数値相対論の展開
Progress & Status of Numerical Relativity

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1 General Relativity & Numerical Relativity

$$G_{\mu\nu} = 8\pi \frac{G}{c^4} T_{\mu\nu}$$

$$\nabla_\mu T^\mu_\nu = 0$$

$$\left(\nabla_\mu G^\mu_\nu = 0 \right)$$

Spacetime dynamics

GR Matter dynamics

Einstein's equation
= 2nd order coupled nonlinear PDE

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi \frac{G}{c^4} T_{\mu\nu}$$

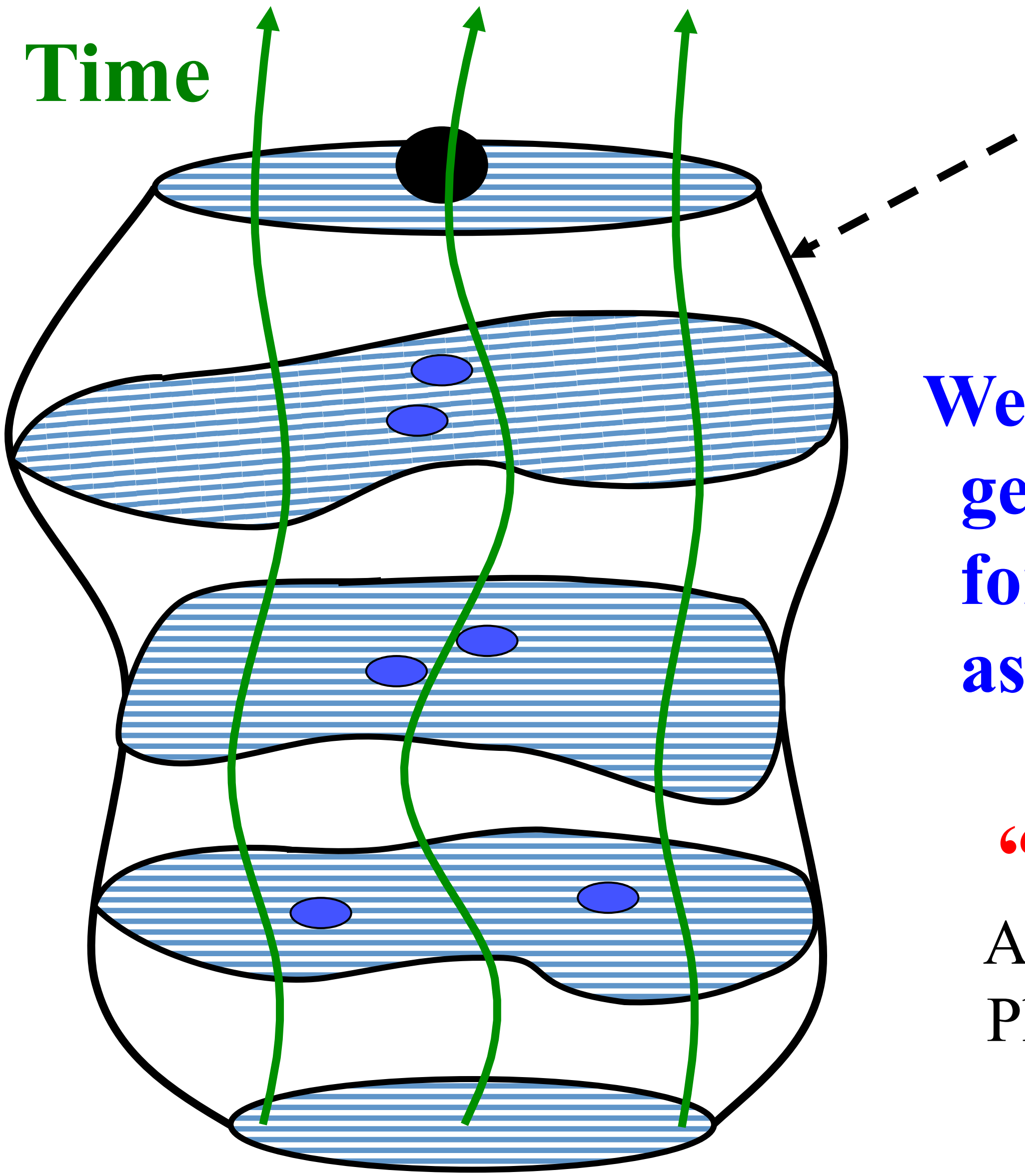
$$R_{\mu\nu} = \partial_\alpha \Gamma^\alpha_{\mu\nu} - \partial_\mu \Gamma^\alpha_{\nu\alpha} + \Gamma^\alpha_{\mu\nu} \Gamma^\beta_{\alpha\beta} - \Gamma^\alpha_{\mu\beta} \Gamma^\beta_{\alpha\nu}$$

$$\Gamma^\alpha_{\mu\nu} = \frac{1}{2} g^{\alpha\beta} \left(\partial_\mu g_{\nu\beta} + \partial_\nu g_{\mu\beta} - \partial_\beta g_{\mu\nu} \right)$$

$R_{\mu\nu}$: Ricci tensor, $\Gamma^\alpha_{\beta\gamma}$: Christoffel symbol

- For general problems, analytic solutions cannot be obtained
- What is the nature of general relativity ?? questions in 1950s → **Numerical relativity**

How to generate dynamical spacetime ?



We have to evolve geometric quantities forward in time as initial-value problem

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"3+1 formulation"

Arnowitt, Deser, & Misner
Phys. Rev. **116**, 1322 (1959)

2 Numerical relativity: history

First attempt of dynamical evolution (1964)

ANNALS OF PHYSICS: **29**, 304–331 (1964)

The Two-Body Problem in Geometrodynamics

SUSAN G. HAHN

International Business Machines Corporation, New York, New York

AND

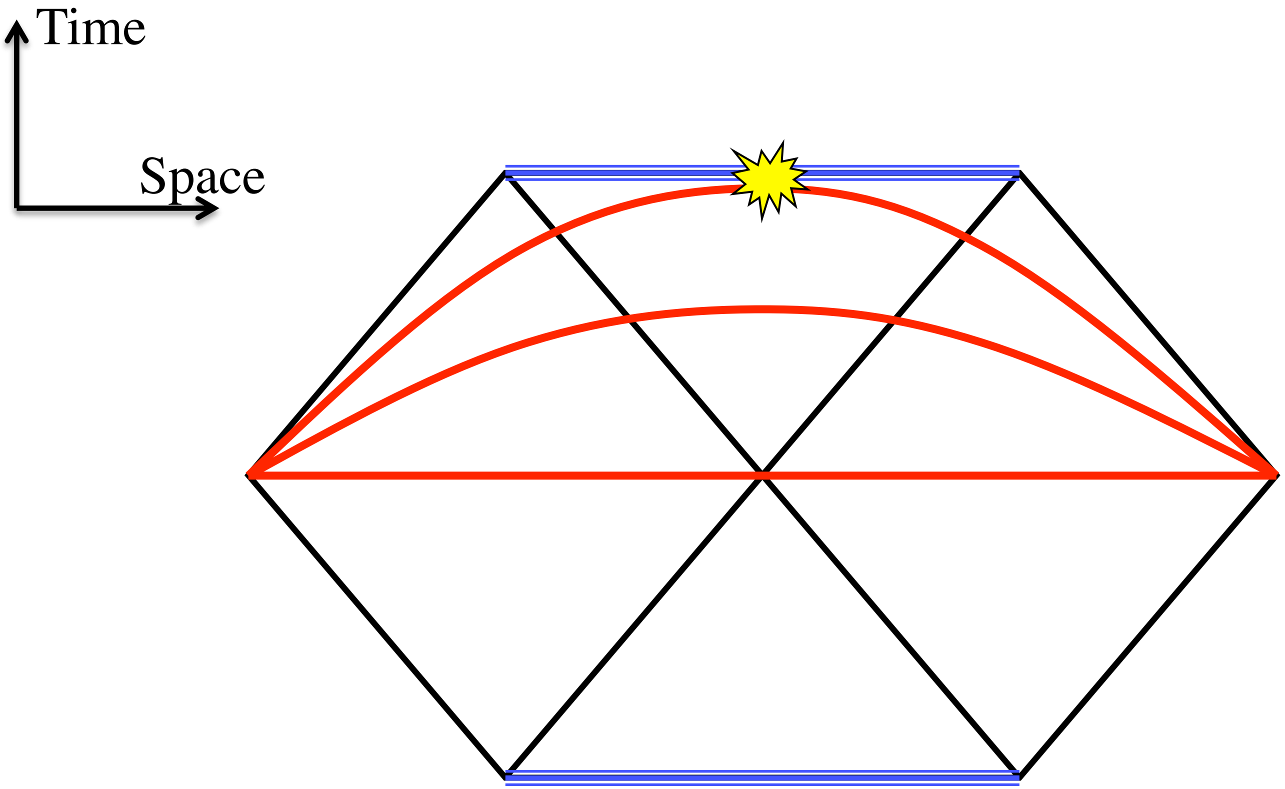
RICHARD W. LINDQUIST

Adelphi University, Garden City, New York

The problem of two interacting masses is investigated within the framework of geometrodynamics. It is assumed that the space-time continuum is free of all real sources of mass or charge; particles are identified with multiply con-

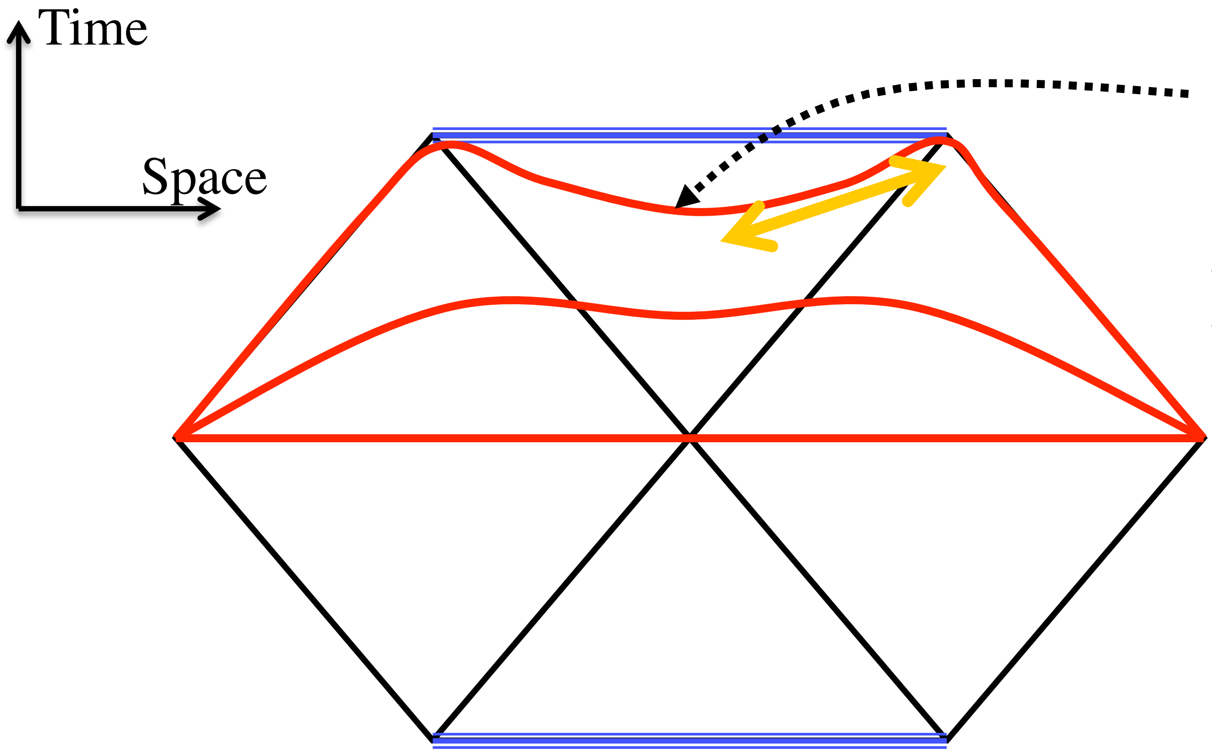
~50th anniversary of Numerical Relativity

Bad slicing



Singularity is not avoided:
This is bad slicing.

Singularity avoiding



Freeze: good !

But, …

First success (1977)

Ann. New York Academy of Sciences **302**, 569, 1977

SPACE-TIMES GENERATED BY COMPUTERS: BLACK HOLES WITH GRAVITATIONAL RADIATION*

Larry Smarr† ← Got PhD in 1975

Center for Astrophysics and
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Cambridge, Massachusetts 02138

The next decade will see the development of a number of new types of sensitive gravitational wave antennae which will probe the universe for a variety of new relativistic sources (see Thorne[1] for an excellent review). As a parallel program, computer programs must be designed that allow theorists to predict the gravity wave signatures of these expected sources. These programs will solve the full Einstein equations of general relativity (or other proposed theories of gravity), to

Embedding diagram of 2-BH collision

Time

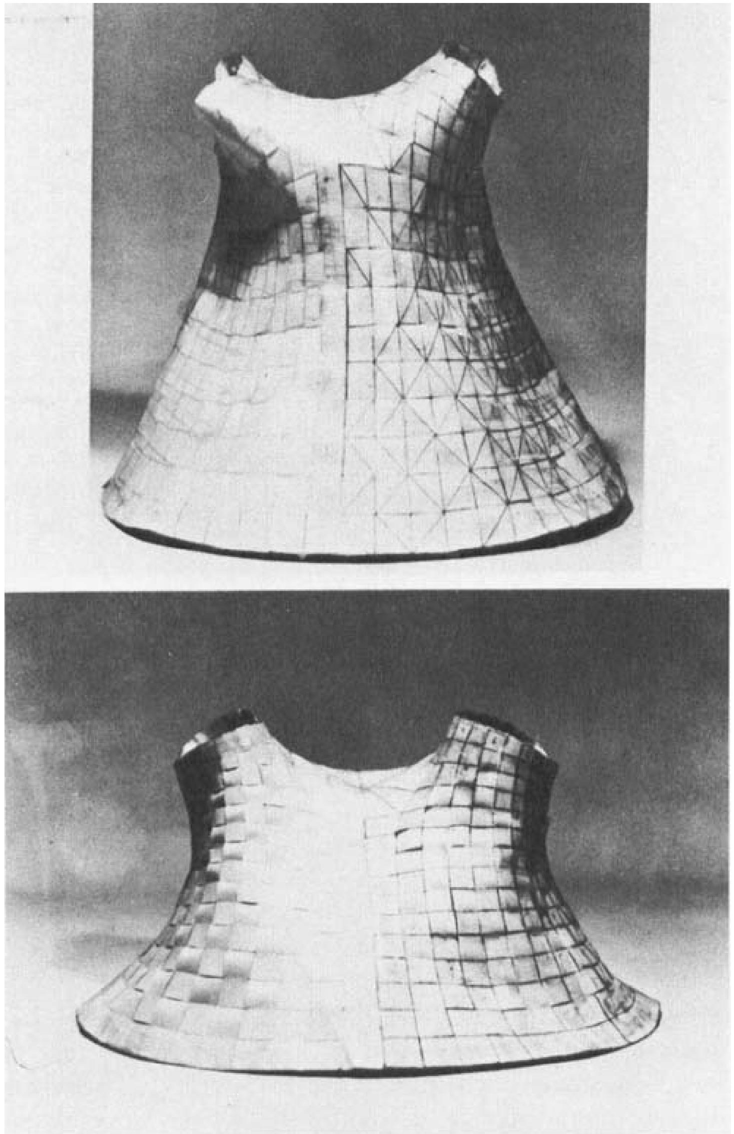

FIGURE 19. These isometric embedding diagrams of the z-ρ plane were constru the Center for Relativity in Austin, Texas, using a program written by Tom Criss. The the two black holes at $t = 0$ (lower) and $t = 9M$ (upper) for the $L_0/M = 3.9$ sion. The shearing at the grid can be seen clearly. The geometry stretches as in Sc... schild[7] and constricts where the new horizon is forming.

Gravitational waves (complex Weyl scalar)

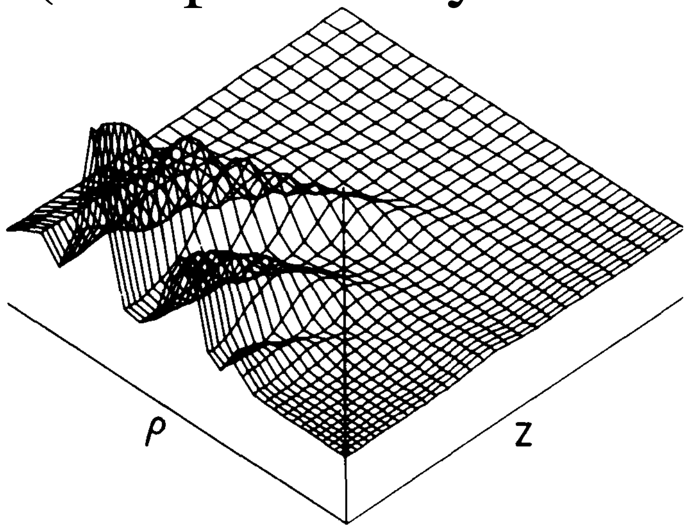

FIGURE 3. The same quantity as in FIGURE 2 except at $t = 5.4$. The "cloud" has dispersed into an outgoing wave train of gravitational radiation. Note there are four pulses, with the middle two the largest.

First multi-D non-vacuum & dynamical solution

1876

Progress of Theoretical Physics, Vol. 65, No. 6, June 1981

General Relativistic Collapse of Axially Symmetric Stars Leading to the Formation of Rotating Black Holes

Takashi NAKAMURA

Research Institute for Fundamental Physics
Kyoto University, Kyoto 606

Born in 09/18/1950

(Received November 1, 1980)

Numerical calculations have been made for the formation process of axisymmetric, rotating black holes of $10M_{\odot}$. The initial density of a star is about 3×10^{13} g/cm^3. Numerical results are classified mainly by q which corresponds to $|a|/M$ in a Kerr black hole. For $q\lesssim0.3$, the effect of rotation to the gravitational collapse is only to make the shape of matter oblate. For $0.3\lesssim q\lesssim0.95$, although the distribution of matter is disk-like, a ring-like peak of proper density appears. This ring is inside the apparent horizon, which is always formed in the case $q\lesssim0.95$. For $q\gtrsim0.95$, no apparent horizon is formed. The distribution of matter shows a central disk plus an expanding ring. It is found that electromagnetic-like field in the $[(2+1)+1]$-formalism plays an important role in a formation of a rotating black hole. Local conservation of angular momentum is checked. Accuracy of constraint equations is also shown to see the truncation error in the numerical calculations.

Stellar collapse → black hole formation or not

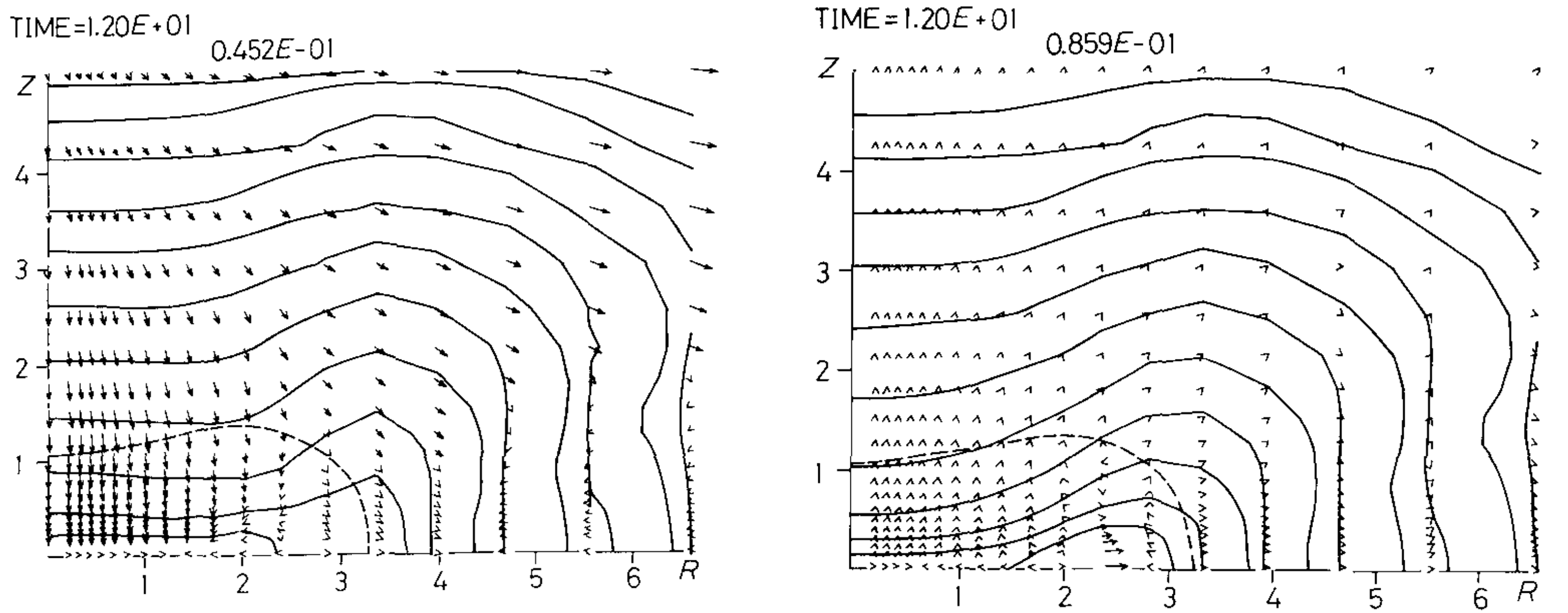

Fig. 3. (a) Contour lines of Q_b for M80 at $t = 12.0$. Each line corresponds to $Q_b = (Q_b)_{\max} \cdot 10^{-n/2}$ where $(Q_b)_{\max} = 4.52 \cdot 10^{-2}$ for $n = 1, 2, \cdots, 11$. Arrows show vectors (J_A/Q_b). The apparent horizon is shown by the dashed line.

(b) Contour lines of proper density (ρ) for M80 at $t = 12.0$. Each line corresponds to $\rho = \rho_{\max} \cdot 10^{-n/2}$ where $\rho_{\max} = 8.59 \cdot 10^{-2}$ for $n = 1, 2, \cdots, 11$. The apparent horizon is shown by the dashed line. Arrows show vectors E^A.

T. Nakamura, PTP 65, 1876 (1981)

See, also, R. Start & T. Piran, PRL 55, 891 (1985):
S. L. Shapiro & S. A. Teukolsky, PRL 66, 994 (1991) etc

Progress in the last quarter of century (1990s ~)

Two major motivations:

- Gravitational-wave detection has become a realistic (not joking) project since early 1990: GWs exist (Hulse-Taylor pulsar) and have to be detected
- High-energy phenomena have been discovered: e.g., gamma-ray bursts ~ dynamical BH + torus

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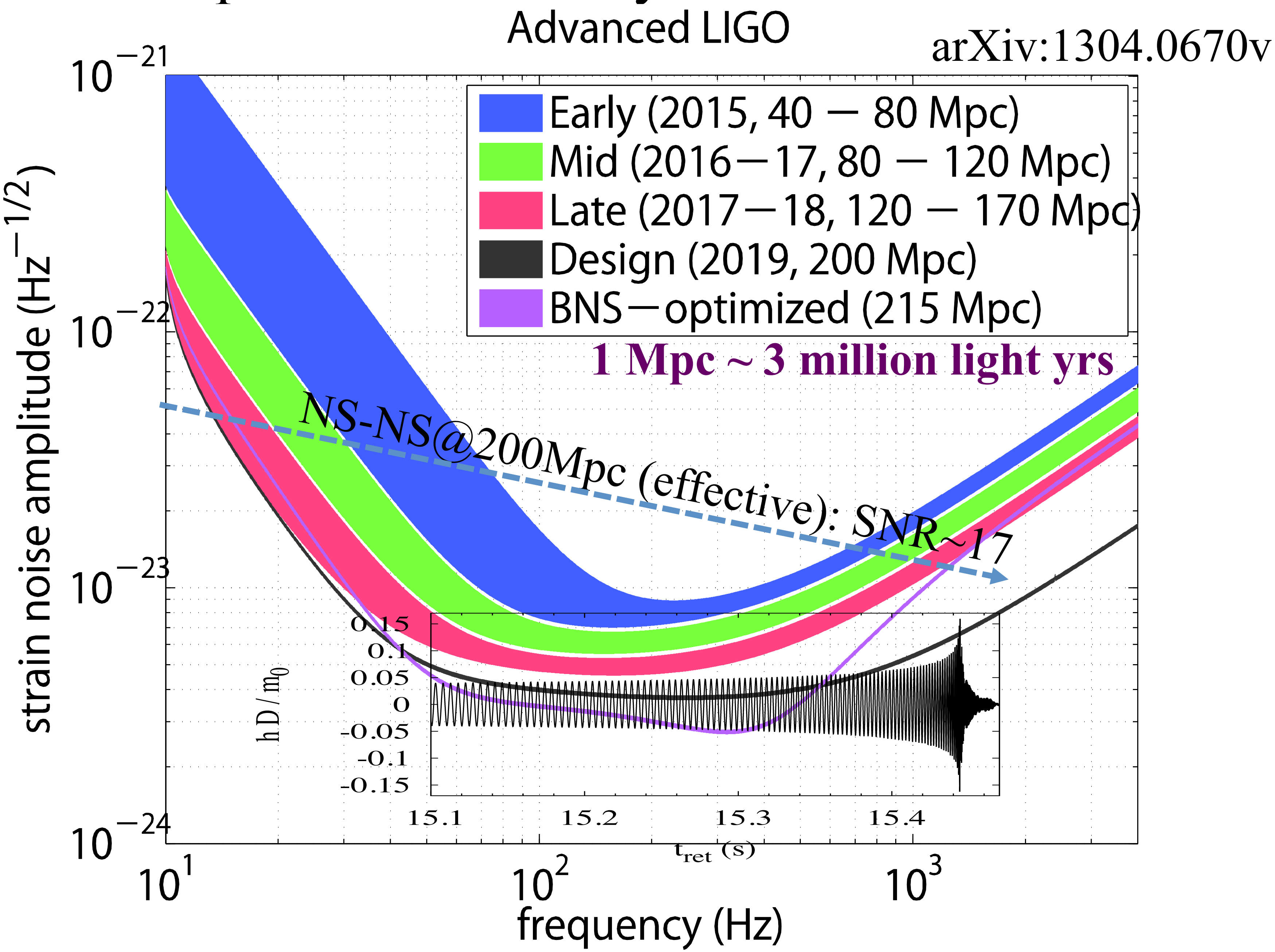
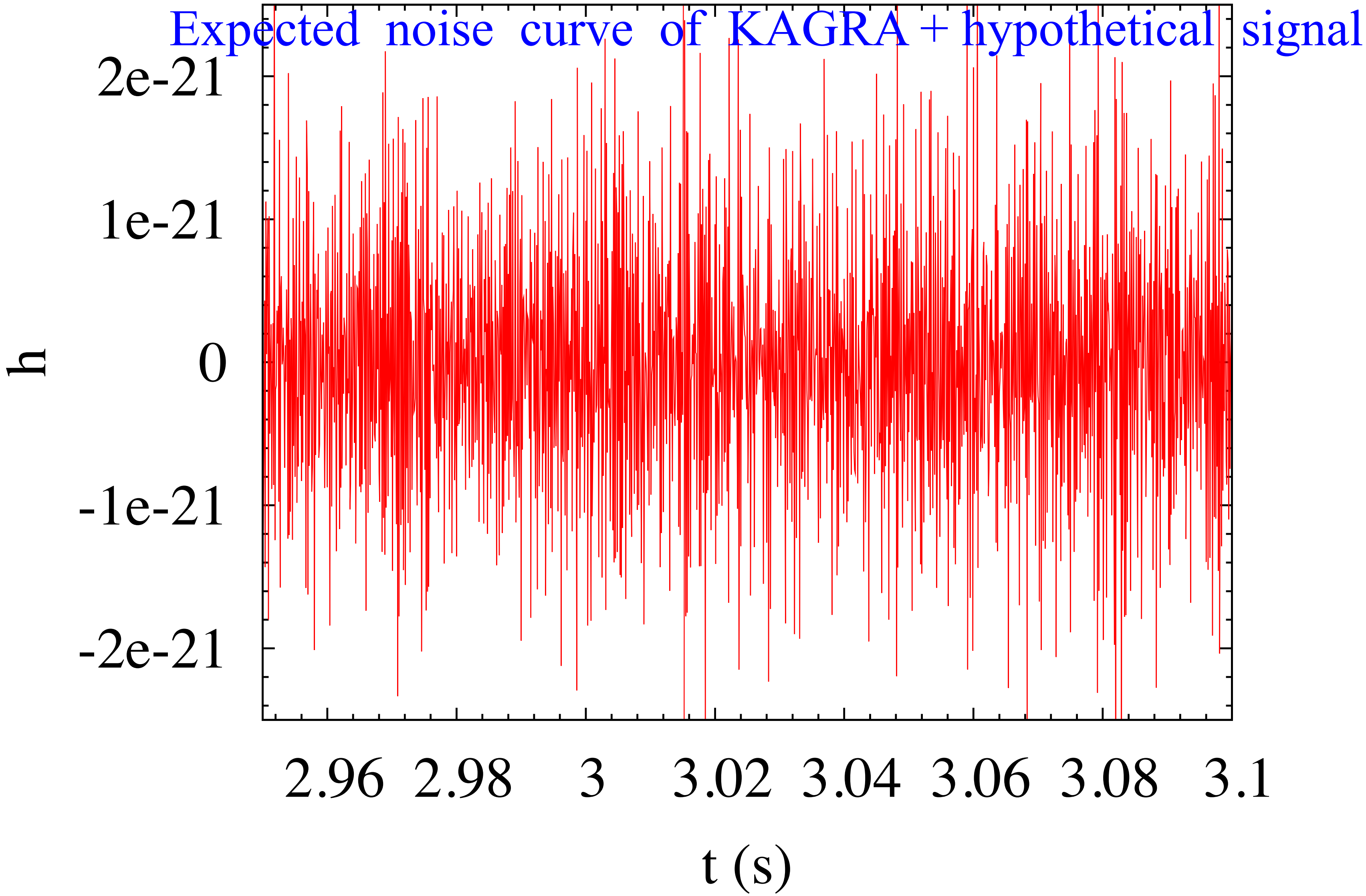
Accurate & physical simulations are required for solid obs. projects: excellent driving force !

Gravitational-wave detectors

Expected sensitivity of adv LIGO



Detecting gravitational waves needs accurate theoretical prediction



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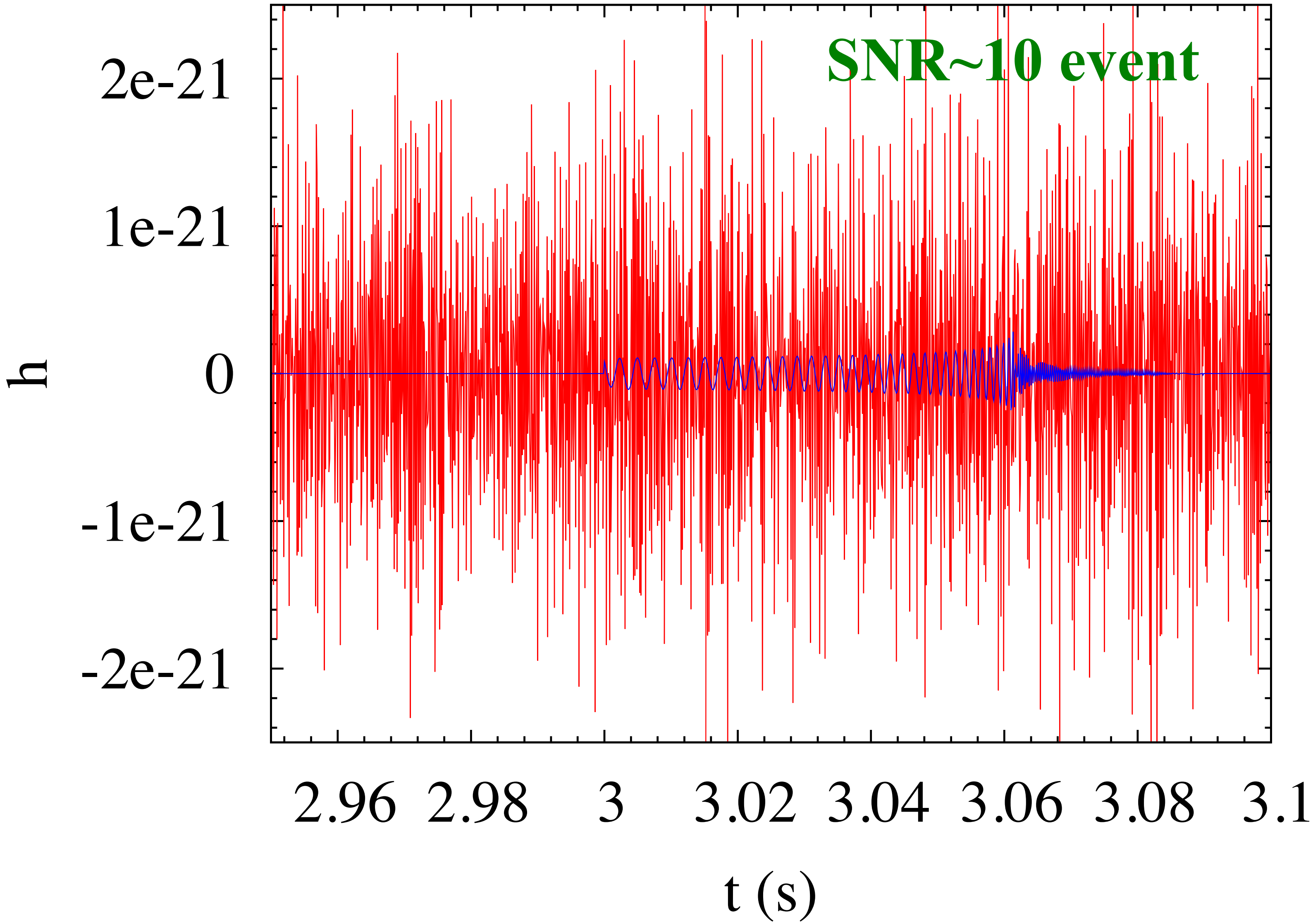
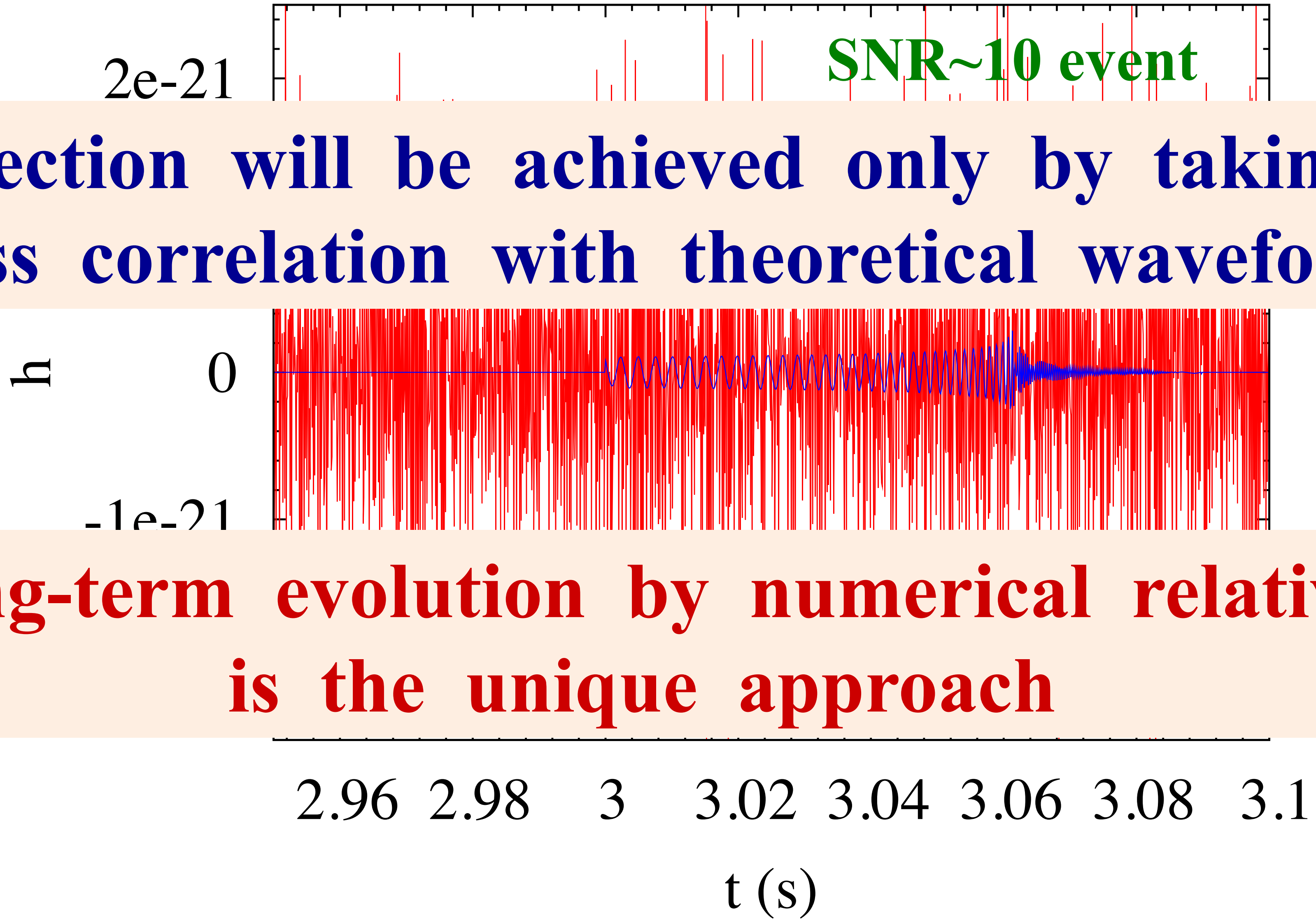


Detecting gravitational waves needs accurate theoretical prediction



Detection will be achieved only by taking cross correlation with theoretical waveforms

Long-term evolution by numerical relativity is the unique approach

III. Numerical relativity: Now

1) High-precession calculation for ***binary black hole*** inspiral, merger, and ringdown
2) Equation-of-state dependence of gravitational waves from ***binary neutron stars***
3) Mass ejection and nucleosynthesis of ***neutron-star binary merger***

1) BH-BH simulations

- Need to solve *vacuum* Einstein's equation
- We have two robust formulations now:

✧ **Modified harmonic gauge formulation**
+ apparent horizon excision (Pretorius 2005)

✧ **BSSN formulations**
+ moving puncture approach
(Shibata-Nakamura 1995, Baumgarte-Shapiro 1998, Campanelli et al. 2006, Baker et al. 2006)

- High-precision simulations are ongoing

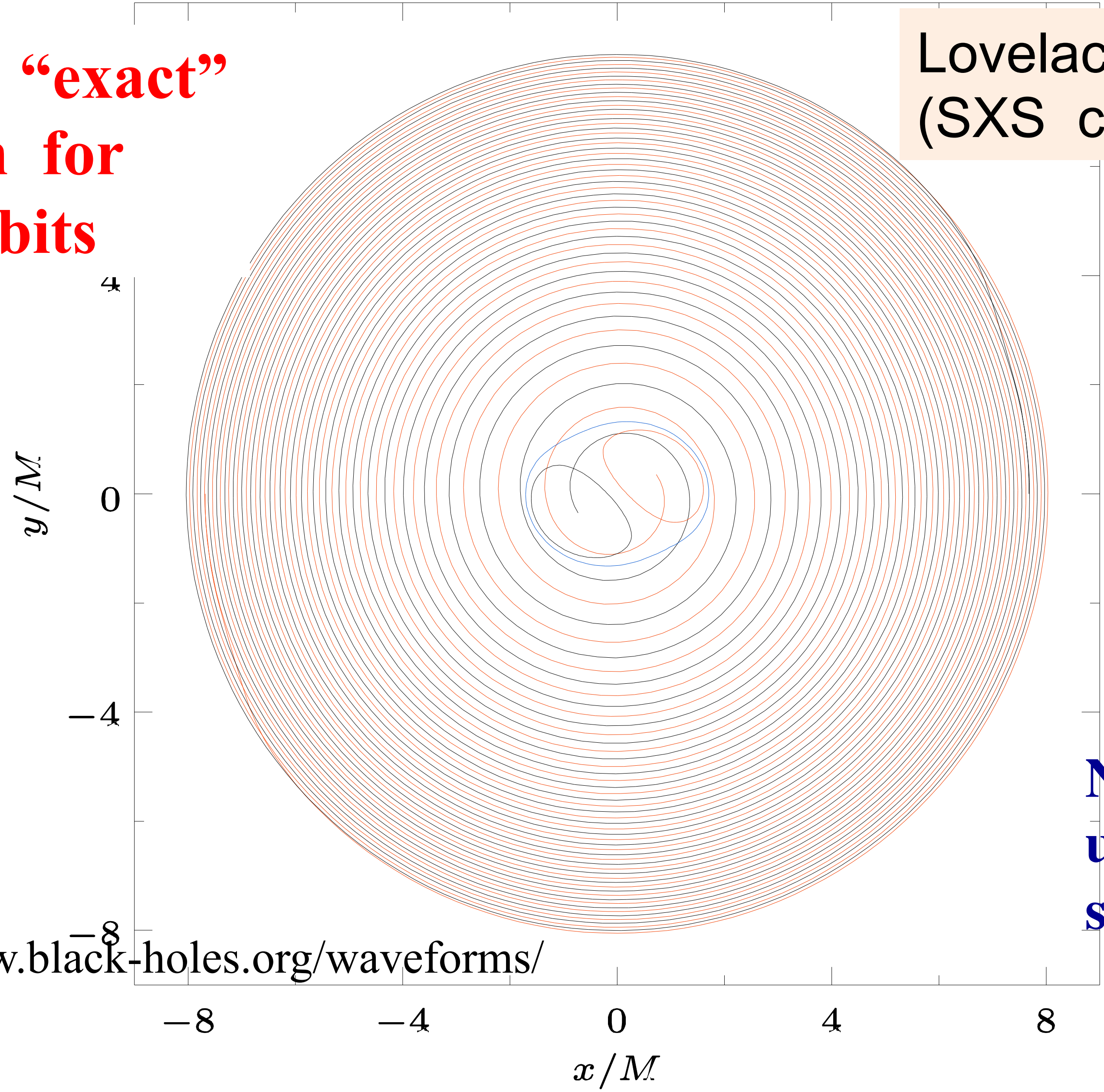
→ High-accuracy waveform

→ Semi-analytic modeling for GW templates

Orbits of BH-BH binary with spin 0.97

Almost "exact" solution for 25.5 orbits

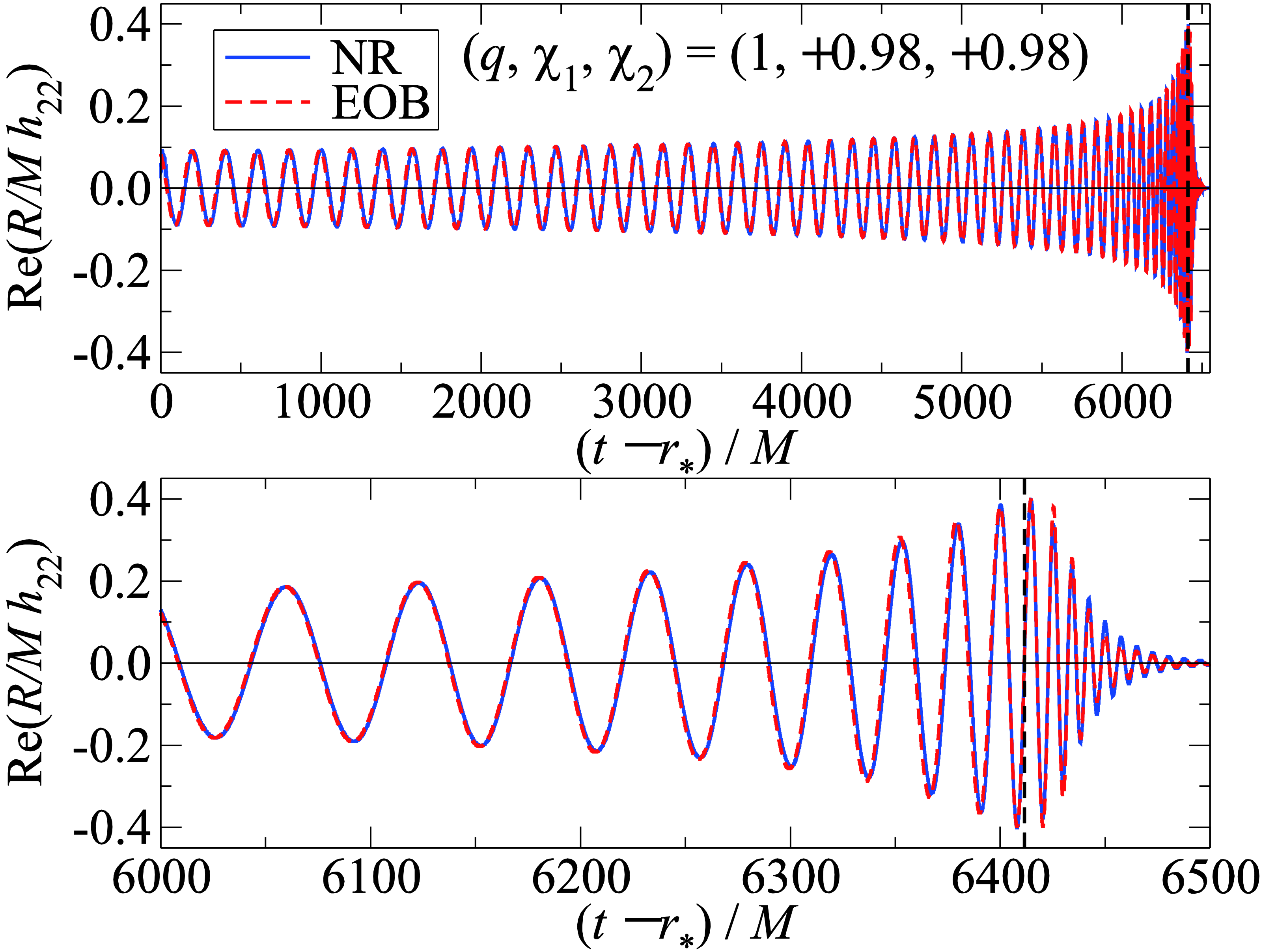
Lovelace+ 2012 (SXS collaboration)



Now feasible up to spin=0.99

https://www.black-holes.org/waveforms/

Modeling by Effective-one-body formalism

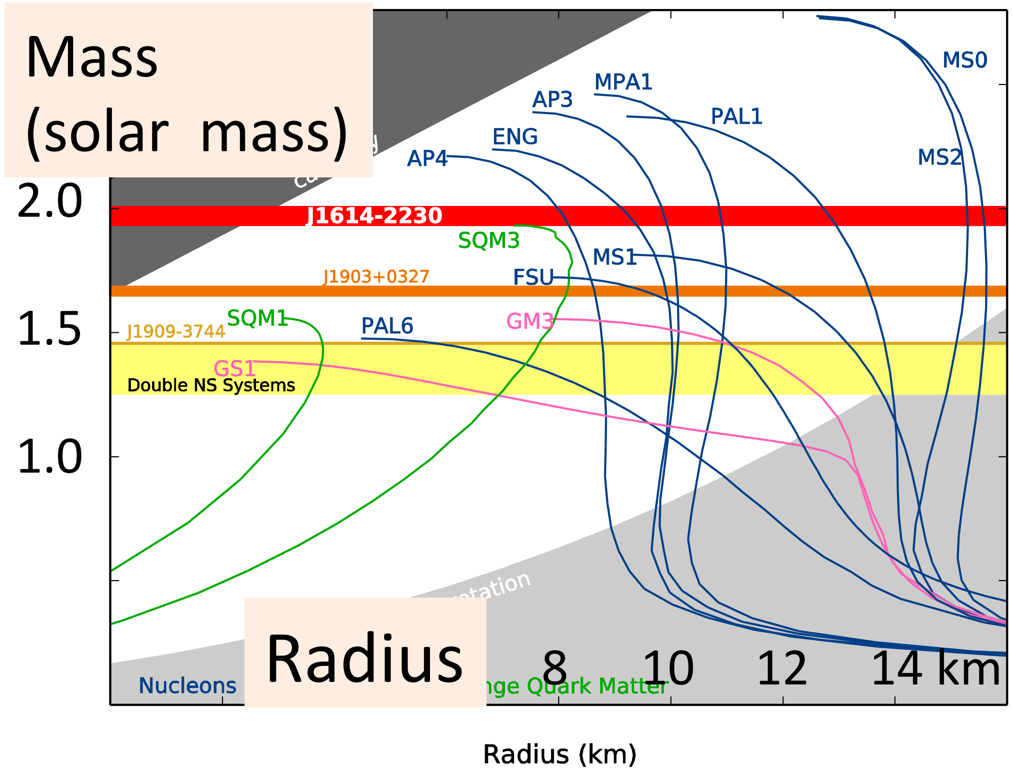


Taraccini et al. PRD**89** (2014)

2) Constraining nuclear-matter EOS

- The EOS for neutron-star matter is still poorly constrained

~ Strong interaction has not been well determined yet



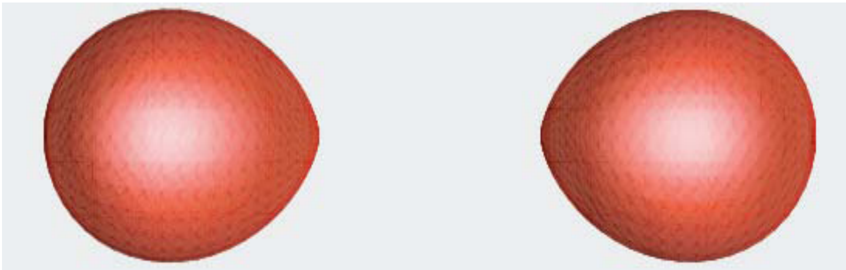
- Merger of neutron-star binary could provide a great opportunity for constraining it because gravitational waves will carry the information

Imprint of EOS on late inspiral waveform

In a binary system, the tides raised on each NS depend on the deformability of that NS:

Stiff EOS = lager radius = large deformability

Soft EOS = small radius = small deformability



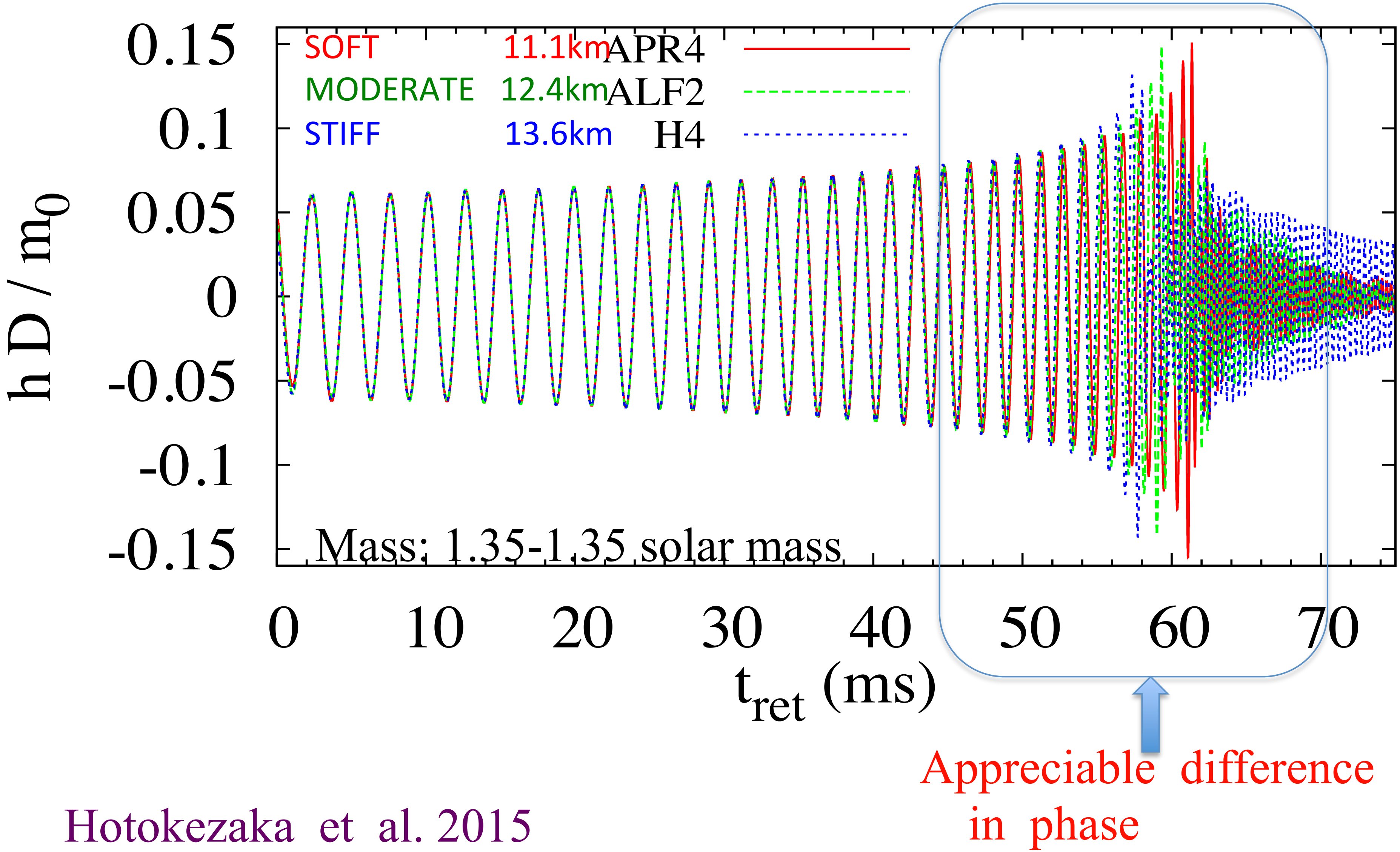
Courtesy J. Friedman

$$\phi \sim -\frac{GM}{r} - \frac{3I_{ij}^{TF} n^i n^j}{2r^3} : I_{ij}^{TF} = O\left(r^{-3}\right)$$

Lai et al. (1994)

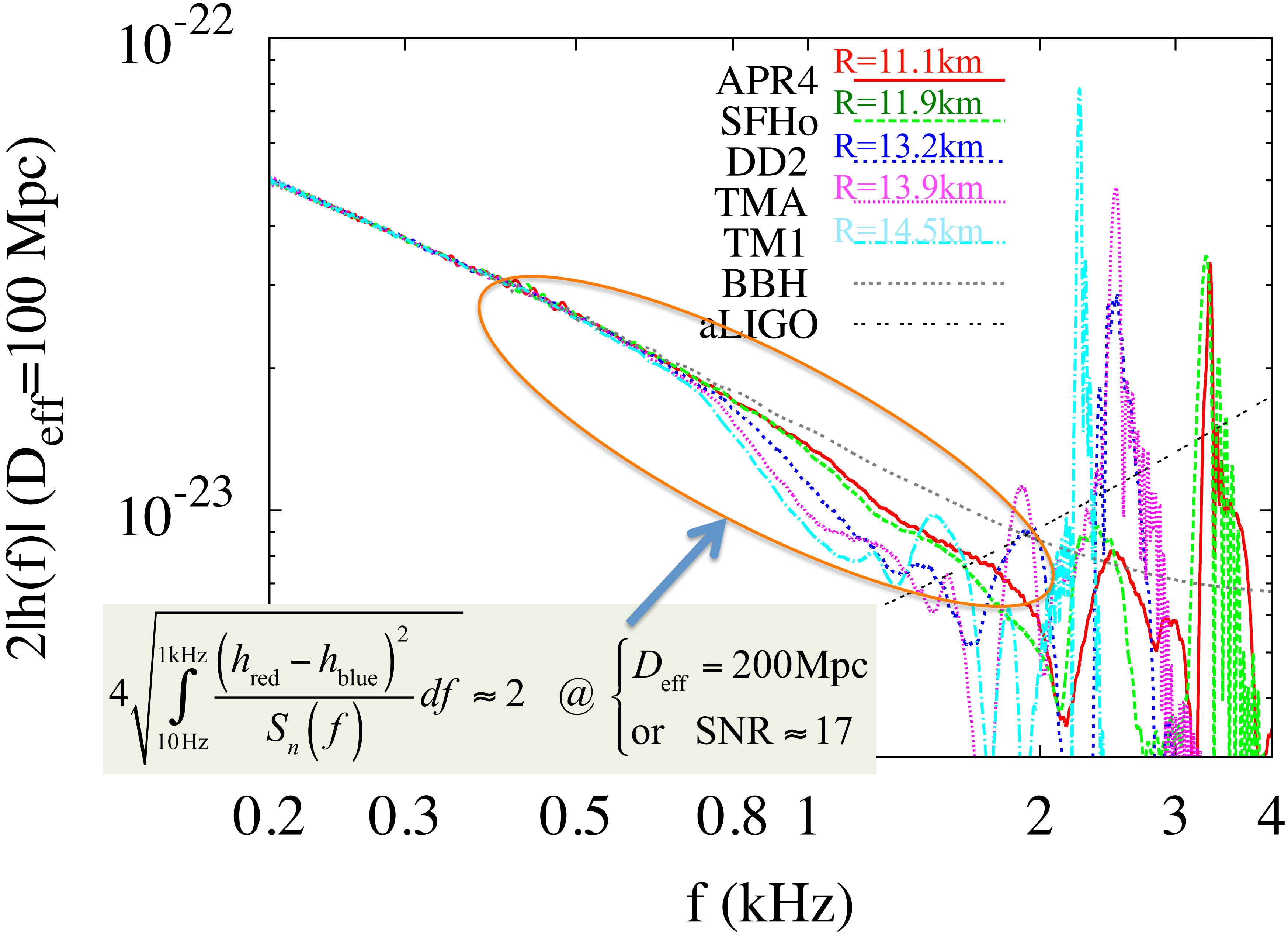
Latest Numerical Relativity Waveforms

Last several phases are different



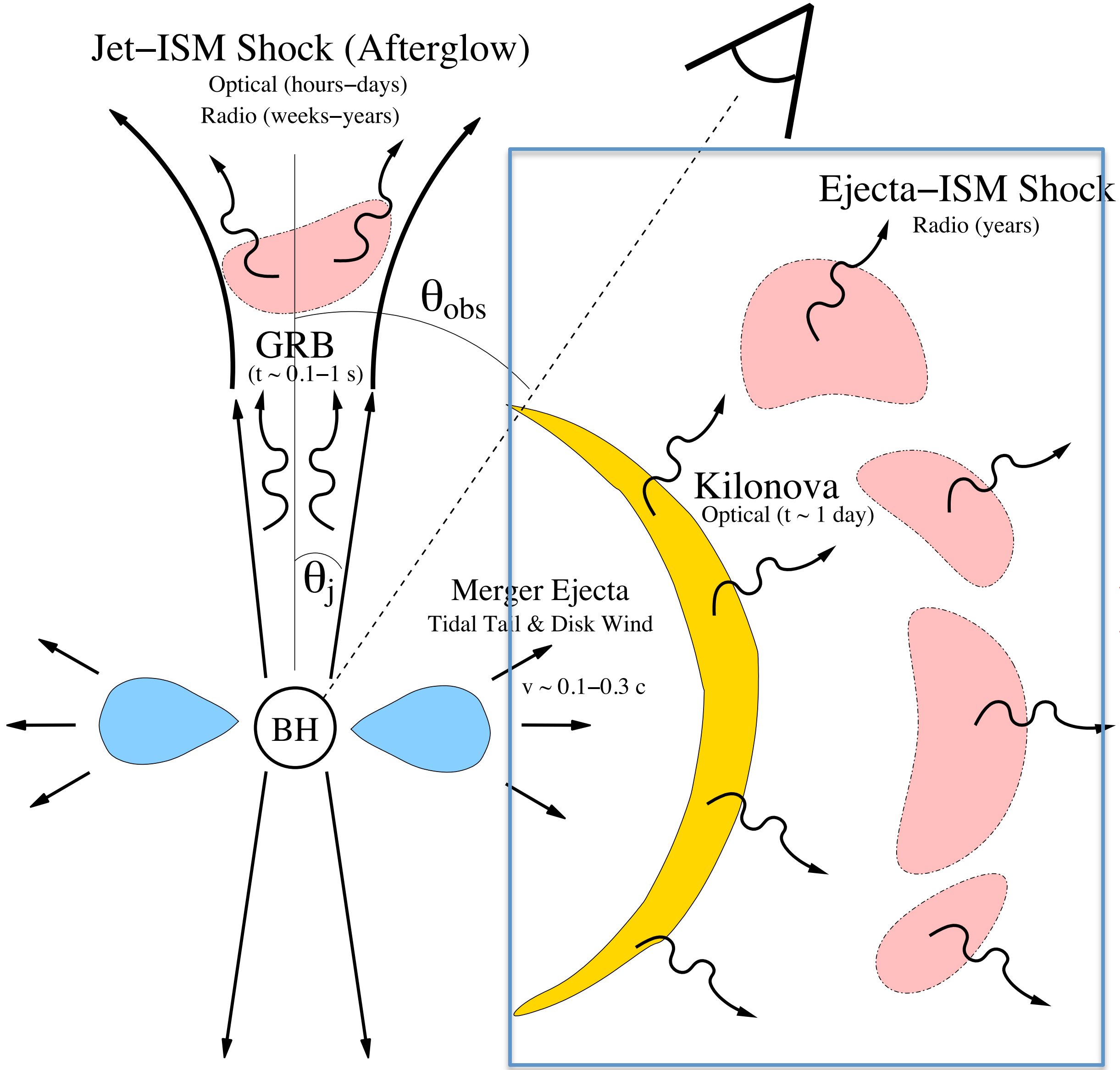
Hotokezaka et al. 2015

Overall spectrum by NR simulations



$$4\sqrt{\int_{10\text{Hz}}^{1\text{kHz}} \frac{\left(h_{\text{red}} - h_{\text{blue}}\right)^2}{S_n(f)} df} \approx 2 \quad @ \begin{cases} D_{\text{eff}} = 200\text{Mpc} \\ \text{or} \quad \text{SNR} \approx 17 \end{cases}$$

3) Mass ejection & EM counterpart



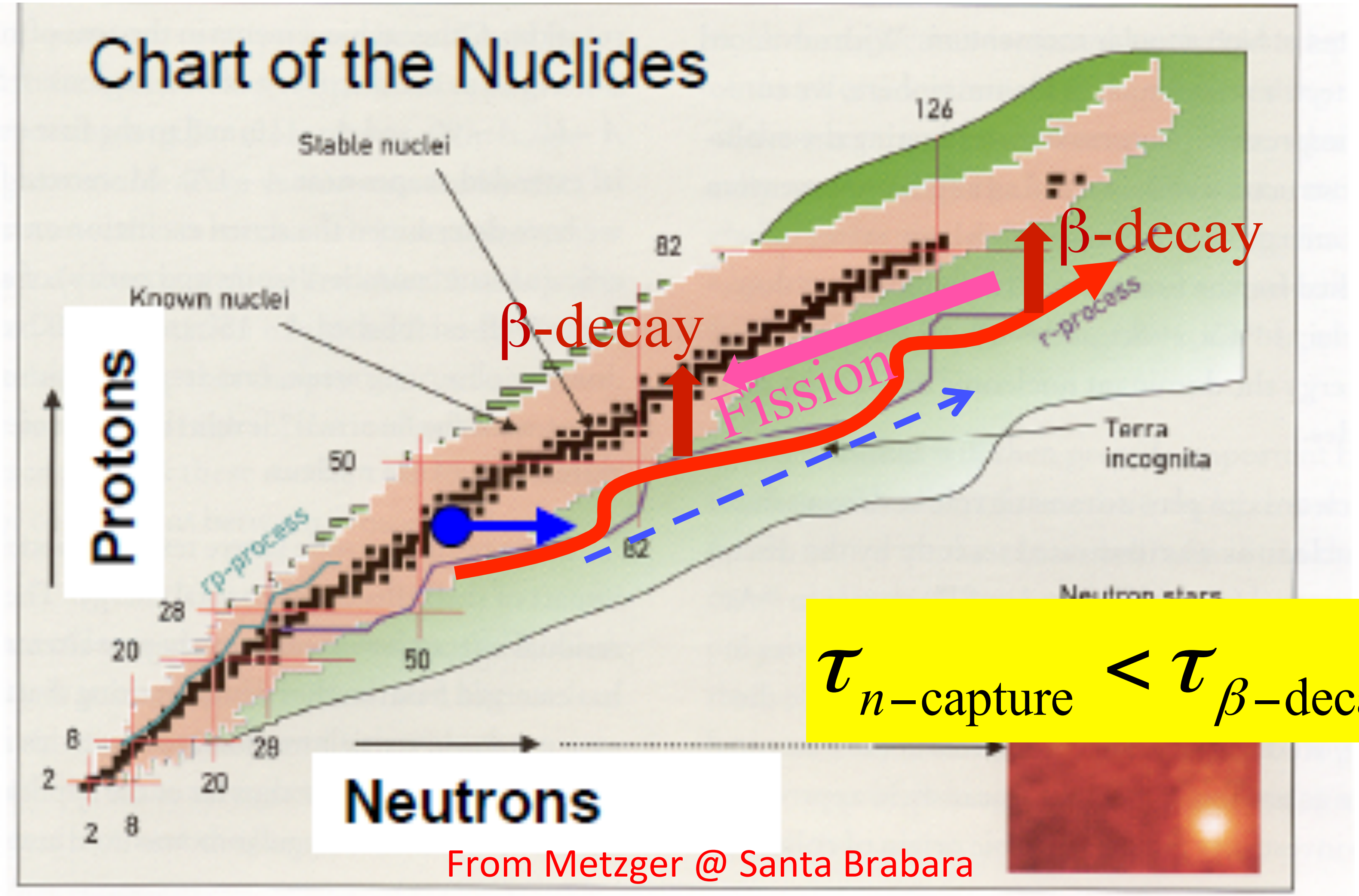
Metzger & Berger 2012

Need to quantify for near-future observation

Key of ejecta:

- Mass
- Velocity
- neutron richness
  → opacity

Mass ejection of neutron rich matter →
nucleosynthesis by rapid neutron capture →
β-decay/fission → heat up → UV ~ IR (Li-Paczynski '98)



$$\tau_{n-\mathrm{capture}} < \tau_{\beta-\mathrm{decay}}$$

Goal in Numerical Relativity

$$\nabla_\mu T^\mu_\nu = 0$$

Continuity equation

$$\nabla_\mu \left(\rho u^\mu \right) = 0$$

Composition evolution

$$\nabla_\mu \left(\rho u^\mu Y_l \right) = Q_l$$

Maxwell's eq

$$\nabla_\mu F^{\mu\nu} = -4\pi j^\nu$$

$$\nabla_{[\mu} F_{\nu\lambda]} = 0$$

Radiation transfer

$$p^\alpha \partial_\alpha f + \dot{p}^\alpha \frac{\partial f}{\partial p^\alpha} = S$$

+EOS

Neutron-star merger
Stellar collapse
= high-density,
high-temperature

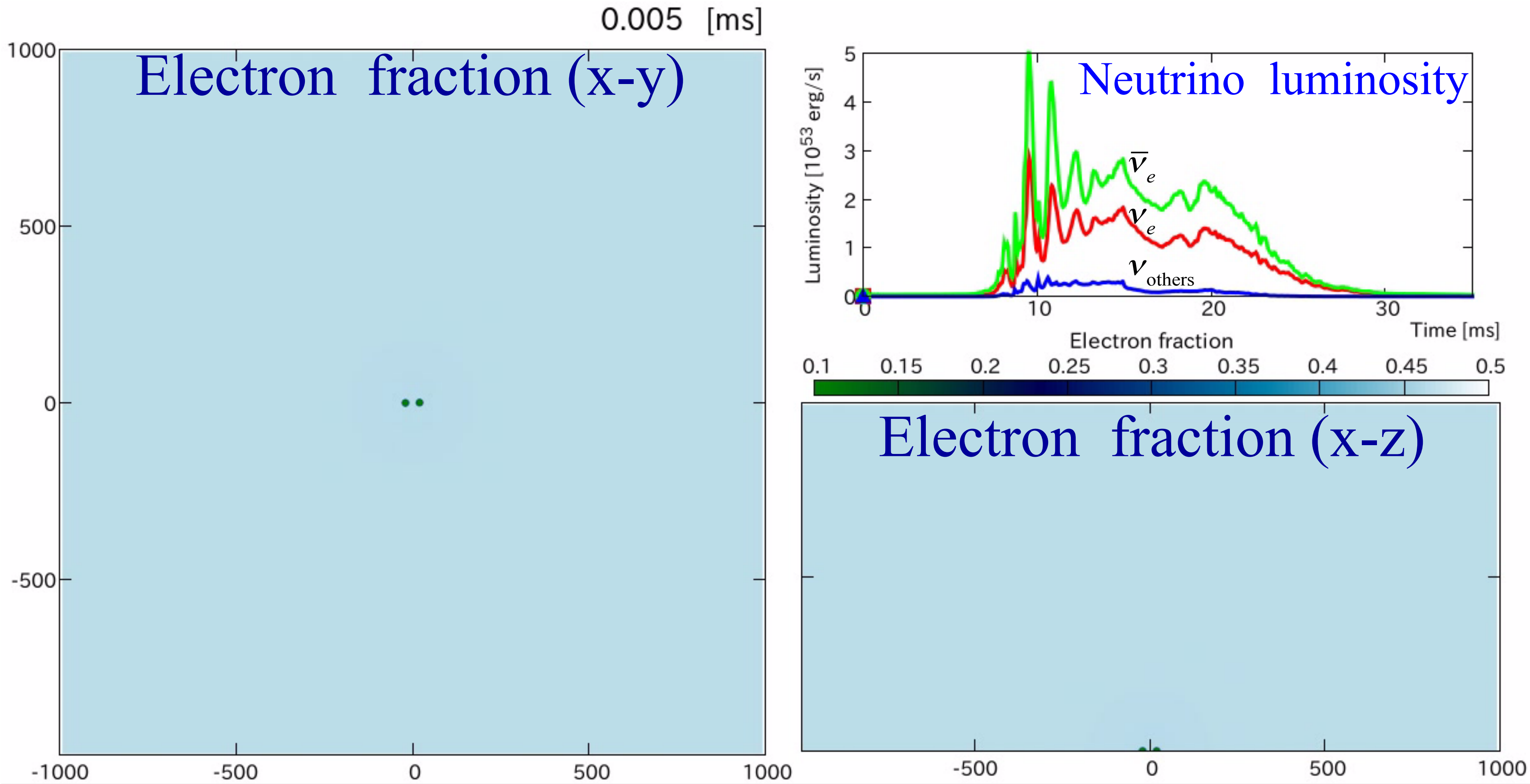
All 4 forces in nature often come into play and hence many eqs. have to be solved:

Ongoing challenge

High temperature $\Rightarrow \gamma\gamma \to e^- + e^+, \quad n + e^+ \to p + \bar{\nu}_e$

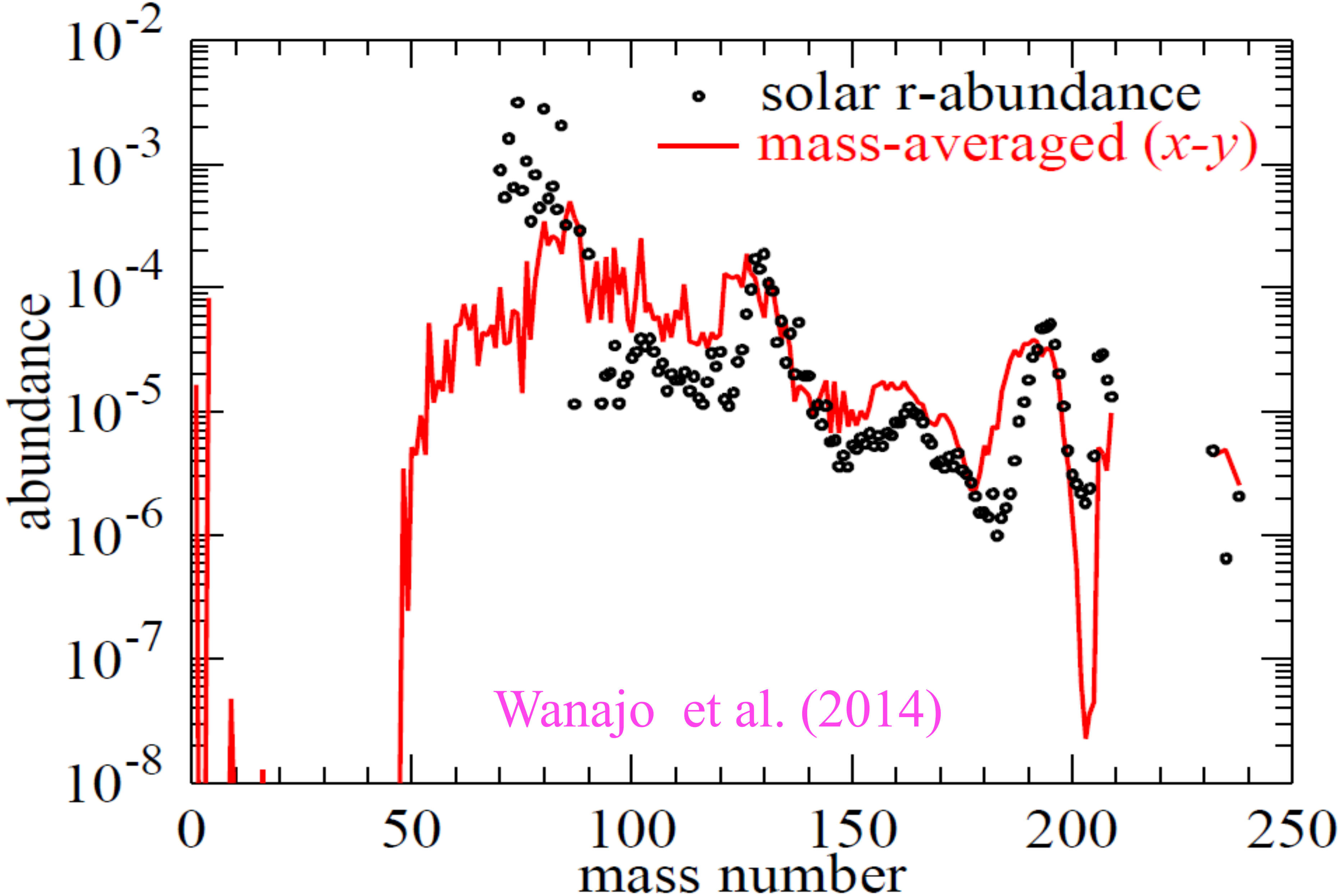
Neutrino emission $\Rightarrow n + \nu \to p + e^-$

Y_e



Sekiguchi et al. (2015)

Consistent with solar abundance pattern



Summary

- After long-term (~50 years) efforts, numerical relativity has become a mature field
- Many "observationally-motivated" simulations are ongoing → Templates of gravitational waves & prediction for EM counterparts
- **Numerical relativity will contribute to solving unsolved issues in GW physics, astronomy/astrophysics & nuclear physics in the next decade**

Estimate by Li-Paczynski (ApJ, 1998)

$$L_{\max} \sim 4\times10^{41}\ \text{ergs/s}\left(\frac{M}{0.01M_\odot}\right)^{1/2}\left(\frac{v}{0.2c}\right)^{1/2}\left(\frac{\kappa}{10\ \text{cm}^2/\text{g}}\right)^{-1/2}\left(\frac{f_{\text{r-proc}}}{3\times10^{-6}}\right)$$

$$\text{at}\ \ t\sim5\ \text{days}\left(\frac{M}{0.01M_\odot}\right)^{1/2}\left(\frac{v}{0.2c}\right)^{-1/2}\left(\frac{\kappa}{10\ \text{cm}^2/\text{g}}\right)^{1/2}$$

3×10^{41} ergs/s $\Leftrightarrow$ M $= -15.0$ mag $\Rightarrow$ m=21.5 mag @ 200Mpc

Observable by
$\sim$ 4--8m telescope

Cf. For sun, L=3.9×10^{33} ergs/s