Dependence of the Air-Fluorescence Yield on Electron Energy

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The air fluorescence yield from electrons is related with several molecular parameters according to well established relationships. Experimental cross sections available in the literature in combination with theoretical data on Franck-Condon factors and branching ratios can be used for the calculation of the fluorescence yield as a function of the electron energy in a wide interval (keV - GeV).

1. Introduction

Air fluorescence is a well established technique for the detection of ultra-high energy cosmic rays [1]. In particular, fluorescence telescopes are a key tool for the energy calibration of the Pierre Auger Observatory [2]. In order to achieve a precise calibration, accurate values of the fluorescence yield of air excited by electrons are required. This need has promoted a number of experiments for the measurement of this parameter in the wavelength interval 300 - 420 nm [3]. In this interval, fluorescence arises from the First Negative System of $\text{N}_2^+$ ($B^2 \Sigma_u^+ \rightarrow X^2 \Sigma_g^+$) and the Second Positive System of $\text{N}_2$ ($C^3 \Pi_u \rightarrow B^3 \Pi_g$) which will be called 1N and 2P Systems respectively.

Not being available yet direct measurements at low energy, the corresponding yield are usually inferred by extrapolating high energy values, assuming that the efficiency for the generation of fluorescence is proportional to the stopping power which follows the well known Bethe-Bloch formula. As far as we know, this assumption has not yet been established neither experimentally nor theoretically. In this paper the dependence of the fluorescence yield with the electron energy is studied using a combination of both, molecular properties and experimental results on the related cross sections.

2. Fluorescence Yield and Stopping Power

The fluorescence yield for a molecular band $\epsilon_{\nu \nu'}$ is defined as the number of fluorescence photons emitted in the molecular transition $\nu \rightarrow \nu'$ per electron and unit path length. At very low pressure $\epsilon_{\nu \nu'} = N \sigma_{\nu} B^{\nu \nu'} = N \sigma_{\nu} \nu'$, where $N$ is the number of molecules per unit volume, $\sigma_{\nu}$ is the excitation cross section for the $\nu$ level, $B^{\nu \nu'}$ is the branching ratio (i.e. the ratio between the partial $A^{\nu \nu'}$ and total $A^\nu$ radiative transition probabilities) and $\sigma_{\nu \nu'}$ is the so-called optical cross section for the transition $\nu \rightarrow \nu'$. As is well known the excitation cross sections $\sigma_{\nu}$ are also proportional to the Franck-Condon factors $q_{X \rightarrow \nu}$, defined as the overlapping integrals of the vibrational wavefunctions of the ground and the excited levels. The relation

$$\frac{\sigma_{\nu \nu'}}{\sigma_{00}} = \frac{q_{X \rightarrow \nu}}{q_{X \rightarrow 0}} \frac{B^{\nu \nu'}}{B^{00}}$$

allows the calculation of any optical cross section from data on a measured transition (e.g. $\nu = 0 - \nu' = 0$).

At high pressure, collisional quenching plays an important role and the fluorescence yield can be expressed by
where \( P'_v \) is the pressure for which the probability of collisional quenching equals that of radiative de-excitation.

The stopping power \( S \) (i.e. the energy loss of an electron per unit length of traversed matter due to both excitation and ionization processes) can be accurately predicted for energies above 10 keV by the well known Bethe-Bloch formula. On the other hand, a cross section \( \sigma_B \), proportional to the stopping power, can be defined by means of

\[
S = N I \sigma_B
\]

where \( I \) is the Bethe parameter (average energy released per collision).

The fluorescence efficiency \( \Phi_{v \to v'} \), defined as the fraction of deposited energy which is emitted as photons of the \( v \to v' \) band, is related with the stopping power by

\[
\Phi_{v \to v'}(P) = \frac{E_{v \to v'} \epsilon_{v \to v'}}{S} = \frac{E_{v \to v'} \epsilon_{v \to v'}}{I \sigma_B} \frac{1}{1 + P/P'_v}
\]

Note that only a fraction of the energy loss is converted in excitation to the level \( v \) of interest, not because of internal processes (as often stated), but due to other allowed direct excitation channels.

3. The First Negative system of \( \text{N}_2 \)

The 1N System has been studied by many authors. In particular many measurements of the optical cross section \( \sigma_{00} \) corresponding to the strongest band at 391.4 nm are available in the literature. At high energy, the cross section for the excitation of the 1N upper level follows a Born-Bethe relativistic law [4].

\[
\sigma = \frac{A}{\beta^2} \left\{ \ln C \beta^2 - \ln(1 - \beta^2) - \beta^2 \right\}
\]

where \( \beta \) is the relativistic speed and, \( A \) and \( C \) are constants. For any function following the behavior of equation (4), a plot of \( \sigma \beta^2 \) versus \( x = \ln(\beta^2) - \ln(1 - \beta^2) - \beta^2 \) (Fano plot) should be a straight line. In figure 1 a significant set of the available experimental results have been represented on a Fano plot. A linear fit of the data over 300 eV predicts a cross section in reasonable agreement with the measurements of Hirs et al. [5]. The Born-Bethe function shown in the figure seems to provide us with a reliable value of \( \sigma_{00} \) in a wide energy range. The cross section for other bands can be easily obtained using relation (1).

On the other hand, it can be easily checked that the stopping power is not linear in a Fano plot. This result should not be surprising since \( \sigma_{v} \) takes into account only one of the many inelastic processes involved in \( \sigma_B \).

4. The Second Positive system of \( \text{N}_2 \)

Many authors have studied the excitation of the 2P System. Experiments carried out on pure nitrogen at low electron energy (\( E < 1 \text{keV} \)) and low pressure (a few mTorr or lower), show a fast \( E^{-2} \) dependence of the fluorescence light (expected from the optically-forbidden nature of the involved transition). For comparison, 1N System exhibits a much slower (logE/E) dependence. Taking into account that, at 1keV energies, the observed 2P fluorescence at low pressure is about three orders of magnitude less intense than that from the 1N system, a simple extrapolation would predict that the 2P fluorescence should be completely negligible at 30keV energies and above (compared to 1N System). On the contrary, experiments carried out at high energies
Figure 1. Optical cross section $\sigma_{oo}$ of the 1N system. Left: Available experimental data in a Fano plot together with a Born-Bethe fit. Right: Optical cross section versus electron energy. The Bethe Block function is shown for comparison.

(up to 1 GeV) show that 2P fluorescence even dominates over the 1N system. The experiments of this second group, mainly devoted to study the fluorescence emission of air, have been performed at much higher pressures (ranging from a few Torr up to 1 atm).

Obviously, other processes take place in the excitation of neutral nitrogen. Very recently a model has been proposed [6] which explains these discrepancies. This model takes into account that each ionization process releases at least one low energy electron for which both the inelastic $\sigma_{inel}$ and the 2P excitation cross sections are very large resulting, for these secondary electrons, in very small mean free paths and large contribution to fluorescence.

An apparent optical cross section $\sigma_{o}^{app}$ can be calculated [6] which turns out to depend on the quantity $\Gamma = N\sigma_{inel}R$ ($R$ is the size of the interaction region). In figure 2, the result for the strongest band of the 2P System (337.1 nm) has been represented against the electron energy for several $N\Gamma$ values together with available experimental data. Measurements at very low pressure and very low energy show a maximum of the cross section at about 20 eV with a fast decrease with energy as predicted by the model. In the limiting case of $N\Gamma >> 1$ (high pressure experiments) the model predicts an apparent cross section given by

$$\sigma_{o}^{app}(E) = \sigma_{oo}(E) + \frac{\sigma_{el}}{\sigma_{in}^{app}} \sigma_{ion}(E)$$

where $\sigma_{ion}$ is the cross section for all ionization processes. As shown in the figure, experimental results at high pressure and high energy are in very good agreement with the model.

5. Conclusions

The optical cross sections for emission of air fluorescence light induced by electrons have been obtained for a wide energy range using both molecular properties and available experimental results. Optical cross sections for 1N system follow the Born-Bethe law. A model which takes into account the contribution of secondary electrons provides the apparent cross section of the 2P system up to high electron energy.
Figure 2. Apparent optical cross section $\sigma_{op}^{2P}$ for 2P system of N$_2$ versus electron energy as predicted by the model of ref. [6] for several $N_R$ values (continuous lines). Data deduced from measurements of Nagano et al. (●) and Kakimoto et al. (○) at high pressure are in good agreement with the model. This model also accounts for experimental results at low pressure (△, □). See [6] for more details.

At high pressure the fluorescence yield can be inferred from these optical cross sections by evaluation of the quenching effect. In a realistic atmosphere, collision of excited molecules with the various quenchers (N$_2$, O$_2$, contaminants) has to be properly determined taking into account a possible dependence of the quenching cross section on energy collision (i.e. temperature).

The application of the above methods for the evaluation of the air fluorescence yield in a wide energy range is in progress and the results will be published soon.

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References