^2$-fitting parameters were $\rho_{CDM}$ and $M_\chi$.

$$\chi^2(M_\chi, \rho_{CDM}) = \sum_{i=1,A} \left( \frac{dF_{exp,i}}{dE} - \frac{dF(M_\chi, \rho_{CDM})}{\sigma_{exp,i}} \right)^2$$

(13.21)

where $\sigma_{exp}$ is written as,

$$\sigma_{exp,i} = 2\sigma \text{ upper limits} \times 0.5,$$
and $F_{\text{exp}}$ is also written as,

$$F_{\text{exp},i} = \sigma_{\text{exp},i} \times 0.1,$$

and $i(=4)$ is the number of data points. The function of $dE/dE$ is defined as,

$$\left[\frac{dF}{dE}\right] = \frac{< \sigma v_{q\bar{q}} > n^2 [V/(4\pi d^2)]}{M_\chi} \cdot \left[ \frac{1}{\sigma_h} \cdot \frac{d\sigma}{d(E/M_\chi)} \right]$$

(13.22)

$$= \frac{< \sigma v_{q\bar{q}} > \rho_{CDM}^2 [\Delta\theta^3/3d^2]}{M_\chi^3} \cdot \left[ \frac{1}{\sigma_h} \cdot \frac{d\sigma}{d(E/M_\chi)} \right],$$

(13.23)

where $\Delta\theta$ is the CANGAROO-III angular cut ($0.2^\circ$) and $d$ is the distance from earth to Cen A (3.5 Mpc). (The various distances were reported for Cen A. So we selected the standard distance, i.e., most cited value.)

The 2-$\sigma$ upper limits for the CDM density were derived from the $d\chi^2 = \chi^2 - \chi^2_{\text{min}} = 4$. We set the CDM density upper limits under the various mass assumptions. The $\chi^2$ under the assumptions of various $M_\chi$ and $\rho_{CDM}$ is shown in Fig 13.5. The Z-axis is $-d\chi^2$.

![Figure 13.5: Relation $M_\chi$ and $\rho_{CDM}$. The Z-axis is $-d\chi^2$.](image)

If we assumed that $M_\chi$ is 1 TeV, the result of 2 $\sigma$-expected flux is shown in Fig 13.6. The result of the upper limit is shown in Fig 13.7. The total mass is in units of $M_\odot/pc^3$. The particle mass is in TeV. Around the TeV region, we obtained an upper limit of several $M_\odot/pc^3$. Considering the size of observed region, our angular cut was $\theta^2 < 0.05$, i.e., $\theta < 0.2^\circ$. The distance to Cen A is 3.5 Mpc, which means that we are seeing the region within 12 kpc from its center. The typical upper limits are several $M_\odot/cm^3$ in
Figure 13.6: Differential flux upper limit of the EGRET and Centaurus A. The arrow in the low energy region is the EGRET result and those in the high energy region are the CANGAROO-III result. The solid line is the 2-$\sigma$ line (Eq. 13.20). The upper limit of $\rho_{CDM}$ was $2.8 < \sigma v B_{qq} > / 3 \times 10^{-26} \, M_{\odot}/pc^3$, when the energy scale ($M_\chi$) was at 1 TeV.

Figure 13.7: Upper limit for the CDM density versus particle mass. It was obtained using 2-$\sigma$ upper limits for out TeV results on Cen A and that by EGRET at 189 MeV.
that volume, approximately a factor times 10 heavier than the estimated density. We, however, derived the upper limit of the CDM mass with a new method and for a new astronomical object. We eagerly hope that future efforts might improve the upper limit.
Chapter 14

Conclusions

CANGAROO-III is one of the major atmospheric Cherenkov telescopes. It has been built for stereoscopic observation of TeV gamma-ray sources with four 10-mφ reflectors. In this thesis, we show the result of the first stereoscopic observation with CANGAROO-III telescopes. The stereoscopic observation can make us to analyze a shower reconstruction event by event. According to MC study, the angular resolution is 0.15° and the energy resolution is about 25 % at 2 TeV in event-by-event basis. The energy resolution becomes better in the detailed analysis. Furthermore, we adopt a new observation mode, previously used by other groups, in which the ON and OFF source data can be simultaneously taken in the same view.

We demonstrated that the stereo mode is working well for the observation of the Crab, which is the standard candle in the TeV energy region. The observation of the Crab was carried out in Dec. 18 ~ Dec. 28 2003 and the observation time was 865 min. A significant excess (5.7 σ) was obtained by the stereoscopic observations at an energy greater than 2.3 TeV. The flux was confirmed to be consistent with the previous measurements and the angular resolution consistent with the Monte-Carlo simulation. The energy threshold of the CANGAROO-III stereo mode was estimated to be about 500 GeV for the zenith observations and the angular resolution is 0.15 °.

After these studies, we observed the Active Radio Galaxy Centaurus A (Cen A). Cen A is the nearest radio galaxy (3.5 Mpc) and has jets. The inclination angle of the jets is estimated to be about 60°. Cen A has been observed in VHE gamma-rays with a non-imaging instrument. In 1975, a positive detection of gamma-rays at 100 TeV
was reported. But recent observations could not confirm this and obtain upper limits. Therefore, it suggests that the emission from Cen A is time variable on a timescale of year. Hence, the possibility of detection of VHE gamma-rays from Cen A still remains.

The observation period was Mar. 16 ~ Apr. 19 2004 and the total observation time was 1050 min. We applied the same analysis of the Crab to Cen A. We, however, could not detect any TeV gamma-ray signal and the 2-σ upper limit was obtained to be $3.2 \times 10^{-12}$ cm$^{-2}$ sec$^{-1}$ at energies greater than 530 GeV. This 2-σ upper limit corresponds to be approximately 7 %-Crab flux. This is an order of magnitude lower than past data. This result is the world best limit in the TeV energy region.

We also gave 2-σ upper limits for time variation of Cen A, i.e., day-by-day upper limits. We could not detect any statistically significant bursts. The upper limits on day 2004 Mar 27 was the highest. Yet this is again significantly lower than the previous positive burst measurements. Considering the peak to valley ratio of the gamma-ray fluxes for typical AGN, ten times of our upper limit can be interpreted as an upper limit at burst situation. Even assuming this, our upper limit is very stringent compared to the previous measurements, and inconsistent with them.

We discussed the theoretical implications especially assuming that Cen A is classified as HBL. We derived physical parameters for an HBL model using our upper limits and multi-wave length spectra. Assuming the lower energy data (E < 100 MeV) are all due to synchrotron radiation, we could estimate the higher energy gamma-ray emissions. The inverse Compton scatterings between the high energy electrons and the synchrotron photons were considered adopting the synchrotron self-Compton (SSC) model. From low-energy data on the luminosity of the synchrotron radiation and from our upper limits on inverse Compton scattering, we derive

$$L_c/L_s < \frac{1}{400} << 1.$$  

Both the denominator and numerator are proportional to the energy densities of the ambient magnetic field and synchrotron photons. Then,

$$U_B > 400U_{sync} = 2400 \text{ eV/cc}.$$  

was derived, where $U_{sync}$ can be estimated from the lower energy photon fluxes. Assuming a volume of the emission region to be that defined by our angular resolution,
\[ B > 210 \mu \text{G}(R/12\text{kpc})^{-1}, \]

was finally obtained. Applying the estimated emission volume in Mkn 421 and 501, this lower limit on the magnetic field strength becomes unreasonably high. Even using a size of an order of a light year, it exceeds one Gauss, a situation which can be hardly understood. We conclude that Cen A is not classified as normal HBL.

Finally we gave an upper limit to the density of Cold Dark Matter (CDM). Around the TeV region, we obtained an upper limit on the order \(M_\odot/pc^3\). The upper limit on total mass inside 12 kpc radius corresponds to several times \(10^{13} M_\odot\), approximately 100 times heavier than the total mass of Cen A. Although this upper limit is not impressive cosmologically, we hope that future efforts might reduce improve the upper limit.
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Appendix A

Observation Mode

In CANGAROO-II, we have observed in an observation mode called LONG ON/OFF until now. In this mode, the observation time is divided into two or three in one night. The On-source run was set to contain the culmination of target. OFF was selected so as to track the same elevation regions. The problem of this observation mode is the reduction of observation time by a factor of two. However, the background is easy to estimate.

A different observation mode was used in HEGRA collaboration ([30]). The telescope pointing was shifted in Declination by ±0.5 degrees every 20 min (Wobble mode). In this way, the ON and OFF data can be simultaneously taken in the same field of view as shown in Fig A.1. Even if target position shifts from the center of the FOV by 0.5°, the acceptance reduction is estimated to be negligible small. The background position was selected to have the same acceptance as the target position, i.e., points of the same radius (0.5°) circle as the target position.

If Wobble mode is chosen, it will become possible to take many background points. Then the statistics for the background (i.e., OFF source data) increase. We chose the Wobble mode and took six background points considering the angular resolution of telescope.
**Figure A.1:** The scheme of Wobble mode. The Red cross is the position of observation target. The blue circles are background positions. The black circle are the camera edges.
Appendix B

Differential Cross section of

\( pA \rightarrow \gamma X \)

The differential cross section is written as

\[
\frac{d\sigma_{\pi}(E_{\pi},E_p)}{dE_{\pi}} = \langle \xi \sigma_{\pi}(E_p) \rangle \frac{dN_{\pi}(E_{\pi},E_p)}{dE_{\pi}},
\]

where \( \xi \) is the multiplicity, and \( \langle \xi \sigma_{\pi}(E_p) \rangle \) is the inclusive cross section for the reaction \( p + p \rightarrow \pi^0 + X \) (anything), respectively. These are determined from experimental data. \( dN_{\pi}(E_{\pi},E_p)/dE_{\pi} \) is the normalized distribution function. For \( dN_{\pi}(E_{\pi},E_p)/dE_{\pi} \) near the threshold energy of \( \sim 1.2180 \text{ GeV} \), the isobaric model where \( \pi^0 \)'s are produced through the excitation of the \( \Delta_{3/2}(1232) \) isobars as

\[
p + p \rightarrow p + \Delta_{3/2}(1232) \\
\Delta_{3/2}(1232) \rightarrow p + \pi^0 \quad \text{(B.2)} \\
\pi^0 \rightarrow 2\gamma.
\]

The distribution of the secondary \( \pi^0 \)'s is written as

\[
\frac{dN_{\pi}(E_{\pi},E_p)}{dE_{\pi}} = \int_{m_p+m_{\pi}}^{\sqrt{s-m_p}} dm_\Delta B(m_\Delta) f(E_{\pi};E_p,m_\Delta),
\]

where \( \sqrt{s} \) is the total energy in the center-of-mass system. \( B(m_\Delta) \) is the isobar mass spectrum given by the Breit-Wigner distribution written as

\[
B(m_\Delta) = \frac{1}{\pi} \frac{\Gamma}{(m_\Delta - m_0)^2 + \Gamma^2},
\]

\( \Gamma \)
where $m_0$ is 1.232GeV, and $\Gamma = \frac{1}{2} \times 0.115$GeV. $f(E_\pi; E_p, m_\Delta)$ is the energy distribution of $\pi^0$s for given $m_\Delta$ and protons with the energy of $E_p$. Assuming isobars decay isotropically in the center-of-mass system, the $\pi$ spectrum in the laboratory frame can be calculated as

$$f(E_\pi; E_p, m_\Delta) = \frac{p_\pi}{4m_\pi\gamma_\Delta\beta_\Delta^*\gamma_\pi^*\beta_\pi^*} \int_0^1 d\cos\theta_\pi \frac{1}{\sqrt{[\gamma_c(E_\pi - \beta_c p_\pi \cos\theta_\pi)]^2 - m_\pi^2}} \times H[\gamma_c(E_\pi - \beta_c p_\pi \cos\theta_\pi); \gamma_\Delta^*(E_\pi^* - \beta_\Delta^* p_\pi^*), \gamma_\Delta(E_\pi^* + \beta_\Delta p_\pi^*)], \quad (B.5)$$

where $H[x; a, b] = 1$ if $a \leq x \leq b$ and otherwise 0. The (*) denote the Lorentz factor in the center-of-mass system, and the (′) denote that in the rest frame of the $\Delta$ isobars, and $C$s denote the Lorentz factor of center-of-mass system with respect to the laboratory frame.

For higher energies, the scaling model was used. Using the Lorentz invariant cross section, the distribution of the secondary $\pi^0$s in the laboratory frame is written by

$$\frac{dN_\pi(E_\pi, E_p)}{dE_\pi} = \frac{2\pi\sqrt{E_\pi^2 - m_\pi^2}}{\langle \xi \sigma_\pi(E_p) \rangle} \int_0^1 d\cos\theta_\pi \left( E_\pi^* \frac{d^3\sigma_\pi^*}{d^3p_\pi^*} \right). \quad (B.6)$$

The invariant cross section is determined from experimental data of $\pi^0$ production.
Appendix C

Definitions of the Image Parameters
(Hillas Parameters)

We define the index of each pixel, the center of each pixel, and the amount of photons in each pixel as \( i \), \( (x_i, y_i) \), and \( s_i \), respectively. The averages are defined as follows:

\[
\langle x \rangle = \frac{\sum s_i x_i}{\sum s_i}, \quad (C.1)
\]
\[
\langle x^2 \rangle = \frac{\sum s_i x_i^2}{\sum s_i}, \quad (C.2)
\]
\[
\langle x^3 \rangle = \frac{\sum s_i x_i^3}{\sum s_i}, \quad (C.3)
\]
\[
\langle y \rangle = \frac{\sum s_i y_i}{\sum s_i}, \quad (C.4)
\]
\[
\langle y^2 \rangle = \frac{\sum s_i y_i^2}{\sum s_i}, \quad (C.5)
\]
\[
\langle y^3 \rangle = \frac{\sum s_i y_i^3}{\sum s_i}, \quad (C.6)
\]
\[
\langle xy \rangle = \frac{\sum s_i x_i y_i}{\sum s_i}, \quad (C.7)
\]
\[
\langle x^2 y \rangle = \frac{\sum s_i x_i^2 y_i}{\sum s_i}, \quad (C.8)
\]
\[
\langle xy^2 \rangle = \frac{\sum s_i x_i y_i^2}{\sum s_i}. \quad (C.9)
\]

The coordinate \((\langle x \rangle, \langle y \rangle)\) correspond to the centroid of the image. The following is further definitions:

\[
\sigma_{x^2} = \langle x^2 \rangle - \langle x \rangle^2, \quad (C.10)
\]
\[ \sigma_{y^2} = \langle y^2 \rangle - \langle y \rangle^2, \quad (C.11) \]
\[ \sigma_{xy} = \langle xy \rangle - \langle x \rangle \langle y \rangle, \quad (C.12) \]
\[ \sigma_{x^3} = \langle x^3 \rangle - 3 \langle x^2 \rangle \langle x \rangle + 2 \langle x \rangle^3, \quad (C.13) \]
\[ \sigma_{y^3} = \langle y^3 \rangle - 3 \langle y^2 \rangle \langle y \rangle + 2 \langle y \rangle^3, \quad (C.14) \]
\[ \sigma_{x^2y} = \langle x^2y \rangle - 2 \langle xy \rangle \langle x \rangle + 2 \langle x \rangle^2 \langle y \rangle - \langle x^2 \rangle \langle y \rangle, \quad (C.15) \]
\[ \sigma_{xy^2} = \langle xy^2 \rangle - 2 \langle xy \rangle \langle y \rangle + 2 \langle y \rangle^2 \langle x \rangle - \langle y^2 \rangle \langle x \rangle. \quad (C.16) \]

Here we introduce \( d = \sigma_{y^2} - \sigma_{x^2} \) and \( z = (d^2 + 4 \sigma_{xy})^{1/2} \). Length and Width are defined as follows:

\[ \text{Width} = \left( \frac{\sigma_{x^2} + \sigma_{y^2} - z}{2} \right)^{1/2}, \quad (C.17) \]
\[ \text{Length} = \left( \frac{\sigma_{x^2} + \sigma_{y^2} + z}{2} \right)^{1/2}. \quad (C.18) \]

If an assumed source position in the field of view is \((x_s, y_s)\) and the Distance vector \( \vec{D} = (x_D, y_D) \) is defined as

\[ \vec{D} = (x_s - \langle x \rangle, y_s - \langle y \rangle), \quad (C.19) \]

then

\[ \text{Distance} = (x_D^2 + y_D^2)^{1/2}. \quad (C.20) \]

If an unit vector of the major axis, \( \vec{u} = (x_u, y_u) \), is

\[ \vec{u} = \left( \frac{z - d}{2z} \right)^{1/2}, \text{sign}(\sigma_{xy}) \left( \frac{z + d}{2z} \right)^{1/2}, \quad (C.21) \]

then

\[ \alpha = \cos^{-1} \left( \frac{x_u x_D + y_u y_D}{\text{Distance}} \right). \quad (C.22) \]

The Asymmetry vector \( \vec{A} \) is

\[ \vec{A} = -\sigma_A \vec{u}, \quad (C.23) \]

where

\[ \sigma_A = (\sigma_{x^3} \cos^3 \phi + 3 \sigma_{x^2y} \cos^2 \phi \sin \phi + 3 \sigma_{xy^2} \cos \phi \sin^2 \phi + \sigma_{y^3} \sin^3 \phi)^{1/3} \quad (C.24) \]

and \( \phi \) is the angle of \( \vec{u} \) with respect to the x axis. Asymmetry is defined as

\[ \text{Asymmetry} = \text{sign}(\vec{A} \cdot \vec{D}) \frac{|\sigma_A|}{\text{Length}} = \frac{\vec{A} \cdot \vec{D}}{\text{Distance} \cdot \text{Length} \cos \alpha}. \quad (C.25) \]
Appendix D

Mono-analysis of Cen A

We show the results of a mono-analysis on Cen A. We have three data which correspond to those by T2, T3, and T4. The analysis method is similar to that of stereo-analysis. The difference is that we use $\alpha$ (image orientation angle) in order to determine the excess flux. Also we do not check the coincidence flag in the data. For the background estimation, we used the OFF source runs.

After "Cloud cut", the available observation times were 1036(924), 1197(1000), and 1114(982) minutes for T2, T3, and T4, respectively. The value in the parentheses are the times for OFF source runs.

The results of $\alpha$ plots are shown in Fig D.1. From left to right, the results of T2, T3, and T4 are shown. The lower figures are the background subtracted events. In the upper figures, the point with error bars are obtained by the ON-source runs and the histograms are by the OFF-source runs. Comparing them with the $\theta^2$ plots of the stereo-analysis, one can see a very high background level, which might worsen the sensitivity. In any of telescopes, we do not see a statistically significant excess.

Using those, we calculated $2\sigma$ upper limits and show them in Fig D.2. The red arrows are our upper limits. The other data points are those of the previous measurements. The line is a standard Crab flux. The upper limits correspond to about 10% crab. Here only the statistical errors are considered. As estimated from the huge background, this result is a factor of two worse than that by the stereo-analysis. We, therefore, concentrated ourselves in the stereo-analysis in this thesis.
**Figure D.1:** $\alpha$ distributions. From left to right, the results of T2, T3, and T4 are shown. The lower figures are the background subtracted events. In the upper figures, the points with error bars are obtained by the ON-source runs and the histograms are by the OFF-source runs.

**Figure D.2:** Integral flux obtained by the mono-analysis. The red arrows are our upper limits. The other data points are those of the previous measurements. The line is a standard Crab flux.